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ANALYSIS OF A DIGITIZED COLLECTION OF SOLVED PROBLEMS OF GENERAL ASTRONOMY BY VOJISLAV MIŠKOVIĆ

Abstract. The first collection of solved astronomical problems written in Serbian is „Zbirka zadataka iz opšte astronomije“ (A collection of solved problems of General Astronomy) written by professor Dr Vojislav Mišković. This collection of solved problem was divided in two separate books due to technical reasons. The first one was published in 1956. in Belgrade, while the second is still in form of a manuscript. Both of them have been digitized and are available in the Virtual Library of the Faculty of Mathematics. More than 60 years have passed since this problem book has been published which gives us an opportunity to analyze differences between given solutions and todays approaches, and also to note importance of this problem book as part of Serbian history of astronomy and science in general.

1. Introduction

In this paper we present an analysis of the first astronomy problem book written in Serbian by professor Vojislav Mišković: “Zbirka zadataka iz opšte astronomije” (A collection of solved problems of General Astronomy) [1, 2]. This book was approved for printing as a textbook for students of the Faculty of Sciences through an act of the Textbook Commission of Belgrade University No 896 on August 10, 1956. Due to technical reasons this collection was divided into two separate books. The first one was published in 1956 by Naučna Knjiga in Belgrade, whereas the second one has never been published. The second part of the collection exists only in manuscript form and it was digitized less than decade ago. Both of these books are included in the Virtual Library of the Faculty of Mathematics of the University of Belgrade at the Internet address http://elibrary.matf.bg.ac.rs [6].

This paper is organized as follows: in the second section we give a short biography of the author, Professor Vojislav Mišković. In the third section we present a brief overview of the contents and structure of the Collection. Fourth section covers analysis between given solutions in the problem book and todays approaches along with some examples. Conclusion is given in the final, fifth section.

2. Biography of professor Vojislav Mišković (1892-1976)

Professor Vojislav Mišković was born in Fužine (Croatia) in 1892 [4, 5, 7]. After graduating form High school, he started studying astronomy at the University of Budapest and Goettingen. During his studies, he was involved in youth movements and also took an active part in the First World War. After the war he continued his interrupted studies at the University of Marseille where he graduated in 1919. Afterwards, he worked as an astronomer at Observatories in Marseille and Nice. In this period he obtained PhD degree (1924) at the University of Montpelier (Theses: “Etudes de statistique stellaire”). During this period he published various scientific papers about observation and computation of the orbits of planetoids and comets.

Following an invitation by Professors Mihailo Petrović Alas (1868–1943), Milutin Milanković (1879-1958) and Bogdan Gavrilović (1864-1947) he came to Belgrade in 1926. At this time he was promoted to Associate Professor at the Faculty of Philosophy in Belgrade and became a Director of the Astronomical Observatory in Belgrade. As a Director, he intended to build a new professional astronomical observatory in Belgrade. He got the sufficient funds for a new building in 1929. The construction began in 1930. and the new astronomical complex was finished in 1932. That was the first, independent building for Serbian astronomers, hence Mišković is often nicknamed as "the builder of the observatory".

As a young professor and a scientist he was elected a corresponding member of the Serbian Royal Academy (1929), and ten years later he became a full member. Among his extensive scientific and academic achievements Vojislav Mišković started various publications (Bulletin of Astronomical Observatory, Nautical almanac etc.) and was interested in the history of astronomy. He published various university books and was active in popularization of the astronomy.

Vojislav Mišković died in 1976 in Belgrade.

3. Content and structure of the book

The first, printed part of the Collection of solved problems in astronomy has 150 pages and it is divided in a few sections: *Preface*, *Introduction* and two chapters. First chapter named *Problems* is about problems concerning three fields: *Spherical Trigonometry*, *Earth as a celestial body* and *Apparent diurnal motion of celestial sphere.* The second chapter *Solutions* has complete and detailed solutions for 126 problems which are given in the first chapter.

Similar organization can be seen in the unpublished manuscript (second part of the Collection). It has three sections: *Introduction, Problems* and *Solutions.* Problems contain 136 problems in three fields *Astronomical Refraction, Elements of theory of motion of planets and comets* and *Apparent annual motion of the Sun*, which are solved in the third chapter.

Field	**Number of problems**
Spherical Trigonometry	33
Earth as a celestial body	42
Apparent diurnal motion of celestial sphere	51
Astronomical refraction	23
Elements of theory of planet and comet motion	36
Apparent annual motion of the Sun	74
Total	**262**

Table 1. Different astronomical areas and total number of solved problems presented in the Collection of solved problems of General astronomy.

Overall, the problem book covers six different areas and has 262 completely solved astronomical problems (see Table 1). More information about the content of this problem book can be seen in Pejović, 2009 [8]. As this book is the first and the only book in Serbian about solved astronomical problems, in the next chapter we present a discussion of given solutions.

4. Analysis of a few selected solutions from the book

This problem book is well known to Serbian astronomers as they have used it for many years as a course material for the first year subject General Astronomy. Form its publication this book is still in use, as there is no other available problem book of solved astronomical problems in Serbian language. We have been using this Collection in our exercise classes of General Astronomy which give us a great opportunity to analyze differences between the methods used almost 60 years ago and the ones used nowadays.

One of the main differences that can be immediately seen is a given method for numerical calculations. In the Collection, for the numerical calculations, the logarithmic tables are extensively used. The main reason behind using logarithms for calculation are its properties. One of the important properties of logarithms are following identities: $log_b(xy) = log_b(x) + log_b(y)$; $log_b(x/y) = log_b(x) - log_b(y)$; $log_b\, x^p = p\, log_b\, x$. This identities allow us to simplify difficult calculations, or in other words, multiplication can be represented as addition, division as subtraction etc. A key tool that enable practical use of logarithms are logarithmic tables, first developed and computed by Henry Briggs (1617). Also, at the beginning of the XX century, in combination with logarithms, mechanical computers had been used in order to perform calculations significantly faster (see Figure 1).

Figure 1. Mechanical computer.

To help university students and even high-school students and professional astronomers to perform these calculation Vojislav Mišković wrote a handbook for logarithmic and numerical tables [3, 7]. This handbook contains different data such as Briggs logarithms, values of trigonometric and hyperbolic functions, special conversion tables etc.

Across the Collection, various problems are calculated using logarithmic calculations. In order to do that starting equations have to be modified to fit logarithmic calculations (see Figure 2). Therefore, additional algebraic transformation are performed. After modifying initial equations, numerical calculations are given in detail (see Figure 3) where usually designation *[x]* is used instead of *log(x)*.

91. Пођи ћемо од образаца Гаусове групе који одређују δ и H кад су познати φ, z и A:

$$\sin\delta = \sin\varphi\cos z - \cos\varphi\sin z\cos A,$$

$$\cos\delta\sin H = \sin z\sin A,$$

$$\cos\delta\cos H = \cos\varphi\cos z + \sin\varphi\sin z\cos A.$$

Да бисмо их подесили за логаритамско рачунање, ставићемо

$$\sin z\cos A = m\sin M,$$

$$\cos z = m\cos M, \quad \text{са } m > 0.$$

Из ових једначина налазимо

$$\operatorname{tg} M = \operatorname{tg} z\cos A \quad \text{и} \quad m = \sin z\cos A\operatorname{cosec} M.$$

Увођењем ових помоћних величина првобитни обрасци постају

$$\sin\delta = m\sin(\varphi - M),$$

$$\cos\delta\sin H = \sin z\sin A,$$

$$\cos\delta\cos H = m\cos(\varphi - M).$$

И тако добивамо непознате величине из једначина

$$\operatorname{tg} H = \sin M\operatorname{tg} A\sec(\varphi - M) \quad \text{и} \quad \operatorname{tg}\delta = \operatorname{tg}(\varphi - M)\cos H.$$

Figure 2. Part of the solution of the problem number 91 from the first book of the Collection. In this part it can be seen how starting equations are modified to fit logarithmic calculations.

Nowadays, we are using computers and calculators which are much faster than calculations with logarithmic tables or mechanical computer. Because of that starting equations do not have to be modified, which significantly simplify overall procedure.

Development of a personal computers had a large influence on performance of different calculations, especially numerical algorithms. One example is visible in a problem number 158 (manuscript): *Solve Kepler's equation if eccentricity is e=0.147 and mean anomaly M = 136^o 25' 32".4.* In the Collection this problem is solved by applying Kepler's equation: $E - e\sin(E) = M$. Obviously, Kepler's equation is a transcendent equation, hence it cannot be solved analytically. Therefore, in order to solve Kepler's equation we have to apply some numerical procedure (method of successive approximation, Gauss method etc.). In the given solution, method of successive approximations is used, starting from the $E_0 = M + e\sin(M)$. and result is obtained after fourth iteration. It is evident that this approach can be very difficult for manually calculations, especially if larger precision is required. Today, with help of computers we can perform this calculation in matter of seconds even though high precision is required. One pseudocode for this solution is given bellow.

```
double keplerequation (M, eo, eps) {
      double E0 = M;
      while ( abs(E0 - M - eo * sin (E0)) > eps ) {
            E0 = M + eo * sin (E0);
      }
      return E0;
}
```

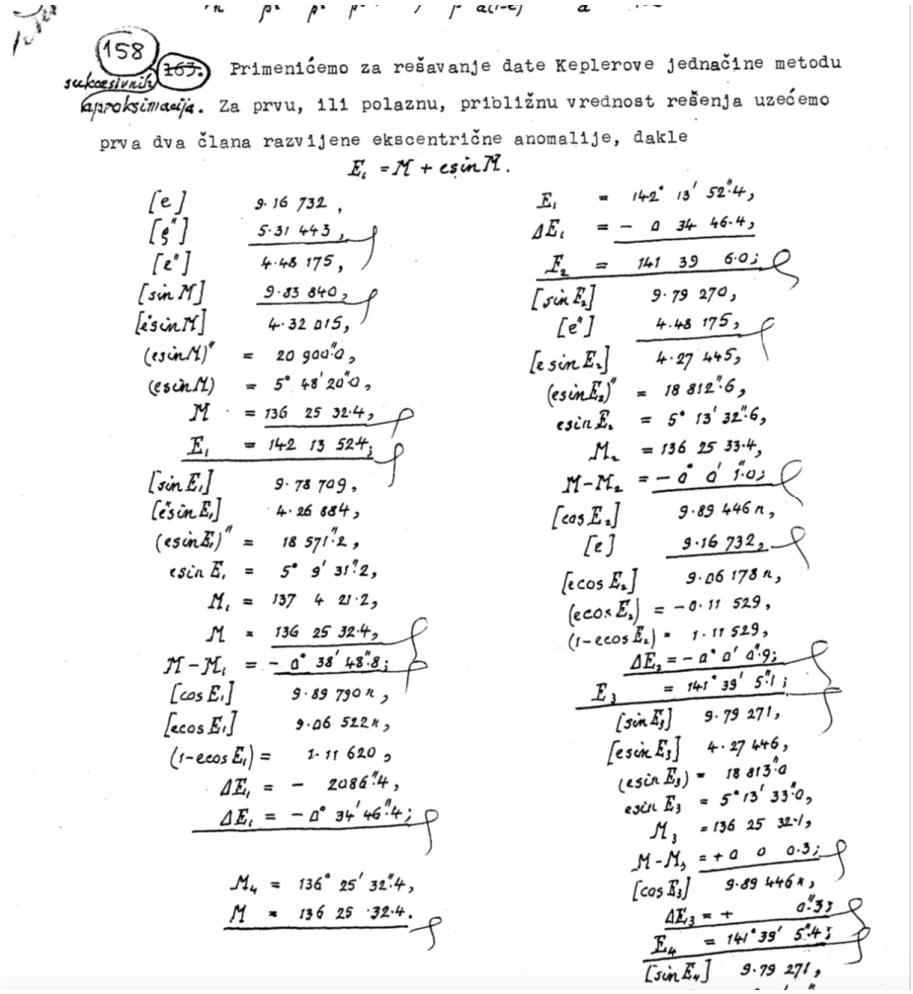
158
sukcesivnih ~~167.~~ Primenićemo za rešavanje date Keplerove jednačine metodu
aproksimacija. Za prvu, ili polaznu, približnu vrednost rešenja uzećemo
prva dva člana razvijene ekscentrične anomalije, dakle
$E_1 = M + e\sin M$.

Figure 3. Solution of the problem number 158 from the manuscript of Collection. In this solution we can see performed numerical procedure in detail.

Across the Collection different designations for common coordinates are used. For instance, the hour angle is usually represented with a small letter *t* while in the Collection capital letter *H* is used. More examples exist in the second part (manuscript), where for instance the right ascension is marked as ℝ, while greek letter α is usually used.

5.Conclusion

As discussed above the Collection of solved problems of general astronomy written by Professor Vojislav Mišković has a crucial place in our astronomical literature. It is the first, clearly written problem book with solutions in astronomy. Also, this book contains some completely solved problems that cannot be find in a foreign literature. Therefore, its importance is immense for astronomy in general.

This book is used at the Belgrade University for the different astronomy courses (General Astronomy, Celestial mechanics, Introduction to Astronomy, Selected chapters of astronomy). Differences between today's methods for numerical calculations and the ones given in the book are important for our history of astronomy and mathematics. They show us how much effort had to be invested and to appreciate more general development in various fields. The complete digitized version can be found in the Virtual Library of the Faculty of Mathematics.

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