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TWENTY FOUR MANUSCRIPTS IN THE VIRTUAL LIBRARY OF THE FACULTY OF MATHEMATICS IN BELGRADE

Abstract: The aim of this paper is to present 24 almost forgotten handwritten manuscripts that were recently found in the library of the Mathematical Institute of the Serbian Academy of Sciences and Arts. These writings belong to the prominent professors of the Belgrade University who lectured pure and applied mathematics at the beginning of the previous century: Dimitrije Danić (1862-1932), two manuscripts on applied mathematics; Kosta Stojanović (1867-1921), two manuscripts on physics and geometry; Mihailo Petrović (1868-1943), 14 manuscripts on mathematics, mostly on differential calculus and algebra; Milutin Milanković (1879-1958), five writings on applied mathematics, physics, and theoretical mechanics and one manuscript by an unknown author. Digital copies of these writings are deposited in the Virtual Library of the Faculty of Mathematics of the University of Belgrade.

Keywords: manuscripts, mathematical sciences, handwritings, lectures

1. Introduction

Almost ten years ago the Virtual Library of the Faculty of Mathematics in Belgrade was founded [12]. It is the largest database of digitized texts in Serbia which is open for public use. At the time of writing this article, it contained about 3000 digitized items and several important collections, including doctoral dissertations in mathematics and rare books from the 18^{th} and 19^{th} century. In this paper we present a collection of the recently digitized 24 manuscripts dating at the end of the 19^{th} and the beginning of the 20^{th} century. They are handwritten notes of lectures delivered by the most outstanding mathematicians and professors of the Belgrade University. Digital copies of manuscripts are deposited in the Virtual Library.

2. Manuscripts

The original manuscripts belong to the Library of Mathematical Institute in Belgrade. Their origin and existence is closely related to the emergence and the work of the Library. Therefore we believe that a short history of the Library and circumstances how manuscripts come to the existence and how they were discovered could be interesting to the reader.

Today the Library has the status of a specialized and central library in the field of mathematics and related disciplines such as mechanics, astronomy and computer science. Besides the library of the Faculty of Mathematics in Belgrade, it is the most important source of printed scientific literature for Serbian mathematical community. Therefore, its main goal is to collect and keep the necessary scientific literature, documentation and information related to mathematical sciences. Library holdings consist of books, journals, and data-bases, as well as various electronic editions. Even if the origin of the Library goes back to the end of the 19^{th} century, a few books and journals in the Library are dated before the Second World War. This shall be explained later.

The library of the Mathematics Seminar, the forerunner of the Library of the Mathematical Institute, was established already in 1894. It provided generations of mathematicians of the Belgrade University with ample opportunities for scientific work. Until the First World War the library was led by outstanding university professors of mathematics, Bogdan Gavrilović and Mihailo Petrović, and later other mathematicians participated. The Library had a rich collection of books, sets of journals, monographs and other mathematical literature. Until the fifties of the 20th century, the Department of Mathematics was a part of the Faculty of Philosophy of the Belgrade University. All mathematical activities, studies and scientific work were conducted there. So, the Mathematics Seminar and its library as well, were placed there. When the scientists of the Faculty of Philosophy moved in 1938 to a new building built next to the old building of the Faculty in Captain Miša's building, the Library was moved to the same place. Unfortunately, only two days before the liberation of Belgrade, on October 18, 1944, the retreating enemy army set fire to the library and destroyed it. Only a few books that were borrowed at the time by individuals were saved. The saved first book of inventory up to the 1907, shows that Bogdan Gavrilović himself manually recorded the books, up to the number of 110, and Mihailo Petrović over that number up to 301. Due to these historical circumstances, now there are only a few old books and manuscripts in the library. Hence, we were quite astonished when we found almost fortuitously, in the autumn of 2012, twenty four bound manuscripts sitting lonely at one not so easily accessible rack. These writings were handwritten notes of lectures in mathematics, mechanics and theoretical physics which were delivered at the Belgrade University by prominent university professors and mathematicians Dimitrije Danić, Kosta Stojanović, Mihailo Petrović and Milutin Milanković. There is an additional manuscript in this collection, dated 1846. It covers lectures on astronomy and is written in French. The name of the author or the owner of the book is written on the first page, but it is illegible.

By inspection of the manuscripts assigned to Danić and Stojanović, we found that they were written by the authors. On the first page of the notes of Petrović' and Milanković' lectures Borivoj J. Pujić is written. As all signed copies were written in the same beautiful handwriting, one may suppose with certainty that Pujić took the notes himself. Searching old archives of the Faculty of Mathematics, we found that Pujić finished studies of mathematics at the Belgrade University in 1914. Therefore we can determine quite accurately that the notes of Petrović' and Milanković' lectures were created in the period 1910-1914. Studying the writing style, language and the used mathematical terminology, we concluded that Danić and Stojanović wrote their manuscripts at least a decade before, around the beginning of the 20th century. All these books were registered in the library catalog in the sixties of the 20th century. Hence, we may assume that the manuscripts, at least those signed by Pujić, were donated to the Mathematical Institute, most probably by Pujić himself. We do not know much about Pujić, neither had we found traces of his name in other historical or archival documents. He signed as a "Bor. J. Pujić, prof.", so most certainly he was a professor of mathematics in a secondary school. But despite the fact that Pujić was unbeknown, he left us important and very complete documents that witness the status and the level of mathematical sciences at the Belgrade University a century ago.

These lectures cover almost all courses of mathematics and mechanics taught at that time at the Belgrade University. They do not contain only the notes of lectures delivered by Bogdan Gavrilović, another outstanding university professor and academician. This is understandable, since his two university text-books on linear algebra and analytic geometry were already printed at the end of the 20th century. We do not want to enter here into description of the manuscripts. It should only be mentioned that they cover all main topics in mathematics and applied mathematics at the level of similar courses taught at the

leading European universities of that time. Our explanation is that all authors of the manuscripts were educated and they defended their doctoral dissertations at the leading European university centers: in Germany, Austro-Hungary and France. Also, their mentors were prominent mathematicians of their time. This is also noted by other authors, for example by Robson and Stedall [10].

We digitized all manuscripts and they are now deposited in the Virtual Library of the Faculty of Mathematics[1] and we invite the reader to browse them. The reader will find that some of the lectures were rather advanced, for example Petrović' lectures on differential equations and the theory of functions and Milanković' lectures on theoretical mechanics. Looking into the titles and contents of the courses, one can conclude that pure mathematics was not the only goal of the university studies of mathematics, but also applications of mathematics to other sciences.

3. Authors

We provide only short biographical notes on authors; much more complete biographies of these prominent professors and outstanding men one can find in references and supplied links. All manuscripts, except that one written by an unknown author are written in Serbian, but we also supplied translations of their titles into English. Digital copies of manuscripts can be found in the Virtual Library of the Faculty of Mathematics.

Dimitrije Danić

Dimitrije Danić (1862 – 1932) is the first Serbian mathematician with a Ph.D. degree (Jena University, 1885). He was professor of mathematics at the Military Academy, but he also lectured occasionally at the Belgrade University. Manuscripts were written by the author himself around the beginning of the 20th century.

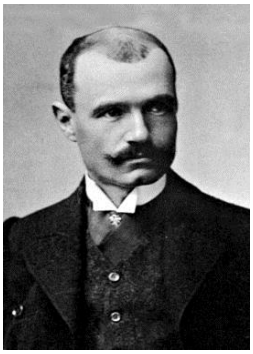
1. Теорија конформног снимања и њена примена у Картографији и Вишој Геодезији (Theory of conformal mapping and its application to Cartography and Higher Geodesy), IV+147, Belgrade, end of XIX Century.
2. Теорни основи Методе најмањих квадрата (Foundation of the least square method), II+78, Belgrade, end of XIX Century.

Kosta Stojanović

Kosta Stojanović (1867 – 1921) was the professor of mechanics appointed at the Belgrade University until 1905. After that he turned to politics and advanced to the position of the Minister of economics. He wrote probably the best Serbian book ever on mathematical economy (*Foundations of the Theory of Economic Values*, published in 1910). The book is inspired by phenomena appearing in mechanics and physics and involves high mathematical formalism. Manuscripts were written by the author himself around the beginning of the 20th century.

[1] http://elibrary.matf.bg.ac.rs. In the electronic version of this paper we supplied electronic hyperlinks to the manuscripts and so the reader will have the opportunity to browse and read them online.

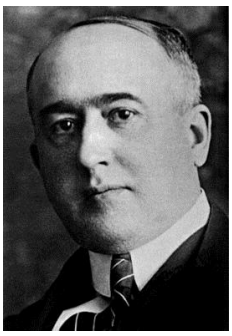
1. О полу и полари код кривих линија (On poles and polars of curved lines), III+127, 8 tables, Belgrade, end of XIX Century.
2. Предавања на Универзитету из Математичке физике (University lectures on Mathematical Physics), 470p, Belgrade, beginning of XX Century.



Mihailo Petrović

Mihailo Petrović Alas (1868 – 1943) is probably the most famous Serbian mathematician, also known as the founder of the world-wide recognized Belgrade Mathematical School. Besides several hundred scientific papers in mathematics, he also wrote several books related to philosophy, as well as on fishing and water-life. He was a great traveler and a member of expeditions which visited and examined distant places in South and Nordic sees. His books on these travels are still popular in the general reading public. Petrović delivered lectures to students of mathematics at the Mathematical Department of the Belgrade University. Notes of all lectures, except of "Теорија алгебарских једначина" were taken by Petrovic' student Borivoj J. Pujić around 1910.

1. Основи теорије детерминаната (Basics of the theory of determinants), II+103.
2. Аналитична геометрија. I део: у равни (Analytic geometry. Part I: in plane), II+382.
3. Аналитична геометрија. II део: у простору (Analytic geometry. Part II: in space), II+384.
4. Теорија алгебарских једначина (Theory of algebraic equations), II+254.
5. Теорија извода са применама (Theory of [function] derivatives with applications), II+227.
6. Интегрални рачун (Integrals), II+384.
7. Диференцијални рачун (Differential calculus), II+278.
8. Геометријске примене диференцијалног рачуна (Geometrical applications of differential calculus), II+246.
9. Геометријске примене интегралног рачуна (Geometrical applications of integrals), II+110.
10. Линеарне диференцијалне једначине (Linear differential equations), II+116.
11. Обичне диференцијалне једначине (Ordinary differential equations), II+116.
12. Геометријске примене диференцијалних једначина (Geometrical applications of differential equations), II+82.
13. Парцијалне диференцијалне једначине (Partial differential equations), II+126.
14. Теорија функција (Theory of functions), II+126.



Milutin Milanković

Milutin Milanković (1879 – 1958) is one of the Serbian greatest and most cited scientists of all times. His theory on the Ice age that accurately explains the change of climate on a large time scale is accepted world-wide. In recognition of his scientific achievements, a crater on the Moon, another one on the Mars and a planetoid were named after him. Besides his the most famous work *Kanon der Erdbestrahlung und seine Anwendung auf das Eiszeitenproblem* (published in 1941) and many other scientific papers, he also wrote an excellent book on celestial mechanics and books that popularize the science. He is also known as a good civil engineer. Notes of all lectures were taken by Borivoj. J. Pujić, a student of mathematics, around 1910.

1. Модерне теорије о електрицитету и магнетизму (Modern theories on electricity and magnetism), II+280.
2. Општа теорија физикалних поља (General theory of physical fields), II+294.
3. Рационална механика, I deo (Rational mechanics, Part I), II+277.

4. Рационална механика, II deo (Rational mechanics, Part II), II+372.
5. Векторска анализа са применама из Теориске физике (Vector analysis with applications to Theoretical Physics), II+250.

4. Unknown author

Cours d'Astronomie et de Geodesie, (Course on Astronomy and Geodesy), Ecole Polytechnique, II+578, Paris, 1846. This is an interesting but unfortunately unsigned hand-written book on astronomy and geodesy. A copy of this manuscript can be found in several French university libraries (Lyon and Bordeaux) and in the Indiana University Library. The catalogue record of this book is available in OCLC[2] Worldcat (no. 25637690). The book is written in French and according to metadata the author is unknown, even the country of origin is unknown. However, there is a possibility that the book was written by a French astronomer and geodesist, later the minister of education, Hervé Auguste Etienne Léopold Alban Faye[3] (1814-1902). Namely, he published several books in the second half of the 19th century with similar titles and contents. Another argument in the favor of this thesis is that he was first a student and later a lecturer (1848-1854) at l'Ecole Polytechnique of a course of geodesy.

5. Conclusion

We presented an invaluable collection of digitized manuscripts belonging to the Mathematical Institute in Belgrade, dating from the end of the 19th and the beginning of the 20th century. The manuscripts are handwritten notes of university lectures which were held by prominent Serbian mathematicians at the Belgrade University. This collection is an important part of scientific heritage and also the testimony of how mathematical sciences were taught at the Belgrade University at the beginning of the 20th century.

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[11] Trifunović, D. *Chronicles of life and work of Mihailo Petrović* (*Letopis života i rada Mihaila Petrovića)*. Beograd, SANU, 629p, 1969.
[12] *Virtual Library of Faculty of Mathematics in Belgrade*. Available at: http://elibrary.matf.bg.ac.rs (2014-01-18)

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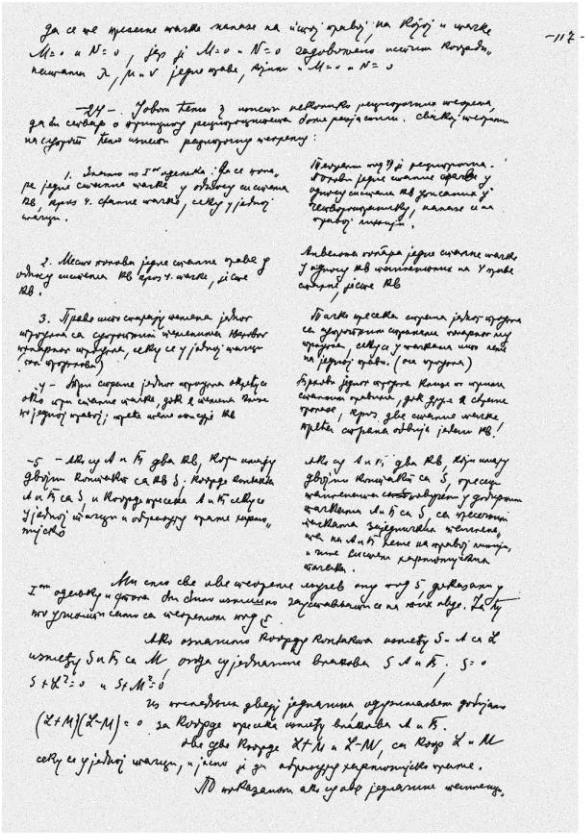
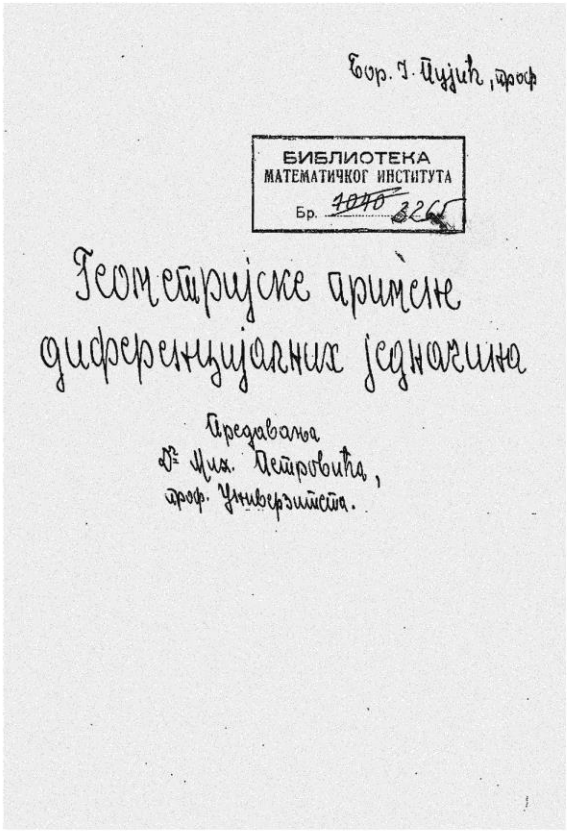
Pages from manuscripts

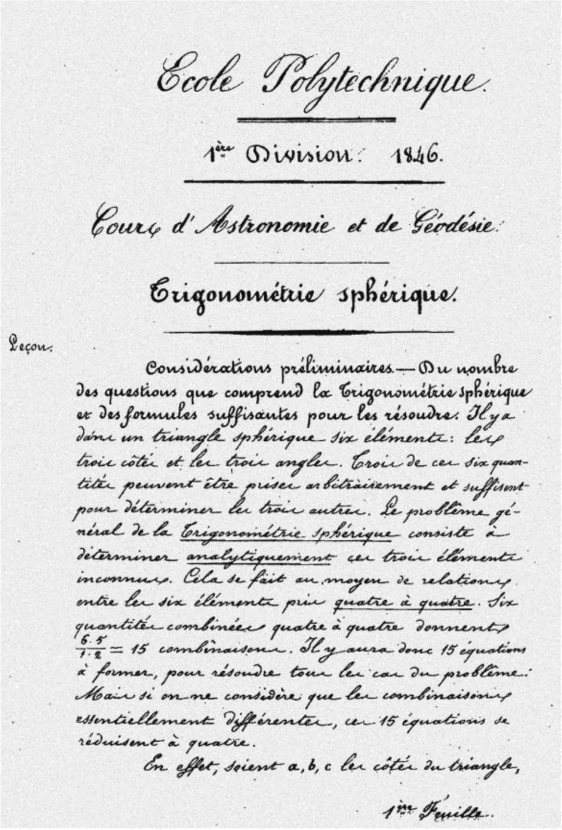


A page from Stojanović
On poles and polars of curved lines



First page from Petrović'
Geometrical applications of differential equations

Pages from manuscripts



Ecole Polytechnique.

1re Division. 1846.

Cours d'Astronomie et de Géodésie.

Trigonométrie sphérique.

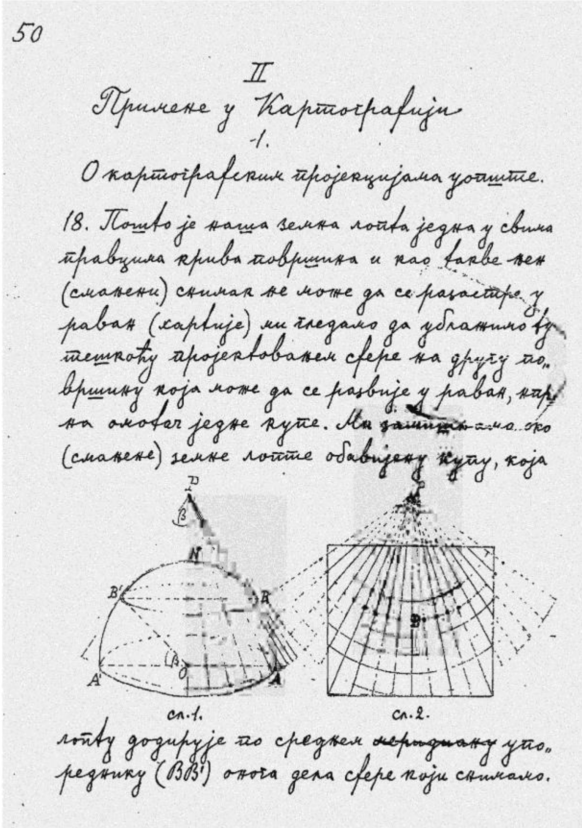
Leçon.

Considérations préliminaires.— Du nombre des questions que comprend la Trigonométrie sphérique et des formules suffisantes pour les résoudre. Il y a dans un triangle sphérique six éléments: les trois côtés et les trois angles. Trois de ces six quantités peuvent être prises arbitrairement et suffisent pour déterminer les trois autres. Le problème général de la Trigonométrie sphérique consiste à déterminer analytiquement ces trois éléments inconnus. Cela se fait au moyen de relations entre les six éléments pris quatre à quatre. Six quantités combinées quatre à quatre donnent $\frac{6.5}{1.2} = 15$ combinaisons. Il y aura donc 15 équations à former, pour résoudre tous les cas du problème. Mais si on ne considère que les combinaisons essentiellement différentes, ces 15 équations se réduisent à quatre.

En effet, soient a, b, c les côtés du triangle,

1re Feuille.

First page from *Cours d'Astronomie et de Geodesie*



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II

Примене у Картографији

1.

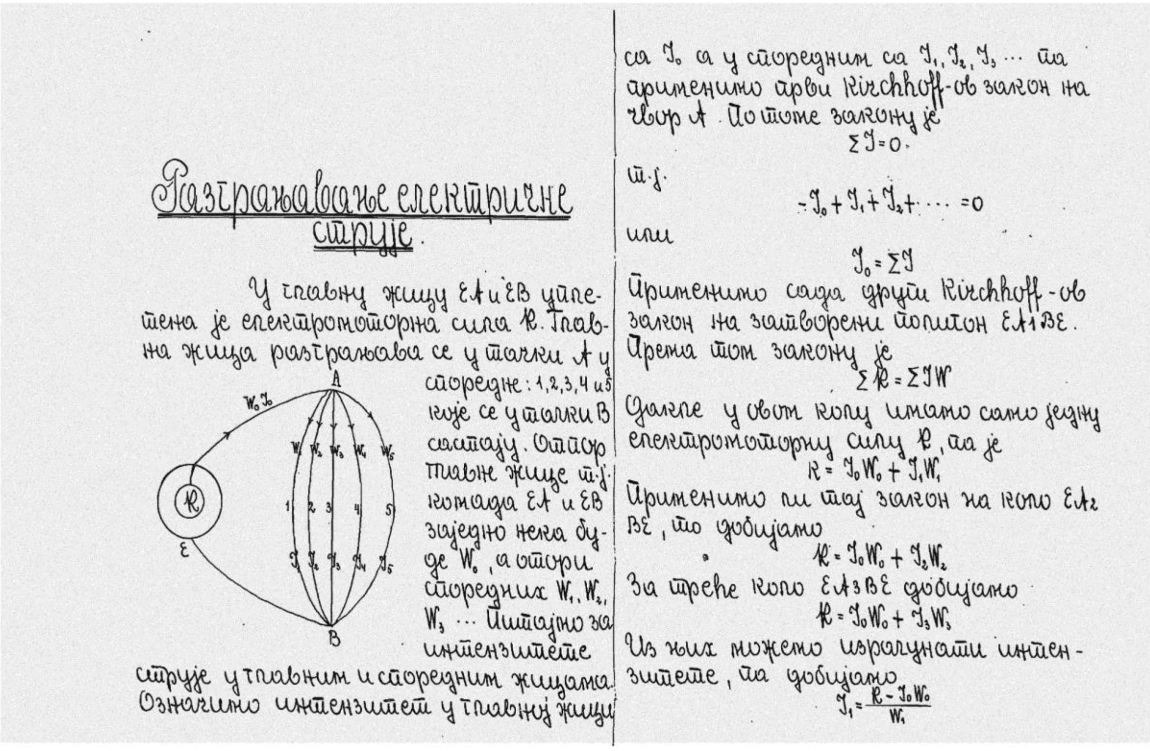
О картографским пројекцијама уопште.

18. Пошто је наша земна лопта једна у свима правцима крива површина и као такве њен (смањени) снимак не може да се разастре у раван (хартије) ми гледамо да ублажимо ту тешкоћу пројектовањем сфере на другу површину која може да се развије у раван, нпр. на омотач једне купе. Ми замишљамо око (смањене) земне лопте обавијену купу, која

сл.1. сл.2.

лопту додирује по средњем ~~меридијану~~ упоредику (BB') оног дела сфере који снимамо.

A page from Danić' *Theory of conformal mapping*



Разграњавање електричне струје.

У главну жицу ЕАиЕВ уплетена је електромоторна сила К. Главна жица разграњава се у тачки А у споредне: 1,2,3,4 и 5 које се у тачки В састају. Отпор главне жице т.ј. комада ЕА и ЕВ заједно нека буде W_0, а отпори споредних $W_1, W_2, W_3 \cdots$ Интензитет струје у главним и споредним жицама. Означимо интензитет у главној жици

са J_0 а у споредним са $J_1, J_2, J_3 \cdots$ па применимо први Kirchhoff-ов закон на чвор А. По томе закону је

$$\Sigma J = 0.$$

т.ј.

$$-J_0 + J_1 + J_2 + \cdots = 0$$

или

$$J_0 = \Sigma J$$

Применимо сада други Kirchhoff-ов закон на затворени полигон ЕА1ВЕ. Према том закону је

$$\Sigma K = \Sigma JW$$

Дакле у овом колу имамо само једну електромоторну силу К, па је

$$K = J_0 W_0 + J_1 W_1$$

Применимо ли тај закон на коло ЕА2ВЕ, то добијамо

$$K = J_0 W_0 + J_2 W_2$$

За треће коло ЕА3ВЕ добијамо

$$K = J_0 W_0 + J_3 W_3$$

Из њих можемо израчунати интензитете, па добијамо

$$J_1 = \frac{K - J_0 W_0}{W_1}$$

Pages from Milanković' *Modern theories on Electricity and Magnetism*