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DIGITIZED WORKS OF PROFESSOR DRAGOMIR SIMEUNOVIĆ

Abstract. In this paper we present digitized works of Dragomir Simeunović (1930-2021), a former professor of mathematics at Belgrade University. We consider that his works should be interesting to the wider audience for two reasons. The first one is that he initially finished astronomy and was one of the last students of the famous Serbian astronomer and applied mathematician Milutin Milanković. Secondly, he also graduated in mathematics and after that his scientific work was completely devoted to geometry of polynomials, a discipline founded in Serbia by Mihailo Petrović Alas, the father of modern mathematics in Serbia. Digital copies of Simeunović's works are deposited in the Virtual Library of the Faculty of mathematics, elibrary.matf.bg.ac.rs.

Keywords. Dragomir Simeunović, digitized works, geometry of polynomials.

1. Introduction

This paper belongs to a series of articles related to the digitization of scientific works of prominent Serbian scientists from the past who were working in mathematical sciences. Some recently published papers were on Jovan Karamata, [1], Dimitrije Danić, [2], Bogdan Gavrilović, [3], Vuk Marinković, [4], Mihailo Petrović Alas, [5], Slaviša Prešić, [6] and Emilijan Josimović, [7]. For a general overview of the project and general activities in digitization of the scientific heritage in Serbia, one may consult [10], [11], [12] and [13], while for the technical details one can see [8] and [9]. Digitized works are deposited in the Virtual Library of the Faculty of Mathematics in Belgrade at the address <http://elibrary.matf.bg.ac.rs>.

For this occasion we decided to present scientific works of Dragomir Simeunović, a prominent professor of mathematics of the Belgrade University. We believe that his works could be of an interest to the wider audience for two reasons. He first finished astronomy and was one of the last students of the famous Serbian astronomer and applied mathematician Milutin Milanković. Secondly, he also graduated in mathematics and after that his scientific work was completely in the field of geometry of polynomials, a discipline founded in Serbia by Mihailo Petrović Alas, the founder of the Belgrade mathematical school. Interestingly, he was one of the last Serbian mathematicians working in this field.

2. Short biography of Dragomir Simeunović

Dragomir Simeunović (Figure 1) was born on February 21, 1931 in Prilike near Ivanjica to father Milan and mother Novka. He finished the four-year primary school in Prilike in 1942 as the best pupil, excelling especially in mathematics. Then he received a nickname from his friends after the famous ancient Greek mathematician Pythagoras. He finished the elementary school in 1946 in Ivanjica, and gymnasium in 1950 in Belgrade.



Figure 1. Dragomir Simeunović

He completed his studies in astronomy at the Faculty of Natural Sciences and Mathematics (PMF) in Belgrade in, 1955, as the last graduate of Professor Milutin Milanković. Under Milanković's guidance he wrote his graduating thesis *The force of perturbation and its field with application in sea tide theory*. In 1960, he graduated in mathematics at the same faculty. He defended his master's thesis in mathematics *Localization of polynomial's zeros* at the Faculty of Natural Science in 1967. At the same faculty in 1969 he defended his doctoral dissertation *On the boundaries of the roots of algebraic equations and some of their applications* under the mentorship of Professor Đuro Kurepa.

Following graduation in astronomy, he served his military service. After that he began working in the State Department of Occupational Medicine of the Hygiene Institute, where he remained until the transfer for an assistant to the Faculty of Mining and Geology in Belgrade (RGF) in February 1962. At the same faculty, he was elected assistant professor in 1970, associate professor in 1977 and full professor in 1982. He stayed at RGF until retirement, where he taught mathematics at undergraduate and postgraduate studies as well. During his work at RGF, he was the head of the Chair of Applied Mathematics for almost 20 years. He also taught at higher schools and several faculties in Belgrade: the Faculty of Economics, the Faculty of Veterinary Medicine and the Military Academy, but also outside Belgrade, in Bor, Priština and Tuzla. He has written a dozen books: textbooks, scripts, and collections of mathematical problems. He has published the textbooks *Mathematics for Postgraduate Studies* (published by RGF) and *Higher Mathematics for High School Students*, and is the coauthor of the textbooks *Mathematics II - Tasks and Fundamentals of Theory* for students of the Faculty of Economics.

Professor Simeunović was very active in his academic work at the University of Belgrade. He was a member of the defense commissions for 24 doctoral dissertations in mining, geology, mathematics and astronomy. He was also a member of the commissions for the defense of a number of master's and graduating theses. He was a member of GAMM (Gesellschaft für angewandte mathematik und mechanik - Society

for Applied Mathematics and Mechanics) for more than twenty years. During his studies of astronomy, Simeunović met his colleagues Jovan Simovljević and Jovan Lazović, who later became professors at the Department of Astronomy at the Faculty of Mathematics and Science. Then a great friendship was formed between the three of them which lasted throughout their lives. Due to this friendship Simeunović often visited seminars at the Department, attended also by other members of the Department, where interesting and professional discussions in astronomy, mathematics and mechanics were held.

Professor Dragomir Simeunović passed away in Belgrade at the age of 90, on June 2, 2020.

3. Scientific work

Professor Simeunović's work in mathematics is mostly related to the geometry of polynomials - the distribution of roots in a complex plane of polynomials with complex coefficients. This area is close to complex analysis, especially the theory of the entire functions, which are in way generalizations of complex polynomials and share many properties with them. After Gauss's¹ proof of the basic theorem of algebra that every polynomial of degree greater than or equal to one has at least one root in a complex plane, the key step in the further development of algebra were Abel's² and Galois's³ proofs that roots of polynomials of degree five or higher cannot be represented by radicals in general. In connection with this, a new area appears between algebra and complex analysis, which deals with domains in a complex plane in which all zeros of a given polynomial are located, or a part of the roots with prescribed properties. The results of this new mathematical discipline, now known as geometry of polynomials, can often be successfully applied to entire functions, i.e. complex functions that can be represented by a Taylor series with an infinite radius of convergence. Mihailo Petrović Alas was the first Serbian mathematician who dealt with the geometry of polynomials and made important contributions to this field. Many of his students also worked in this area, e.g. Jovan Karamata, Miodrag Tomić, Šefkija Rajević and Dragoljub Marković. Results of these mathematicians are quoted in Marden's⁴ famous monograph [14]. Professor Simeunović published 24 papers in this field in leading Serbian journals (Glas SANU, Matematički Vesnik and Publications de l'Institute Mathématique) as well as in reputable foreign journals (ZAM - *Z. angew. Math. Mech.*, *Mathematica Moravica* and *Revue d' numerical analysis and the theory of approximation*). In his works, he dealt with the localization of zeros of real and complex polynomials, the estimation of the modulus of the roots of polynomials and studied iterative procedures for determining the approximate numerical values of the roots. He continued his scientific work primarily of Professor Dragoljub Markovic, but also of several foreign mathematicians, e.g. the famous Greek mathematician Panagiotis Zervos (1878-1952). In his scientific work, he showed that he has an excellent mathematical technique and erudition. It should be mentioned that Serbian mathematicians later turned to the other areas of mathematics, so, in a sense Simeunović was the last representative of the old Serbian school of mathematics founded by Mihailo Petrović Alas.

¹Carl Friedrich Gauss (1777-1855), famous German mathematician, astronomer and physicist .

²Niels Henrik Abel (1802-1829), Norwegian mathematician.

³Évariste Galois (1811-1821), French mathematician.

⁴Morris Marden (1905-1991), American mathematician who introduced the term "Geometry of polynomials" for this field of mathematics.

Simeunović also dealt with numerical methods for determining the approximate values roots of polynomials and the properties of Riccati's differential equation. He published scientific papers mostly as the only author until 2014. He also has a large number of papers on the application of mathematics, especially in mining and in the exploitation of ore deposits.

He also wrote a large article about Radivoje Kašanin (1892 - 1989), a distinguished professor at the Faculty of civil engineering of the University of Belgrade and one of the first members of the Serbian Mathematical School. The manuscript was published in the book *Life and work of Serbian scientists* published by Serbian Academy of Sciences and Art.

Professor Simeunović's scientific work had a response in the foreign scientific public. For example, it is quoted by several mathematicians. Only in the last few years, a full 25 years after his retirement, he has been quoted in reputable journals: M. Gil (*Mathematica Moravica*, 2019), F. Dubeau, C. Gnanang (*Journal of Mathematical Analysis*, 2020), F. Dubeau (*Numerical Functional Analysis and Optimization*, 2021) and even in an Indonesian journal (E. Susilawati, S. Putra, Z. Zulkarnain, 2014).

4. Textbooks

Professor Simeunović wrote a dozen books, mostly textbooks, collections of assignments and scripts, for teaching purposes at the Faculty of Mining and Geology, the Faculty of Economics, the Faculty of Veterinary Medicine and the Military Academy in Belgrade, as well as the colleges where he taught. In the penultimate publication *Selected Papers*, Simeunović collected 34 scientific papers, which he published in mathematical journals. *Selected Papers* were published it in 2016. In all of these works, except for one, he is the only author.

The last book, *Radovi* (Works), contains all his works, including those that were published in the previous publication. There is a total of 70 scientific and professional papers in this book. In addition to 34 scientific papers published in mathematical journals, it also contains 36 scientific and professional papers published in various journals and proceedings of conferences, symposia and congresses.

5. Graduate thesis

Simeunović's diploma thesis entitled *The force of perturbation and its field with application in sea tide theory* is of a hystorical importance, considering that it was the last diploma thesis done under the mentorship of the great scientist Milutin Milanković. Simeunović defended his diploma thesis in astronomy on June 28, 1955, before a commission: Professor Milutin Milanković, mentor and professor Vojislav Mišković.

Let us remind that Milanković was one of the first theoretical mechanics who introduced vector analysis into celestial mechanics and with that mathematical apparatus created a vector-scalar system of elements of planetary orbits. Therefore, this system is also called Milanković's system of elements. After the introduction of Milanković's path elements, Anton Bilimović introduces partial gradients of the perturbation function into the classical theory of perturbation. This systematic treatment of the basic tasks of celestial mechanics has created a recognizable school at the University of Belgrade. Simeunović's graduation thesis, written in the 1950s, is an important and beautiful example of the newly established Belgrade School of Celestial Mechanics.

The complexity of the problem dealt with in the graduate thesis can be seen from the theoretical derivation of the equations used for computing of the height of the sea tide

caused by the Moon's and the Sun's attractive action. In this paper, all the apparent and right motions of these celestial bodies are taken into account. For example, due to the Earth's rotation, there is an apparent daily lunar and apparent daily solar movement, which on the other hand causes half-day and day tides.

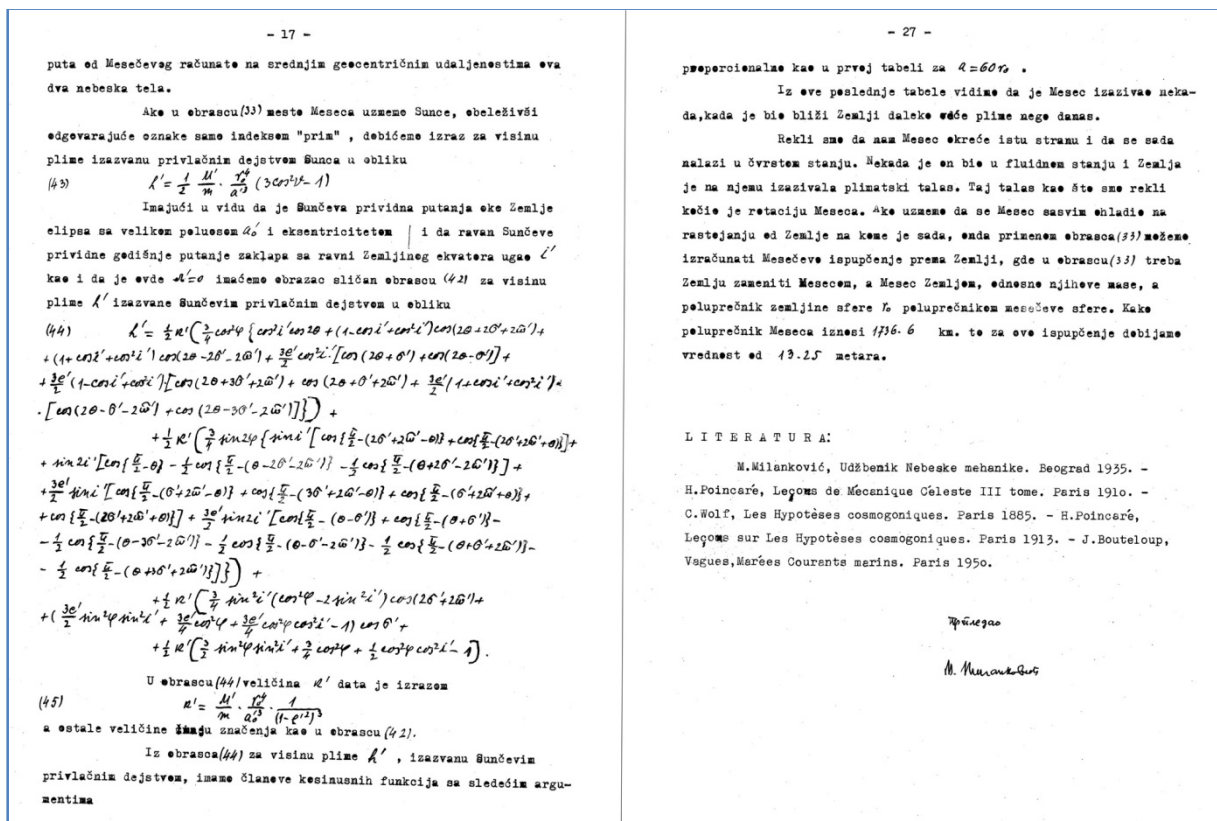


Figure 2. Pages 17 and 27 from Simeunović's graduate thesis
The page 27 contains the Milanković's signature

Furthermore, the true motion of the Moon around the Earth and the formation of crescent and lunar tides, the apparent annual motion of the Sun due to the Earth's revolution lead to semi-annual and annual tides, while the movement of the Moon's apse line or major axis creates direct tides with a period of 8.85 years. There are other phenomena that celestial mechanics take into account when calculating the tide. For example, the movement of the Moon's node line in the retrograde direction leads to a tide with a period of 18.66 years, as well as the movement of the Earth's apse and the solstice line (the line connecting the spring and autumn equinoxes). Changes in the tilt of the Moon's orbit, the tilt of the Earth's orbit, and changes in the eccentricity of these orbits are also included in Simeunović's considerations. He also considered the change of tide with the change of latitude on the Earth, taking into account some additional factors. Under the guidance of his mentor Milutin Milanković, Simeunović presented successfully such a complex phenomenon of the sea tide with the use of a non-trivial mathematical apparatus. Milanković possessed an extraordinary ability to express every natural phenomenon in the language of mathematics, which he successfully passed on to his student on this occasion.

We would like to mention the word "sicigia" (in latin "syzygia", or English "syzygy"), which is mentioned in the diploma thesis (p. 22) and which is almost forgotten in Serbian professional astronomy. Syzygy stands for the mutual positions of

the Moon and the Sun in conjunction (new moon) and in opposition (“uštap” or full moon).

However, even if that was an excellent diploma thesis, Simeunović did not continue work in astronomy. Professor Milanković shortly after that retired, and Simeunović switched to mathematics. He received his bachelor's, master's and doctoral degrees in mathematics and devoted his entire working life to mathematics. However, near the end of his life, Simeunović returned to astronomy, and began writing a manuscript on the theory of gravitation. Unfortunately, this work failed to complete.

6. Conclusion

Dragomir Simeunović gave important contributions to Geometry of polynomials, a discipline of mathematics dealing with distribution of roots of complex polynomials, by which the Serbian mathematical school was recognized in the world. He also was a respectable professor of the Belgrade university, teaching mathematics and related sciences at the highest educational institutions in Belgrade in the second half of XX century. All his published works are digitized and they are now a part of his digital legacy in the Virtual library of the Mathematical Faculty in Belgrade. We believe that this material in the Library will serve to better understand Simeunović's life and work, but the works of Serbian mathematicians in the second half of the XX century as well.

Works by Dragomir Simeunović

The collected works of Professor Simeunović can be found in the Virtual Library (VB) at <http://elibrary.matf.bg.ac.rs>, in: Communities & Collections> Mathematical Sciences> Collected Works, that is, at the address <http://elibrary.matf.bg.ac.rs/handle/123456789/4569>.

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