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MATHEMATICAL MODELLING OF MONETARY MINTING IN MEDIAEVAL BULGARIA

Abstract. The article presents an approach to "measuring" the dynamics of monetary circulation in mediaeval Bulgaria, considering archaeological finds of both Bulgarian and foreign coins in our country. We construct appropriate functions, presenting the dynamics of monetary circulation during the 13th and the 14th centuries. We use a spreadsheet program such as Microsoft Excel to construct the functions and their graphs, which illustrate the latter one. As a data source we use a comprehensive historian's article on archaeological finds of coins in Bulgaria. There are a lot of coin finds, dated during the 13th and the 14th centuries. Therefore the historians consider that during the 13th and the 14th centuries the trade in Bulgaria was well developed. This is obvious, but using mathematical methods one can go further. We use a principle, which historians apply and which can be formulated as follows: more coin finds to a period, more coins in circulation were during this period. We believe our methods can help the historical research by presenting all the data about coin finds on a single graph. Thus it becomes possible to specify the periods (decades) of economic growth and decline. Further the investigators of our past can combine the visual picture obtained by our method with other information and reasoning.
Keywords: monetary circulation, Microsoft Excel, volume function

1. Introduction

The application of mathematical methods in historical research could enable historians to raise and test hypotheses on some dependencies or subjects, which couldn't be noticed and traced with the standard historical methods of research.

Work in this trend has been done for a couple of decades. We can mention here the attempts of mathematicians in introducing mathematical methods in historical research. A. Fomenko, S. Rachev and V. Kalashnikov [1, 2, 3, 4, 5] and J. Tabov [8] applied "volume functions" in exploration of historical texts. As far as we know, this approach has not been applied to the study of numismatic collections yet. Our work presents an attempt in this direction.

On the other hand, in the field of applying mathematical methods to the study of numismatic collections we could mention Cristian Gazdac [6] and his *Apulum project* where histograms are used to present distribution of Roman coin finds over several provinces. While he collected numerical values on the coin findings, he did not consider any specific study of their chronological distribution.

We decided to study the chronological distribution of coins from Mediaeval Bulgaria. For our research we used data from the article "Monetary finds from 13–14th centuries as a source on the foreign trade of mediaeval Bulgaria" by Zdravko

Plyakov [7]. Our goal was to create a model (function) of the chronological distribution of the finds of coins, used during a given period of the Middle ages in a given territory.

In order to reach it, we first entered the data into Excel worksheet. On the second step, we constructed the so-called volume functions and their graphs. Finally we made some conclusions and hypotheses presented in this paper.

2. Description of the method

2.1. Extracting and organising data from the basic source. We chose Plyakov's article [7], because it gives a comprehensive overview of almost all coin finds in Bulgaria in the studied period. It also includes conclusions on the economic status of our country in the past, namely that in 13th–14th century period the trade and the feudal manufacture were well developed in Bulgaria because of the many archaeological finds of mediaeval coins during and close to this period. This is the Plyakov's main and only conclusion. For the need of his research in [7] Plyakov sometimes didn't specify the number of found coins and the accuracy of their dating, which is needed for our purposes. Our study requires more precise information. We need both *dating* and *number of coins* to construct our functions. Because of that reason, we extracted from [7] data only on the finds, where dating and quantity were both appropriately specified. Fortunately, most data in [7] satisfy our requirements; the incomplete data are in the minority and are randomly spread over different periods. Since we wish to obtain a rough assessment of the monetary circulation, it is not a problem to omit these data; it does not change significantly the final results.

The data on the coins in the article [7] are grouped in a table by the place of the respective finds, for example: "Yambol – (year1936) – Joan V[1] 7 coins, Manuel II 3 coins, coins from Ivan Alexander[2] and Turkish coins from 15th century", or: "Lukovit – (no year of excavation is given in the respective column) – Alexius III[3] 2160 coins, 16 Bulgarian and 31 Latin coins, all from the12th century". We regrouped the coins not by the place, but by the ruler, under whom the coins had been struck. We considered both Bulgarian and foreign coins (e.g. Byzantine, Venetian, Serbian, Turkish etc.), because they both were in circulation in mediaeval Bulgaria. On some foreign coins there are stamps of Bulgarian rulers, verifying that coins' everyday use in Bulgaria was allowed. These coins were supposedly stamped in the mints in the capitals Vidin, Turnovo or those of the principality of Dobrotitza. Furthermore the presence of foreign currencies is indication for foreign trade, which is both result and premise for developed economy. The changes in the economic status of mediaeval Bulgaria is the object we explore via the changes of the monetary circulation. Hence we are interested in all the coins.

2.2. Function of Monetary Volume (FMV). For every coin we define Individual Unit-step Function (IUF) as follows:

$$f(t)=\begin{cases}60/n, if\ t\in[X_1;X_2]\\ 0, if\ t\notin[X_1;X_2]\end{cases}$$

[1] Joan V (1341-1391) was a Byzantine Emperor.
[2] Ivan Alexander (1330-1371) was a Bulgarian Tzar (King).
[3] Alexius III (1195-1203) was a Byzantine Emperor.

where:

$[X_1; X_2]$ is the period of governing of the ruler, who emitted this coin;

t is not a particular year but a decade;

n is the number of decades in the period $[X_1; X_2]$.

We consider decades as smallest basic time units. Thus n is the "length" of the interval $[X_1; X_2]$. That's why we round periods like 1331–1343 using in this case 1330–1340. Thus each coin has a total contribution of 60 to all decades in which it was in use, i.e. all coins have equivalent contribution. The number 60 is devisible by 2, 3, 4, 5, and 6. This is convenient to be applied to periods of length 2, 3, 4, 5, and 6 decades respectively in order to obtain integer values of f.

Given a ruler N. N emits k coins with IUF= $f_1(t)$ each. Define R(N)= $k.f_1(t)$. R(N) is the total contribution of the coins of N for the decade t. It's $\frac{60k}{n}$ for a decade in the interval of government and zero in the other case. Thus R(N) is similar to $f_1(t)$.

Summing up R(Z) for all the rulers from 13th–14th century period, we obtain the value of the total contribution of all the coins from 13th–14th centuries, as needed. We call the sum of all R(Z) *Function of Monetary Volume (FMV)*. The so obtained function is a mathematical model of monetary circulation.

2.3. Table creation in Excel. We first find the number of coins emitted by every ruler. Then we create in Excel a table with 40 columns, the first of which corresponds to 1100–1110 decade, second – to 1110–1120 decade and so on. The explored period is 13th–14th centuries, but we consider the comprehending period 1100–1500. The latter gives us opportunities to explore better some tendencies of monetary circulation at the endpoints of the period of interest. The number of rows equals the number of rulers we consider. For every ruler is allocated a row in the table. We enter in the cells of the row the values of R(N) for the respective decades. Summing up the column elements we obtain FMV. Excel performs the last operation automatically. Thus the task is done in an easy way. Consider that the archaeologists find a new quantity of coins, struck under the Byzantine Emperor Theodore I (1204–1222), for example. We already have this emperor in our table and just add the number of these new coins to the old number of coins from him. It is another task we do easily with Excel.

2.4. Graph constructing of FMV. When the table is finished, the last row contains the values of FMV. We insert a new row above it, where we enter in the cells the decades between 1100–1500. Then we construct automatically graphs based on the values in the table. Here (Fig. 1) is a graph, which we obtained:

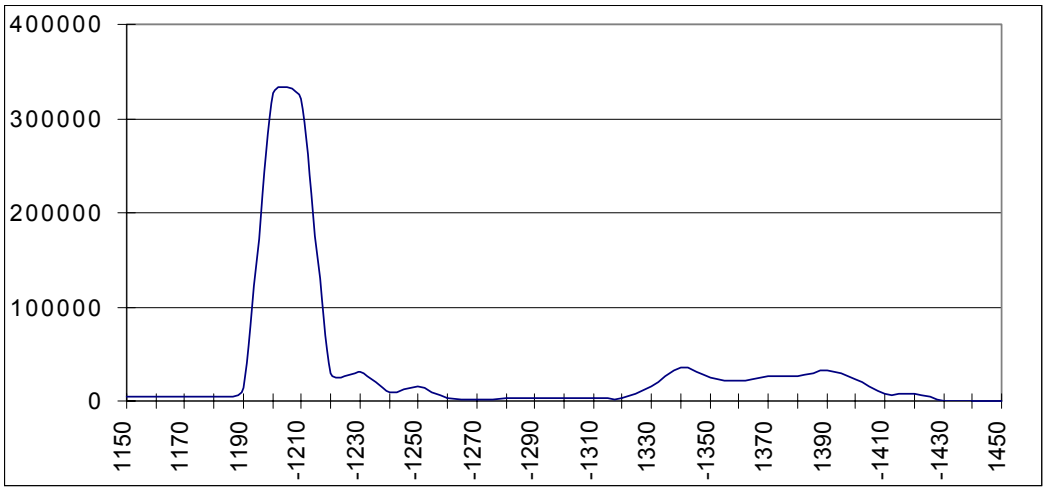

Fig. 1. The graph of FMV in the interval (1100-1500)

3. Analysis and hypotheses

Let us recall that our source of data is the article [7]. The author's main and only conclusion there is that in 13^{th}–14^{th} century period the trade and the feudal manufacture were well developed in Bulgaria because of the many archaeological finds of mediaeval coins during and around this period.

Our methods give opportunities to explore the dynamics of monetary circulation. It is obvious from Fig. 1 that in the period 1190–1220 the FMV has a global maximum, equal to 327420. There are two local maxima at the decades 1330–1340 and 1380–1390 as well. The first peak of FMV is never reached again. This function value is about eleven times higher than the rest of the maxima. To explore better the period, including the two other mentioned maxima, we constructed the graph of FMV in the interval 1250–1500 (see Fig. 2).

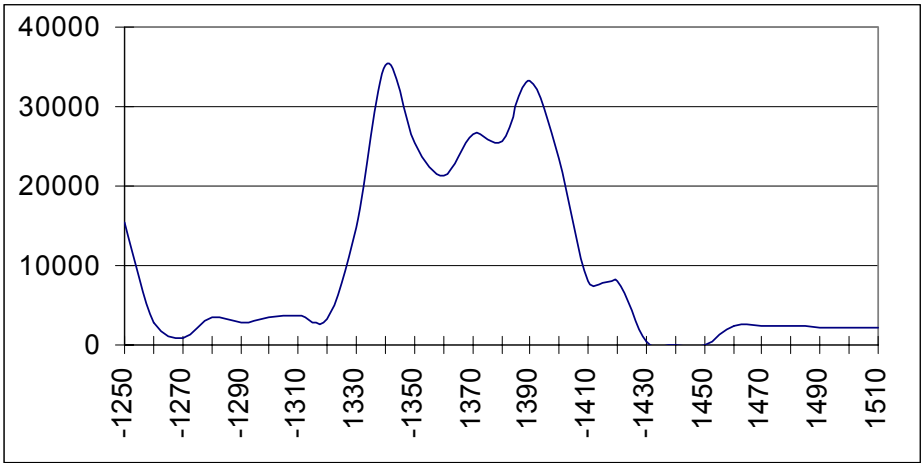

Fig. 2. The graph of FMV in the interval (1250-1500)

It is important to note, that the data in [7], which we use in our investigations, are limited in the interval 1200–1400, and therefore the picture before 1180 and after 1420

does not reflect the real situation). The vertical scale is more appropriate in Fig. 2. The decrease after 1350, between the two maxima, is distinct and sudden there. It contrasts with the tendencies in the period 1300–1400. There is a serious reason for this crisis. In 1348 began a bubonic plague epidemic, which little by little spread over all South Europe. The sickness and death of numerous people caused falling in the manufactory and the trade. This unexpected fall of the function values is not so distinct on the graph in Fig. 1.

The values of FMV in the interval 1260–1290 are low. During this period there was Tartar domination in Eastern Europe, including Bulgarian lands, attended with robberies, conflagrations, insecurity and panic among the people, risk for the traders etc. The respective values after 1380 are low too. This could be explained with the Turkish invasion and conquest of Bulgaria.

Let us finally revert to the global maximum. There are some possible interpretations.

- Some coins are not identified (attributed to a specific ruler or a period of time) by different reasons. They could change the graph at a certain extent, if they become recognized and attributed to the period 1200–1400.
- Some of the coins, dated this period, belong in fact to another period.

The results, described by the graph in Fig. 1, pose two problems:

1. Why the number of the coin finds from the last decade of the 12-th and the first two decades of the 13-th centuries is so large (compared to that of the other periods)?
2. Why the number of the coin finds from the interval 1220–1250 is so small? Should we expect that in fact among the coin finds in Bulgaria there are coins struck in the interval 1220–1250, but they are among the non-recognized coins?

In conclusion, it is important to recall, that the above method gives only a rough, approximate picture of the real situation. Although the graph reflects correctly the appropriate archaeological data from the initial paper [7], the connection between the number of the coin finds from a given period of the past and the coins in the real circulation during the same period is based on the laws of the probability theory and therefore is limited by these laws. Hence our conclusions are not proved, they are just rather probable.

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