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A REVIEW OF MODEL THEORY IN SERBIA

ABSTRACT. In this article we present a historical overview of research in the area of model theory in Serbia. We will concentrate only on model theory of the first order structures and briefly summarize some of the most interesting results achieved in this area by Serbian mathematicians.

It is generally agreed that the rise of modern logic in Serbia is closely related to the work of the Seminar on Mathematical Logic in Belgrade. Accordingly, the interest for model theory in Serbia appeared in the beginning of 1970's when the members of the Seminar on Mathematical Logic of the Mathematical Institute in Belgrade delivered lectures on various topics in mathematical logic, including model theory. Lectures on model theory covered basic model theoretic concepts and techniques known by then and there were no research papers on model theory published at that time. One of the members, Žarko Mijajlović, showed a particular interest in model theory. His influence turned out to be decisive in this early stage and in the successive years.

Mijajlović spent 1973–1974 as a graduate student at the University of Wisconsin in Madison working under supervision of Jerome H. Keisler; he also spent 1978 there at postdoctoral studies. At that time Madison was the world center of model theory because in 1973 the famous book ‘Model Theory’, written by C.C. Chang and J.H. Keisler, was published. Mijajlović was on the source of actual happenings and developed a particular research interest in model theory, especially in Boolean algebras and models of arithmetic. He continued his research in Belgrade and it resulted in his PhD Thesis *A Contribution to Model Theory and Boolean Algebras* (in Serbian), it was defended in 1977 at the University of Belgrade and the supervisor was Slaviša Prešić; this is the first published research work in model theory in Serbia. He also published the first research article: *A note on elementary end extension*. Publications de l’Institut Mathématique (Beograd) 1977; there he generalized the well-known result which says that every countable model of ZF has an elementary end extension. Mijajlović was the first to observe that every nonstandard model of arithmetic is partially recursively saturated, afterwards this result is often called Robinson’s property (after Abraham Robinson). He also proved that semantics of the generalized quantifier “there are uncountably many” can be reduced to model theory of the first order logic with an extra binary relation. In 1987 he published a book *An introduction to model theory* Univerzitet u Novom Sadu, Institut za Matematiku, Novi

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Sad, 1987. Alan Slomson wrote in the review: ... *This book would serve as an excellent text for an introductory course in model theory. A note in the book indicates that only 500 copies were produced. It is to be hoped that a way will be found to make it more widely available ...*

In the seventies only two serbian mathematicians published research papers in model theory; the other was Gradimir Vojvodić. In 1978 Vojvodić published a paper *Some theorems for model theory of mixed-valued predicate calculi* in Publications de l'Institut Mathématique (Beograd); there he extended some results of classical model theory to k -models for mixed-valued predicate calculi which were earlier introduced by Helena Rasiowa. In 1979 Vojvodić received a PhD degree from the University of Belgrade and the supervisor was again Slaviša Prešić. The thesis title was *A Contribution to Research of Mixed-Valued Predicate Calculi* (in Serbian).

After his return from Madison, Mijajlović started teaching courses at the Logic seminar and at the Faculty of Mathematics in Belgrade, mainly on model theory (on Boolean algebras, models of arithmetic and nonstandard analysis) and also on the theory of recursive functions. The courses lasted throughout the eighties and nineties, they were always well attended and they attracted several young mathematicians to mathematical logic. In the early eighties they were: Slobodan Vujošević (Heyting algebras and intuitionism), Aleksandar Jovanović (ultraproducts), Miodrag Rašković (nonstandard analysis), Rade Živaljević (nonstandard analysis) and Milan Grulović (model theory). They got their PhD degrees in the period 1982–1984 at the University of Belgrade and the thesis supervisor in all cases was Mijajlović. Grulović, who also spent a year as a graduate student in Madison, was interested in model-theoretical forcing. He introduced n -finite forcing which generalizes Abraham Robinson's finite forcing and proved analogous results; his PhD thesis title was *Forcing in Model Theory*, and it was defended in 1984. I would like to remark here that since then, no PhD thesis in pure model theory has been defended in Serbia.

In the late eighties and early nineties there was a large group of students attending Mijajlović's courses who became interested in mathematical logic; among them were: Žikica Perović (Boolean algebras), Harizanov Valentina (model theory, recursion theory), Aleksandar Ignjatović (formal arithmetic), Predrag Tanović (model theory), Djordje Čubrić (proof theory), Željko Sokolović (model theory, recursion theory), Ilijas Farah (set theory) and others. Almost all of them completed their Master's degree at the University of Belgrade (Mijajlović supervised most of them) and continued their graduate studies in North America where they got their PhD degrees; most of them still live and work there, and only Tanović returned back to Belgrade. It is worth noting that some of the mentioned later switched to other areas of mathematics; Jovanović switched completely to set theory, Živaljević to algebraic topology, Rašković switched to the study of probabilistic quantifiers and formed a large research group which is still very active; Sokolović and Čubrić switched to financial mathematics, then left research in mathematics and made careers in financial business.

Valentina Harizanov obtained a PhD in mathematics in 1987 at the University of Wisconsin in Madison under the direction of Terrence Millar; the thesis title was *Degree Spectrum of a Recursive Relation on a Recursive Structure*. Her main research interest is in computable model theory and computable algebra. She introduced the notion of degree spectra of relations on computable structures and obtained the first significant results concerning uncountable, countable and finite Turing degree spectra. She published several significant articles and book chapters on the subject, for example *Pure Computable*

Model Theory, in Handbook of Recursive Mathematics, vol. I, Yu. Ershov, S. Goncharov, A. Nerode, and J. B. Remmel (editors), North-Holland, Amsterdam, 1998.

Željko Sokolović continued his graduate studies at the University of Notre Dame, Indiana, USA in 1987. Working under the supervision of Anand Pillay he became interested in the model theory of differential fields. In 1992 he received a PhD degree and the thesis title was *Model Theory of Differential Fields*. The most disturbing open problem in the area at that time was to determine the number of countable models of DCF_0 (the theory of differentially closed fields of characteristic 0). Namely, it was known that DCF_0 is \aleph_0 -stable and not \aleph_0 -categorical and, in the early eighties, Shelah proved that any such theory has either \aleph_0 or 2^{\aleph_0} countable models. Thus it was known that DCF_0 has either \aleph_0 or 2^{\aleph_0} countable models, but the exact number was not known. In 1993 Sokolović and Ehud Hrushovski showed that the number is 2^{\aleph_0} and, in the author's opinion, this is the most outstanding result of a Serbian model theorist by now; a curiosity is that it has never been published! It circulates as a preprint: *Minimal subsets of differentially closed fields* (1992). There they completely characterized nontrivial strongly minimal sets definable in DCF_0 : those which essentially differ from the field of constants appear naturally in studying abelian varieties as differential algebraic groups. Then, inside a suitable chosen definable subgroup of an elliptic curve, they found a non-isolated strongly minimal type orthogonal to \emptyset ; by Shelah's results this suffices to conclude that the number is 2^{\aleph_0} . At that time it was really surprising that elliptic curves have something to do with the number of countable differential closed fields!

Predrag Tanović received a PhD from McGill University, Montreal, Canada in 1994 under the direction of James Loveys; the thesis title was *Fundamental Order and the Number of Countable Models*. His research interest is in countable first order structures. For example, he proved that there is no stable Ehrenfeucht theory with an infinite set of constants. Recently he published the first part of the proof of an old conjecture of Anand Pillay: every countably infinite first-order structure has infinitely many non-isomorphic countable elementary extensions: *Types directed by constants*, Annals of Pure and Applied Logic 2010.

At the end we give a list of selected papers which have been not mentioned by now:

Doctoral theses related to model theory in Serbia (all of them were defended at the University of Belgrade under supervision of Žarko Mijačlović):

- Slobodan Vujošević, A Contribution to the Theory of Heyting Algebras, 1982.
- Aleksandar Jovanović, A Contribution to the Theory of Ultraproducts, 1982.
- Miodrag Rašković, Logics with Measure in Leibniz's Universe, 1983.
- Rade Zivaljević, Ten Etides about Hyperfiniteness, 1983.

Selected papers of Žarko Mijačlović:

- Saturated Boolean algebras with ultrafilters, *Publications de l'Institut Mathématique (Beograd)* (N.S.) 26(40) (1979), pp.175–197.
- Regular relations and the quantifier 'there exists uncountably many', with V. Harizanova, *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik* 29 (1983), pp.151–161.
- Submodels and definable points in models of Peano arithmetic, *Notre Dame Journal of Formal Logic* 24 (1983), pp.417–425.
- On a proof of the Erdős Monk theorem, *Publications de l'Institut Mathématique (Beograd)* (N.S.) 37(51) (1985), pp.25–28.

- On the definability of the quantifier “there exist uncountably many”, *Studia Logica* 44(1985), pp.257–264.
- Hilbert Problems and Logic (in Serbian), Matematička Biblioteka 48. Zavod za Udbenike i Nastavna Sredstva, Belgrade, 1986.
- Definable ultrapowers and the omitting types theorem, *Publications de l’Institut Mathématique (Beograd)* (N.S.) 50(64) (1991), pp.14–18.

Selected papers of Milan Grulović:

- On n -finite forcing, *Zbornik radova prirodno matematičkog fakulteta Novi Sad* 13(1983), pp.405–421.
- On n -finite forcing companions, *Zbornik radova prirodno matematičkog fakulteta Novi Sad* 14(1984), pp.211–222.
- On reduced products of forcing systems, *Publications de l’Institut Mathématique (Beograd)* (N.S.) 41(55) (1987), pp.17–20.
- L_t -Horn sentences and reduced products, with M. Kurilić, *Publicationes Mathematicae Debrecen* 48(1996), pp.175–190.
- A comment on the joint embedding property, with A. Ignjatović, *Periodica Mathematica Hungarica* 33(1996), pp.45–50.

Selected papers of Valentina Harizanov:

- Uncountable degree spectra, *Annals of Pure and Applied Logic* 54(1991), pp.255–263.
- Some effects of Ash-Nerode and other decidability conditions on degree spectra, *Annals of Pure and Applied Logic* 55 (1991), pp.51–65.
- Turing degrees of certain isomorphic images of recursive relations, *Annals of Pure and Applied Logic* 93, 1998, pp.103–113.
- Effectively nowhere simple relations on computable structures, in the volume: Recursion Theory and Complexity, M.M. Arslanov and S.Lempp, editors; Walter de Gruyter, Berlin, 1999, pp.59–70.
- Computability-theoretic complexity of countable structures, *Bulletin of Symbolic Logic* 8(2002), pp.457–477.
- Turing degrees of hypersimple relations on computable structures, *Annals of Pure and Applied Logic* 121(2003), pp.209–226.
- Trivial, strongly minimal theories are model complete after naming constants, with S.Goncharov, M.Laskowski, S.Lempp, and C.McCoy, *Proceedings of the American Mathematical Society* 131(2003), pp.39013912.
- Simple and immune relations on countable structures, with S.S. Goncharov, J.F. Knight, and C.F.D. McCoy, *Archive for Mathematical Logic* 42(2003), pp.279–291.
- Turing degrees of nonabelian groups, with M.Dabkowska, M.Dabkowski, and A.Sikora, *Proceedings of the American Mathematical Society*, 135(2007), pp.3383–3391.
- Turing degrees of the isomorphism types of algebraic objects, with W.Calvert and A.Shlapentokh, *Journal of the London Mathematical Society* 73(2007), pp.273–286.
- Effective categoricity of Abelian p -groups, with W.Calvert, D.Cenzer, and A.Morozov, *Annals of Pure and Applied Logic* 159(2009), pp.187–197.
- Degree spectra of the successor relation on computable linear orderings, with J.Chubb and A.Frolov, *Archive for Mathematical Logic* 48(2009), pp.7–13.

Selected papers of Željko Sokolović:

- Superstable differential fields, with A.Pillay, *Journal of Symbolic Logic* 57(1992), pp.97–108.

- A remark on differential algebraic groups, with A.Pillay, *Communications in Algebra* 20(1992), pp.3015–3026.
- A Note On a -Prime Models, with B.Hart and P.Tanović, *Journal of Symbolic Logic* 64(1999), pp.1557–1562.
- Affine covers of Lie geometries and the amalgamation property, with A.Chowdhury, and B.Hart, *Proceedings of the London Mathematical Society* 85(2002), pp.513–563.

Selected papers of Predrag Tanović:

- Stable theories without dense forking chains, with B.Herwig, J.Loveys, A. Pillay and F.Wagner, *Archive for Mathematical Logic* 31(1992), pp.297–303.
- A definable continuous rank for nonmultidimensional superstable theories, with A.Chowdhury and J.Loveys, *Journal of Symbolic Logic* 61(1996), pp.967–984.
- Countable models of trivial theories which admit finite coding, *Journal of Symbolic Logic* 61(1996), pp.1279–1286.
- On the number of countable models of stable theories, *Fundamenta Mathematicae*, 169(2001), pp.139–144.
- On minimal ordered structures, *Publications de l'Institut Mathmatique (Beograd)* (N.S.), 78(92) (2005), pp.65–72.
- On constants and the strict order property, *Archive for Mathematical Logic*, 45(2006), pp.423–430.
- Non-isolated types in stable theories, *Annals of Pure and Applied Logic* 145(2007), pp.1–15.
- Theories with constants and three countable models, *Archive for Mathematical Logic*, 46(2007), pp.517–527.
- Asymmetric RK-minimal types, *Archive for Mathematical Logic* 49(2010), pp.367–377.

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