

Nadežda Pejović, Viktor Radović

Faculty of Mathematics, University of Belgrade, Serbia

PROFESSOR JOVAN LAZOVIĆ AND HIS DIGITIZED UNIVERSITY TEXTBOOK

Abstract. In this article, we present an analysis of digitized university textbook "Fundamentals of motion theory of artificial Earth satellites" written by professor Jovan Lazović. This textbook was the first one where the motion theory of artificial satellites was shown using clear mathematical and astronomical language. The whole textbook is written precisely and systematically. In order to understand this subject, the previous knowledge in advanced mathematics, astronomy and celestial mechanics are required. The digitized version of this book is publicly available in Virtual Library of the Faculty of Mathematics.

1. Introduction

To keep and preserve a large number of rare books, textbooks, dissertations and papers, Virtual Library was developed as a part of the Digitization project of the Faculty of Mathematics in Belgrade and the Mathematical Institute of the Serbian Academy of Sciences and Arts. Virtual Library is freely available via Internet link <http://elibrary.matf.bg.ac.rs> for the general public [1, 2]. This project will allow future generations to understand and appreciate the works of our prominent scientists better, as well as to save past works from oblivion.

In this paper, we present a digitized university textbook written by professor Jovan Lazović, who was an astronomer and respected professor at the University of Belgrade. Title of the textbook is "Fundamentals of motion theory of artificial Earth satellites". It was published by "Naučna knjiga" in 1976 [3]. This textbook was the first one where the motion theory of the artificial satellites was shown using clear mathematical and astronomical language. This book preceded a monography "Introduction to Astrodynamics" by Tatomir Anđelić [4]. Together, these are the only books about the motion of satellites, both artificial and natural.

The whole textbook is written precisely and systematically. In order to understand this subject, the previous knowledge in advanced mathematics, astronomy and celestial mechanics are required. This textbook is primarily intended for students of the fourth year of astronomy who already acquired required bits of knowledge. The digitized version of this book is publicly available in Virtual Library of the Faculty of Mathematics.

The article is organized as follows. In the next section we give a short biography of professor Jovan Lazović. Third section contains a detailed analysis of a digitized textbook and final, fourth, presents a brief conclusion.

2. Biography of professor Jovan Lazović (1931-2019)



Figure 1. Professor Jovan Lazović (1931-2019).

Jovan Lazović (Figure 1) was born in Belgrade on May 22nd, 1931 in the middle-class family. His parents, mother Leposava and father Petar, cherished the tradition of solid education of the younger generation. Therefore, he attended the French elementary school in Belgrade, but not neglecting the origin and national component of the education. Therefore he was dancing in the national folklore assemble and he was playing bagpipes. He graduated in 1954 at the Faculty of Natural Sciences in Bgrade, as part of astronomy group. Ten years after he obtained his bachelor degree, he got his PhD degree (Title: Some essential characteristics in the motion of the quasicomplanar planetoids, 1964). In his dissertation, he analyzed the respective positions of two planetoids with almost the same orbital inclination. Special attention was given to their

proximities (positions when the mutual distance is minimal). For these positions, he derived the original method for their determination, and also he examined the mutual gravitational influence of planetoids. On the board for his defense were academicians Radivoje Kašanin, Tatomir Anđelić and Vojislav Mišković. His dissertation is digitized and available at Virtual Library [8, 9] –

<http://elibrary.matf.bg.ac.rs/handle/123456789/223>.

Professor Lazović was working as a math teacher in two high schools in Belgrade after he had graduated from the University. He was elected as teaching assistant at Department of mechanics and astronomy in 1957, and he was teaching various subjects: General astronomy, Positional astronomy, Theoretical astronomy and Theory and practice of numerical calculus. He was promoted in 1967 to an associate professor for celestial mechanics. Therefore, the Department of mechanics and astronomy obtained its permanent teacher for celestial mechanics after the retirement of academician Milutin Milanković in 1955. In 1982 he was elected as a full professor for subjects: Celestial Mechanics and Motion theory of artificial Earth satellites. He was retired on January 1st, 1997.

Professor Lazović wrote university textbook "Fundamentals of motion theory of artificial Earth satellites." in 1976. It was the first one concerning the motion of artificial satellites published in Serbia. Before this work, academician Milutin Milanković wrote textbooks in the same scientific area: "Celestial mechanics" and "Basics of celestial mechanics".

The main topic of J. Lazović research was celestial mechanics, especially the motion of small solar system bodies and their proximities. He published more than 30 scientific papers and ten professional works. Also, he was giving lectures on various national and international conferences. Additionally, he was a very active observational astronomer, and because of that he participated in the creation of ephemeris for the annual publication "Godišnjak našeg neba".

In 1969 the scientific journal "Publications of the Department of Astronomy" was founded with great efforts from professor Lazović. He was the member of editorial office, and he led all unavoidable works regarding publishing and printing of the journal for 12 years (1970-1981). He actively published his original scientific works in the journal in order to promote its influence and usage. Also, he had active role in

calculations for "Godišnjak našeg neba". Besides his professional activities, professor Lazović was very fond of amateur astronomy.

He had good cooperation with other astronomers at the Department. For example, it is interesting to mention his observations of a total solar eclipse in 1961, which could be seen in our region. He observed that eclipse with his colleague Jovan Simovljević at fortress of Niš. The results of this small expedition were published in two journals; one was a publication of Serbian Academy of Sciences and Arts. He also published several papers with Professor Mike Kuzmanoski which concerned the proximities of small planets of the Solar system [5, 6]. After the Lazović's retirement professor Kuzmanoski took over the lecturing of Celestial mechanics.

Professor Jovan Lazović was quiet and peaceful men but with the capability to stood up to the authorities when it was necessary. One of the authors of this paper had the honor to have J. Lazović as his professor during the studies. Form the author's experience, J. Lazović was a very professional and great teacher who was deeply in love with the subjects he had been teaching.

On Saturday, August 3rd 2019, our famous professor passed away [10]. He left permanent trace on the Serbian astronomy and on his colleagues and students at Faculty of Mathematics, University of Belgrade. This paper is written in his honor.

3. The Textbook

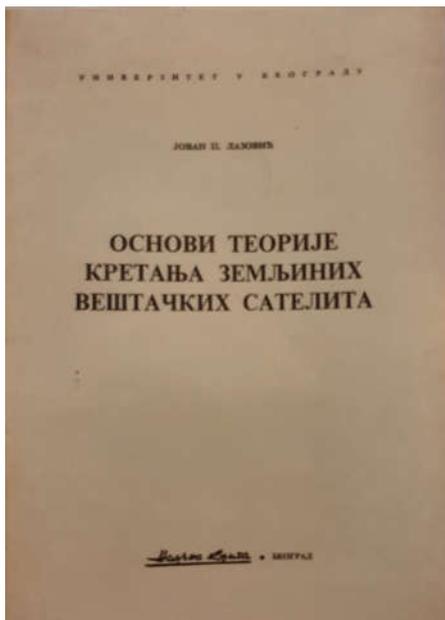


Figure 2. Textbook "Fundamentals of motion theory of artificial Earth satellites" by Jovan Lazović.

"Fundamentals of motion theory of artificial Earth satellites" (Figure 2), published in 1976, is primarily intended for astronomy students on the last, fourth year of their studies at Department of Astronomy [3]. A publisher of this textbook "Naučna knjiga" was established in 1946 and it was one of the oldest publishing corporations in former federal Yugoslavia. It was divided and closed in 1998.

At the time of publishing, it was the only textbook about artificial satellites in the Serbian language. Before, this subject was only explained within Celestial mechanics. In foreign literature, this topic is mainly covered in monographs or chapters of the celestial mechanic's books. This book started the development of this subject in Serbia. For example, Stevo Šegan, lately professor at the Department, got his PhD in this area [7].

The book has one hundred pages with 14 images and two tables. Although it is not extensive, it still demands knowledge of spherical astronomy, celestial mechanic and advanced calculus. In the book, there are many footnotes whose purpose is to give the reader references where he can find the complete derivation of equations which are only given in the textbook. Because of its concision and precision, Lazović textbook was rather difficult for students as it demanded too much time in order to cope with the exposed material. According to references in the book, it was contemporary as it contained all possible facts and information of that time. Considering the given mathematical tools, the "Fundamentals of motion theory of artificial Earth satellites" is an excellent example of a piece from applied mathematics.

In order to discuss the textbook mentioned above, we will divide it into two logically separated parts. The first part (chapters 1, 2 and 3) introduces Laplace's equation, spherical functions, expansion of gravitational potential as well as gravitational acceleration due to Earth flatness. In the second part (chapters 4 and 5) Lazović discusses differential equation of a perturbed motion and secular perturbations of artificial satellites.

3.1. First part. Laplace operator (Laplacian) is introduced using cartesian coordinates in the first chapter. Later in this chapter, we can see definitions of Laplace's equation, gravitational potential, spherical functions, Legendre polynomials and also the expansion of some functions on a spherical surface. Therefore, this chapter gives the required mathematical tools needed for the expansion of gravitational potential.

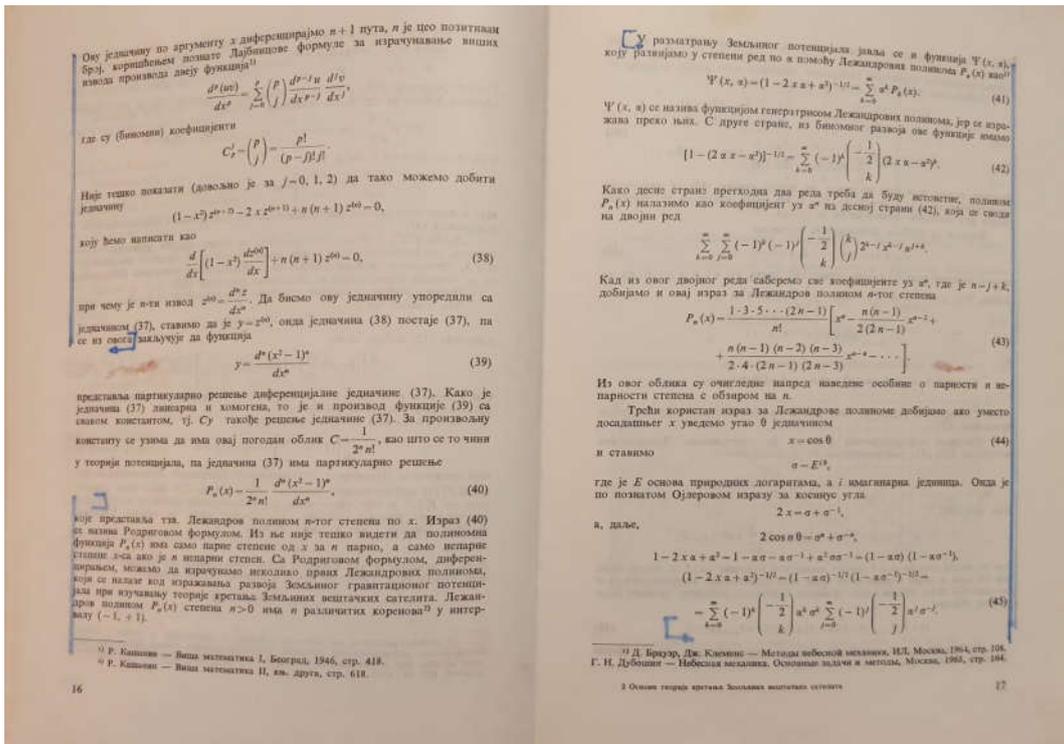


Figure 3. Pages 16 and 17

For instance, on pages 16 and 17 (Figure 3), we can see an example of derivation of Legendre polynomial of the n th degree. Equation number 40 shows Rodrigue's formula or previously called Ivory-Jacobi formula. By differentiation of Rodrigue's formula, a few first Legendre polynomials can be obtained. From this example, it is evident why this book is additionally recognized as a great example of applied mathematics. In footnotes on pages 16 and 17 (Figure 3) J. Lazović gave citations from which he took equations needed for further derivations.

In the second chapter, we can see the expansion of gravitational potential in a spherical harmonics. Next, gravitational potential of a spheroid and gravitation potential of Earth is given. Figure 4 shows pages 46 and 47 from Lazović's textbook. There we could see the general formulae for Earth ellipsoid and standard gravitational potential of Earth. The same figure shows footnotes with literature in Russian on the motion of artificial Earth satellites. In the first table on page 45, a few first values of zonal spherical harmonics are shown.

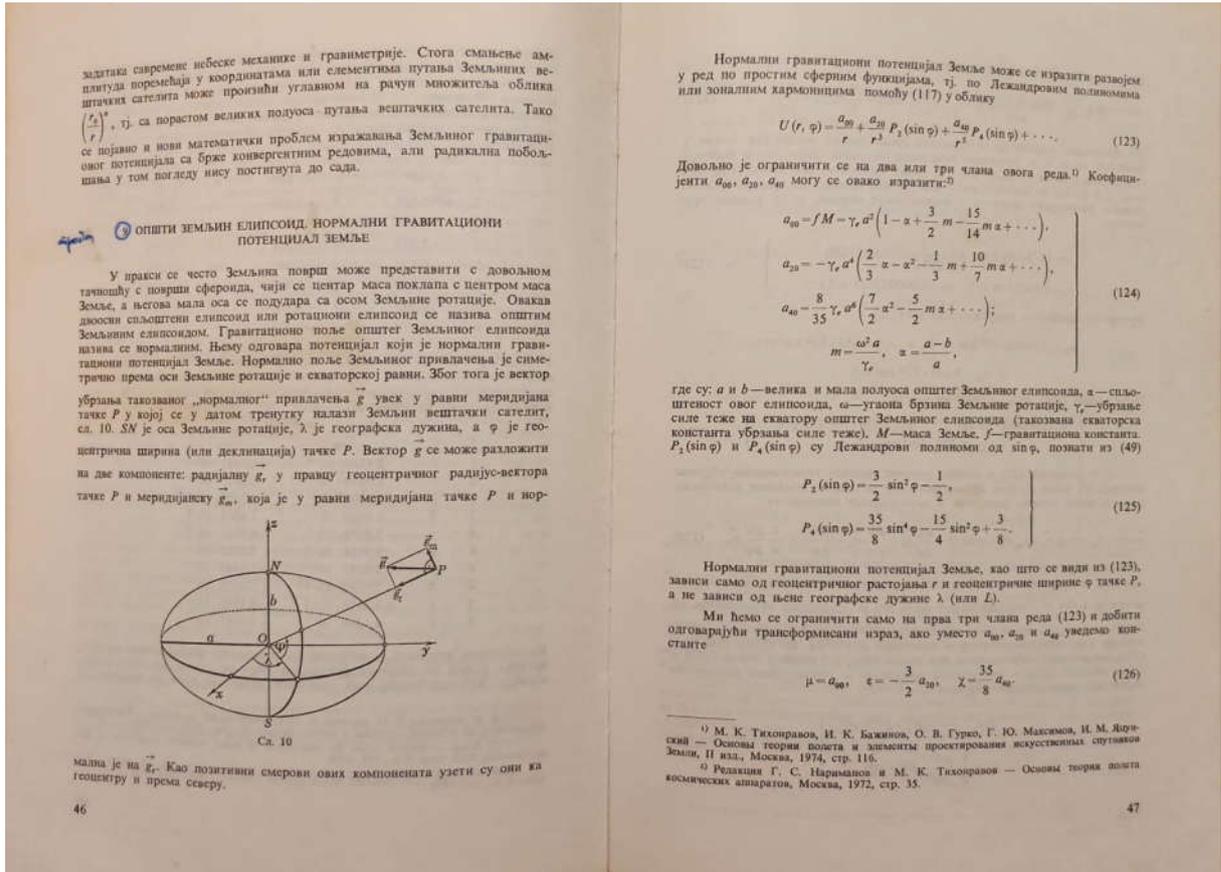


Figure 4. Pages 46 and 47

3.2. Second part. The fourth chapter covers the following topics: differential equations of perturbed motion of an artificial satellite and equation of elliptical elements of the satellite (longitude of the ascending node, a parameter of an ellipse, orbital inclination, eccentricity, true anomaly and argument of perihelion). The given system of equations is solved using the method of successive approximations. Calculations using this numerical algorithm are extensive and demand to use powerful computer resources.

Figure 5 allows us to see the vectorial equation of acceleration of a material point in the moving coordinate system. The last, fifth, chapter shows secular perturbation theory of elliptic orbit of an artificial satellite, as well as the influence of Earth flattening. Also, in this chapter, special attention is given to the period of revolution of a satellite around Earth (draconic and sidereal period). Figure 6 displays secular perturbation of Earth's artificial satellite due to influence of Earth flattening on elliptical elements. These formulae reveal all the difficulty of this calculus. This figure, along with all previous figures, designates the importance of mathematics for this subject. Thus, every successful astronomer has to be a primarily great mathematician in order to cope with problems of celestial mechanics and astrodynamics.

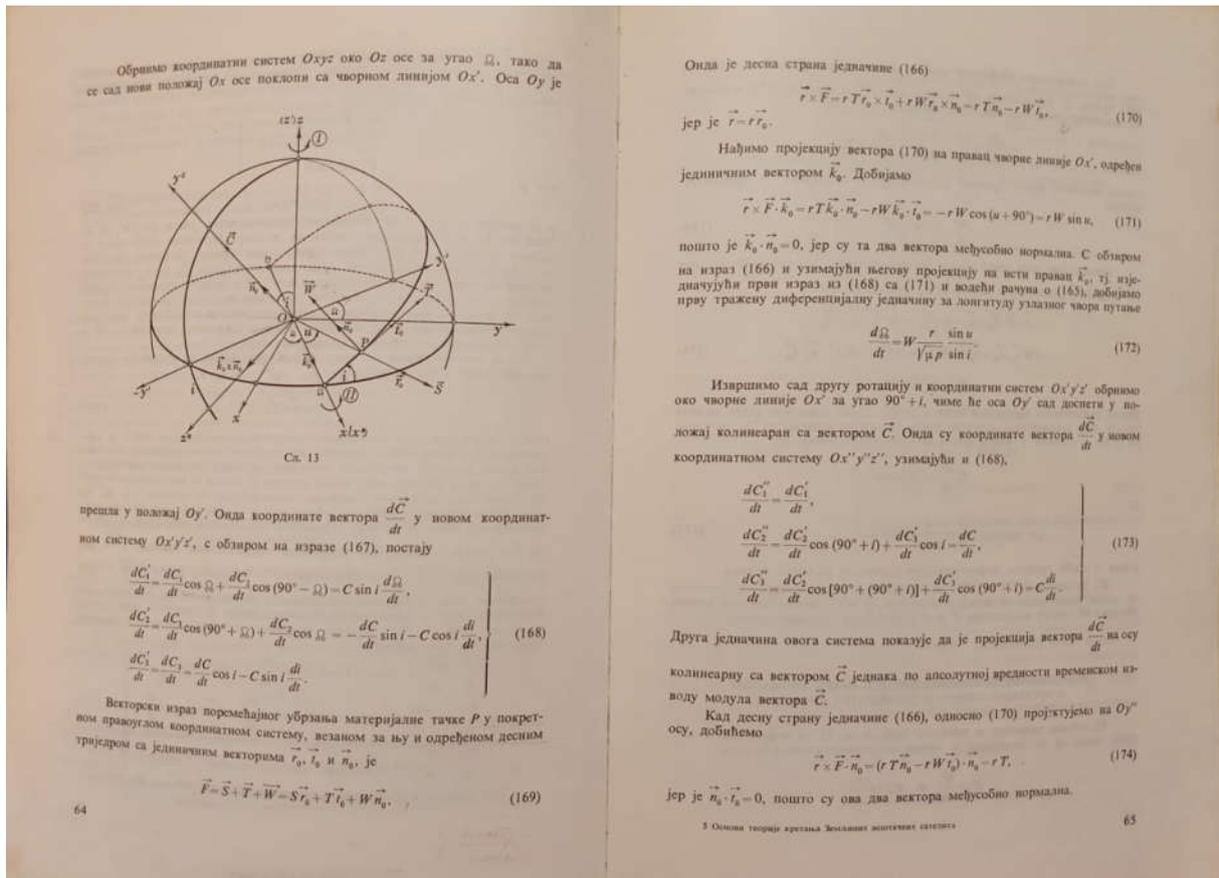


Figure 6. Pages 64 and 65

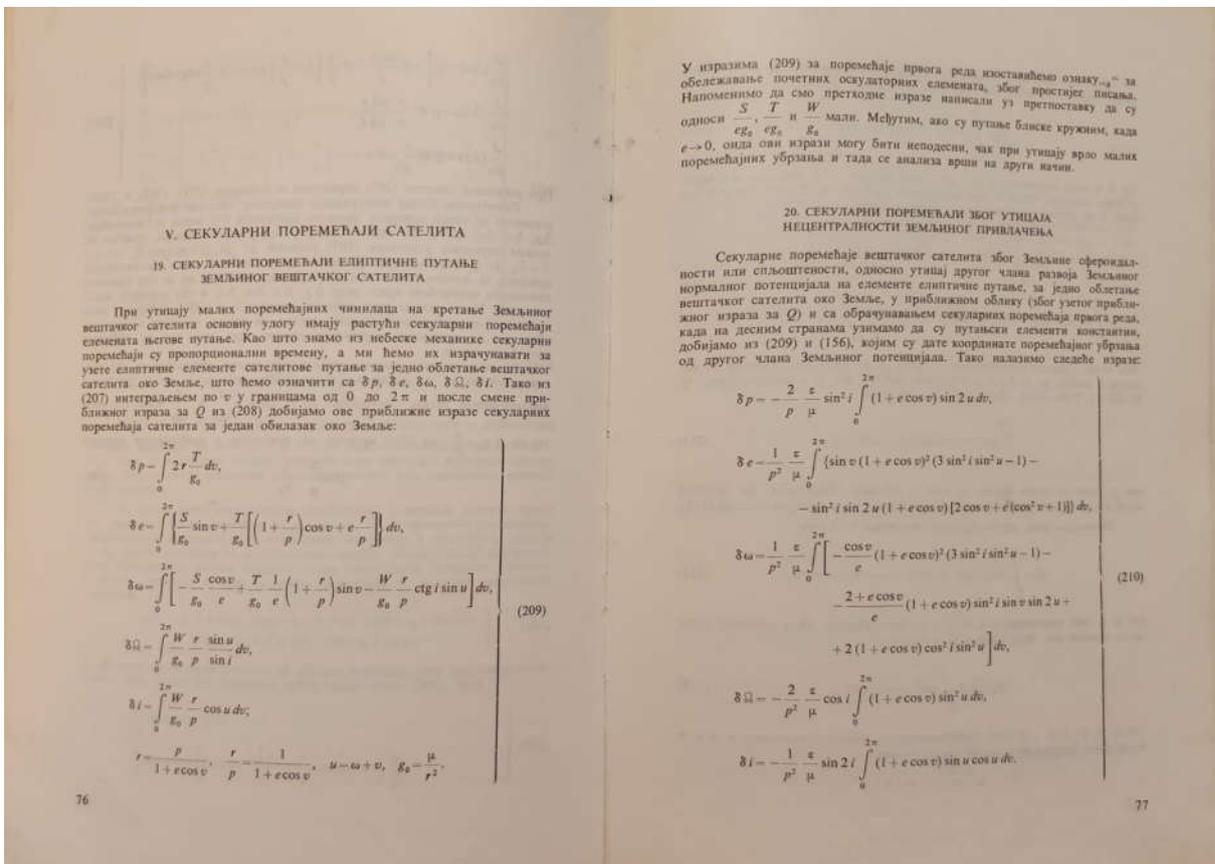


Figure 5. Pages 76 and 77

4. Conclusion

A textbook "Fundamentals of motion theory of artificial Earth satellites" by professor Jovan Lazović represent an essential contribution to the existing literature in astronomy in the Serbian language. As we have already mentioned, this textbook is an excellent example of a textbook in the area of applied mathematics. By digitalization of this textbook, we have made this book publicly available to students and the scientific public. Also, we are currently making a great effort in order to collect all available Lazović's works for making web site where we will honor his legacy.

Acknowledgment

This work was supported by the Serbian Ministry of Education and Science, grant number III 44006.

References

1. Ognjanović Z., "National center for digitization", 1(2003), NCD Review, 3
2. Mijajlović Ž., Pejović N., "National Serbian digitization project: its achievements and activities", Proceedings of the VII BSAC, 2010
3. Lazović J., "Osnovi teorije kretanja Zemljinih veštačkih satelita", 1976, Naučna knjiga, Beograd
4. Anđelić T., "Uvod u astrodinamiku", 1984, Matematički institut SANU
5. Lazović, J., Kuzmanoski M., "Paris of quasicomplanar asteroids considered with respect to mutual inclination of their orbits", Publ. Dept. Astr. Univ., Beograd, 1974, 19–51
6. Lazović, J., Kuzmanoski M., "Groups of quasicomplanar asteroids considered with respect to mutual inclination of their orbits", Publ. Dept. Astr. Univ., Beograd, 1974, 51–85
7. Šegan, S., "Prilog izučavanju kretanja Zemljinih veštačkih satelita u visokoj atmosferi", PhD, 1987
8. Ž. Mijajlović, N. Mitić, S. Malkov, "Digital legacies", NCD Review, 22(2013), 148–152
9. Ž. Mijajlović, Z. Ognjanović, N. Đorđević, T. Zečević, "Virtual library - data base of textual data", NCD Review, 5(2004), 42–48.
10. Pejović, N., "Jovan Lazović – In memoriam", Vasiona, 2019, 4

Other sources

1. Virtual Library of the Faculty of Mathematics in Belgrade, <http://elibrary.matf.bg.ac.rs/>
2. Digital legacies of the Faculty of Mathematics in Belgrade, <http://legati.matf.bg.ac.rs/>

rviktor@matf.bg.ac.rs

nada@matf.bg.ac.rs