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## OVERVIEW OF AUTOMATED REASONING IN SERBIA

### 1. Introduction

The main goals of automated reasoning are understanding different aspects of reasoning and development of algorithms and computer programs that solve problems requiring reasoning. Some subareas of automated reasoning are automated theorem proving, interactive (or formal) theorem proving, automated proof checking, etc. Automated reasoning typically combines results and techniques of mathematical logic, theoretical computer science, algorithmics and artificial intelligence. Logics of interest for automated reasoning include propositional, first-order, equational, higher-order, description, modal, temporal, many-valued, intuitionistic logic, etc. Some of the methods used in automated reasoning are resolution, tableaux, term rewriting, decision procedures, model checking, induction, unification, proof checking, etc. Automated reasoning has applications in software and hardware verification, circuit design, logic programming, ontology reasoning, deductive databases, mathematical software, educational software, robotics, planning and other areas of artificial intelligence. The key modern international forum for automated reasoning is *Association for Automated Reasoning* (AAR).<sup>1</sup> The major conferences for automated reasoning are *Conference on Automated Deduction* (CADE),<sup>2</sup> and *International Joint Conference on Automated Reasoning* (IJCAR).<sup>3</sup> The major journal for automated reasoning is *Journal of Automated Reasoning* (JAR).<sup>4</sup> Outstanding contributions in the field are honored by *Herbrand Award for Distinguished Contributions to Automated Reasoning*.

The history of automated reasoning in Serbia is more than twenty years long. Like in the rest of the world, research in this area is mainly carried out by mathematicians and informaticians, especially those involved in mathematical logic. Most of this research focuses on uniform proof procedures (such as tableaux and resolution methods), decision procedures in automated theorem proving, automated theorem proving in geometry, probabilistic and fuzzy reasoning, and interactive theorem proving. In the rest of this paper, we briefly summarize results achieved in this field in Serbia. There are several PhD and MSc theses in the area of automated reasoning defended in Serbia, but this overview does

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<sup>1</sup><http://www.aarinc.org/>

<sup>2</sup><http://www.cadeinc.org/>

<sup>3</sup><http://www.ijcar.org/>

<sup>4</sup><http://www.springerlink.com/content/100280/>

not list them and, instead, focuses on results presented at research conferences and on papers published in research journals (visible by Internet services such as DBLP<sup>5</sup>). The overview is by no means exhaustive and omissions are possible. In the following text, only publications by Serbian authors are cited (and not the other relevant publications). The home institutions are given for all mentioned authors (either from Serbia or not). The following abbreviations are used for Serbian institutions: EEUBg — School of Electrical Engineering, University of Belgrade; MIBg — Mathematical Institute, Belgrade; MathUBg — Faculty of Mathematics, University of Belgrade; TFZUNS — Technical Faculty „Mihajlo Pupin“ Zrenjanin, University of Novi Sad; FSUKg — Faculty of Science, University of Kragujevac.

## 2. Uniform Proof Procedures

During 1980's, a team led by Dragoš Cvetković (EEUBg, MIBg) developed an interactive system GRAPH for graph theory. The system consisted of several components, including a theorem prover. The theorem prover (working in first order logic) provided support for interactive theorem proving (in natural deduction) and for automated theorem proving (based on the resolution method) [3].

Aleksandar Krapež, Miodrag Kapetanović, Zoran Ognjanović and Tatjana Petrović (all from MIBg), developed a parallel theorem prover PROVER91, based on the tableaux method [18]. Zoran Ognjanović, Dragan Urošević, Tatjana Petrović and Zorana Ristović (all from MIBg) developed a tableaux-based method for modal theorem proving [35], later extended by Zoran Ognjanović [31].

Slaviša Prešić (MathUBg) showed how logic programming could be implemented over arbitrary (not necessarily Horn) clauses [36]. Far beyond this paper, Slaviša Prešić, with his seminal role in development of mathematical logic in Serbia and his broad interests in artificial intelligence, influenced a lot of research and many researches working in the area of automated reasoning.

During 2000s, Petar Hotomski (TFZUNS) and his collaborators have been developing an automated theorem prover ATP based on ordered linear resolution and its extension BASELOG, a system for the deductive databases. These systems were used in practical problems like program verification and scheduling [2, 26, 38].

## 3. Decision Procedures, SAT and SMT

Žarko Mijajlović (MathUBg) constructed a decision procedure for monadic calculus [27] that was later implemented by a team led by Aleksandar Jovanović (MathUBg) [25].

Predrag Janičić (MathUBg), Ian Green (University of Edinburgh) and Alan Bundy (University of Edinburgh) developed a generic framework GS that facilitates implementing, combining, augmenting decision procedures and using them in theorem provers [11, 9]. Portions of this system were used in CLAM and LAMBDA CLAM proof planning systems. Along this line of research, Mateja Jamnik (University of Cambridge), Predrag Janičić and Alan Bundy developed approaches for automated synthesis of certain classes of decision procedures [5, 6, 10].

Filip Marić (MathUBg) and Predrag Janičić implemented a solver ARGOLIB for the *satisfiability modulo theory* problem (SMT), based on the DPLL(T) scheme and with support for several decidable theories and their combinations [21]. The ARGOLIB system successfully took part in the 2007 SMT competition and is used by research groups in

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<sup>5</sup><http://www.informatik.uni-trier.de/~ley/db/>

several countries. Filip Marić has also been developing a solver for the propositional satisfiability problem (SAT) ARGOSAT, with a flexible architecture. Both ARGOLIB and ARGOSAT are under constant development and publicly available.<sup>6</sup> Within a project of developing a successor of ARGOLIB — a system ARGOSMT — Milan Banković (MathUBg) and Filip Marić constructed and implemented a new SMT solver for the theory *all-different* [1].

Mladen Nikolić (MathUBg), Filip Marić and Predrag Janičić developed a methodology for instance-based selection of policies for SAT solvers (as an alternative to fixed policies used for all input instances) and implemented a corresponding SAT solver ARGOSMART [30]. Mladen Nikolić developed a statistically founded methodology for comparison of SAT solvers, as an alternative to the methodology currently used at SAT competitions [29].

There are applications of SAT and SMT solvers in a range of domains: Dejan Jovanović (MathUBg) and Predrag Janičić used a SAT solver for solving a class of cryptographic problems [17], Milena Vujošević-Janičić (MathUBg), Filip Marić and Dušan Tošić (MathUBg) used SMT solvers in software verification [42, 43], while Filip Marić and Predrag Janičić developed a system for uniform reduction of a wide class of constraint satisfaction problems to the theory of bitvector arithmetic [24].

#### 4. Automated Reasoning in Geometry

Predrag Janičić and Stevan Kordić (MathUBg) developed a first automated geometry theorem prover (EUCLID) based on coherent logic [12]. The prover generates human-readable, traditional proofs of conjectures in Euclidean geometry. Sana Stojanović (MathUBg), Vesna Pavlović (MathUBg) and Predrag Janičić later reimplemented and improved this prover, so the new version (ARGOCLP) can be used for any coherent theory and can export machine-verifiable proofs [41].

Since 1996 Predrag Janičić has been developing, in some parts with his collaborators, a geometry system GCLC<sup>7</sup> — a tool for visualizing geometrical objects and reasoning about them [7, 8]. There are three geometry theorem provers built in GCLC, all capable of efficiently proving hundreds of complex geometry theorems: a theorem prover based on the area method, implemented by Predrag Janičić and Pedro Quaresma (University of Coimbra) [14, 15, 37] and theorem provers based on the Gröbner bases method and on Wu's method, implemented by Goran Predović (MathUBg) within his master thesis supervised by Predrag Janičić. The tool GCLC has thousands of users and is taught in a number of high-school and university courses worldwide.

Predrag Janičić, Julien Narboux (University of Strasbourg) and Pedro Quaresma made a rational reconstruction of the area method (one of the most significant methods for automated deduction in geometry) and formally proved its properties within the Coq proof assistant [13].

#### 5. Probabilistic and Fuzzy Reasoning

Miodrag Rašković (FSUKg, MIBg), Zoran Marković (MIBg) and Zoran Ognjanović with their collaborators have been working on probabilistic logics and probabilistic temporal logics and on their applications in default reasoning and in reasoning about evidence [40, 39, 4].

Zoran Ognjanović and his collaborators used genetic algorithms [32, 33] and approaches based on the variable neighborhood search metaheuristic [16, 34] for solving

<sup>6</sup><http://argo.matf.bg.ac.rs>

<sup>7</sup><http://www.matf.bg.ac.rs/~janicic/gclc>

the probabilistic satisfiability problem (PSAT). The approaches used were able to solve largest PSAT problem instances reported in the literature at that time.

Branka Nikolić (TFZUNS) and Petar Hotomski analyzed and compared fuzzy reasoning using H-logical norms and some t-norms [28].

## 6. Interactive Theorem Proving

Filip Marić has been working (in some parts with Predrag Janičić) on formal verification (using interactive, formal theorem proving) of SAT solvers with a goal of producing solvers that are both efficient and fully trusted, which is critical in many applications [22]. This work includes proving correctness of the classical DPLL procedure [23] and verification of modern SAT solvers using three paradigms: Hoare-style verification of an imperative implementation of a solver [19], verification of a solver defined as a set of recursive functions within higher order logic [20] and verification of abstract state transition systems describing a SAT solver. These verification tasks are accompanied with the solver ARGOSAT, with a trusted, automatically generated solver implemented in a functional language and with formal, machine-verifiable proofs constructed by Filip Marić within the Isabelle proof assistant. Given the complexity of SAT solvers and the level of algorithmic details considered, these proofs are major contributions not only in verification of SAT solvers, but also in the wider area of interactive theorem proving and in the area of software verification.

## 7. Activities Related to Automated Reasoning

There is a number of conferences and seminars in Serbia that often host lectures related to automated reasoning, but here we briefly mention only those focused on this research field. Since 2008, the Automated Reasoning GrOuP<sup>8</sup> (ARGO) (MathUBg) organizes and hosts annual international workshop on interactive and automated theorem proving with applications. The same research group organizes a regular seminar on automated reasoning since October 2007.

## 8. Automated Reasoning in Serbian Diaspora

There is a number of researchers originating from Serbia involved in research in automated reasoning in other countries. Most of them obtained their first university degree in Serbia and higher degrees abroad. Here we list some of the researchers (with doctorates) from Serbia actively involved in automated reasoning (without details about their publications): Sava Krstić (Intel Corporation, USA; area: SMT solving), Viktor Kunčak (EPFL, Switzerland; area: SMT solving, program analysis and verification), Ranko Lazić (University of Warwick, UK; area: automata, games, software model checking, infinite-state systems), Maja Miličić (Dresden University of Technology, Germany; area: description logics), Dejan Ničković (Institute of Science and Technology, Austria; area: timed automata and real-time logics).

## 9. Conclusions

Automated reasoning has been attracting a lot of research efforts in Serbia over the last decades and with increasing trends. There are tens of publications in the areas of automated reasoning, including papers presented at leading international conferences and papers published in leading international journals. Apart from researchers working in

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<sup>8</sup><http://argo.matf.bg.ac.rs>

Serbia, there is also a number of researchers originating from Serbia involved in automated reasoning. Maintaining their mutual links gives further promising prospects for the future of automated reasoning in Serbia.

## References

- [1] Milan Banković, Filip Marić: Alldifferent Constraint Solver in SMT. *Proceedings of 8th International Workshop on Satisfiability Modulo Theories*, 2010.
- [2] Ivana Berković, Biljana Radulović, Petar Hotomski: Extensions of Deductive Concept in Logic Programming and Some Applications. *Proceedings of 2nd International Multiconference on Computer Science and Information Technology*, 2007.
- [3] Dragoš Cvetković, Irena Pevac: Man-Machine Theorem Proving in Graph Theory. *Artificial Intelligence*, 35(1), 1988.
- [4] Dragan Doder, Zoran Marković, Zoran Ognjanović, Aleksandar Perović, Miodrag Rašković: A Probabilistic Temporal Logic That Can Model Reasoning about Evidence. *Proceedings of 6th International Symposium on Foundations of Information and Knowledge Systems*, 2010.
- [5] Mateja Jamnik, Predrag Janičić, Learning Strategies for Mechanised Building of Decision Procedures. *Electronic Notes on Theoretical Computer Science*, 86(1), 2003.
- [6] Mateja Jamnik, Predrag Janičić, Can decision procedures be automatically learnt?, *Proceedings of 4th International Workshop on First order Theorem Proving*, 2003.
- [7] Predrag Janičić: GCLC – A Tool for Constructive Euclidean Geometry and More than That. *Proceedings of 2nd International Congress of Mathematical Software*, LNCS 4151, Springer, 2006.
- [8] Predrag Janičić: Geometry Constructions Language. *Journal of Automated Reasoning*, 44(1-2), 2010.
- [9] Predrag Janičić, Alan Bundy: A General Setting for Flexibly Combining and Augmenting Decision Procedures. *Journal of Automated Reasoning*, 28(3), 2002.
- [10] Predrag Janičić, Alan Bundy: Automatic Synthesis of Decision Procedures: a Case Study of Ground and Linear Arithmetic. *Proceedings of 14th Symposium Calculemus: Towards Mechanized Mathematical Assistants*, LNCS 4573, Springer, 2007.
- [11] Predrag Janičić, Alan Bundy, Ian Green: A Framework for the Flexible Integration of a Class of Decision Procedures into Theorem Provers. *Proceedings of 16th Conference on Automated Deduction*, LNAI 1632, Springer, 1999.
- [12] Predrag Janičić, Stevan Kordić: EUCLID — the geometry theorem prover. *FILOMAT*, 9(3), 1995.
- [13] Predrag Janičić, Julien Narboux, Pedro Quaresma: The Area Method: A Recapitulation. *Journal of Automated Reasoning*, to appear, 2011.
- [14] Predrag Janičić, Pedro Quaresma: System Description: GCLCprover + GeoThms. *Proceedings of 3rd International Joint Conference on Automated Reasoning*, LNCS 4130, Springer, 2006.
- [15] Predrag Janičić, Pedro Quaresma: Automatic Verification of Regular Constructions in Dynamic Geometry Systems. *Proceedings of 6th International Workshop on Automated Deduction in Geometry*, LNCS 4869, Springer, 2007.
- [16] Dejan Jovanović, Nenad Mladenović, Zoran Ognjanović: Variable Neighborhood Search for the Probabilistic Satisfiability Problem. *Metaheuristics: Progress in Complex Systems Optimization*, Operations Research Series 39, Springer, 2007.
- [17] Dejan Jovanović, Predrag Janičić: Logical Analysis of Hash Functions. *Proceedings of 5th International Workshop on Frontiers of Combining Systems*, LNCS 3717, Springer, 2005.
- [18] Aleksandar Krapež, Miodrag Kapetanović, Zoran Ognjanović, Tatjana Petrović: Prover 91 — A Parallel Theorem Prover. *Proceedings of 1st International Conference on Analytic Tableaux and Related Methods — TABLEAUX*, 1992.
- [19] Filip Marić. Formalization and Implementation of SAT solvers. *Journal of Automated Reasoning*, 43(1), 2009.
- [20] Filip Marić: Verification of a Modern SAT Solver by Shallow Embedding into Isabelle/HOL. *Theoretical Computer Science*, 411(50), 2010.
- [21] Filip Marić, Predrag Janičić. ARGO-LIB: A generic platform for decision procedures. *Proceedings of 2nd International Joint Conference on Automated Reasoning*, LNAI 3097, Springer, 2004.
- [22] Filip Marić, Predrag Janičić: SAT Solver Verification Project. *Proceedings of 22nd International Conference on Theorem Proving in Higher Order Logics: Emerging Trends*, 2009.

- [23] Filip Marić, Predrag Janičić: Formal Correctness Proof for DPLL Procedure. *Informatica*, 21(1), 2010.
- [24] Filip Marić, Predrag Janičić: URBiVA: Uniform Reduction to Bit-Vector Arithmetic, Proceedings of *5th International Joint Conference on Automated Reasoning*. LNCS 6173, Springer, 2010.
- [25] Filip Marić, Miroslav Marić, Žarko Mijajlović, Aleksandar Jovanović: Theorem Provers Based On The Quantifier Elimination Method. Proceedings of *47th Conference ETRAN*, 2003.
- [26] Branko Markoski, Petar Hotomski, Dušan Malbaški, Danilo Obradović: Resolution Methods In Proving The Program Correctness. *Yugoslav Journal Of Operations Research*, 17(2), 2007.
- [27] Žarko Mijajlović: Decidable theories (in Serbian), *Računarstvo*, 1(1), 1991.
- [28] Branka Nikolić, Petar Hotomski: Comparison of  $H$ -Logical Norm with some  $t$ -Norms. *Informatica*, 14(4), 2003.
- [29] Mladen Nikolić: Statistical Methodology for Comparison of SAT Solvers. *13th International Conference on Theory and Applications of Satisfiability Testing*, LNCS 6175, Springer, 2010.
- [30] Mladen Nikolić, Filip Marić, Predrag Janičić: Instance-Based Selection of Policies for SAT Solvers. Proceedings of *12th International Conference on Theory and Applications of Satisfiability Testing*, LNCS 5584, Springer, 2009.
- [31] Zoran Ognjanović: A tableau-like proof procedure for normal modal logics. *Theoretical Computer Science*, 129(1), 1994.
- [32] Zoran Ognjanović, Jozef Kratica, Miloš Milovanović: A Genetic Algorithm for Satisfiability Problem in a Probabilistic logic: A First Report. Proceedings of *6th European Conference on Symbolic and Quantitative Approaches to Reasoning with Uncertainty*, LNCS 2143, Springer, 2001.
- [33] Zoran Ognjanović, Uroš Midić, Jozef Kratica: A Genetic Algorithm for Probabilistic SAT Problem, Proceedings of *7th International Conference on Artificial Intelligence and Soft Computing*, LNCS 3070, Springer, 2004.
- [34] Zoran Ognjanović, Uroš Midić, Nenad Mladenović: A Hybrid Genetic and Variable Neighborhood Descent for Probabilistic SAT Problem. Proceedings of *2nd International Workshop on Hybrid Metaheuristics*, LNCS 3636, Springer, 2005.
- [35] Zoran Ognjanović, Dragan Urošević, Tatjana Petrović, Zorana Ristović: A Tableaux Related Method for Modal Theorem Proving. Proceedings of *1st International Conference on Analytic Tableaux and Related Methods — TABLEAUX*, 1992.
- [36] Slaviša Prešić: How to Generalize Logic Programming to Arbitrary set of Clauses. *Publications de l'Institut Mathématique*, 61(75), 1997.
- [37] Pedro Quaresma, Predrag Janičić: Integrating Dynamic Geometry Software, Deduction Systems and Theorem Repositories. Proceedings of *5th International Conference on Mathematical Knowledge Management*, LNCS 4108, Springer, 2006.
- [38] Biljana Radulović, Ivana Berković, Petar Hotomski, Zoltan Kazi: The Development of Baselog System and Some Applications. *International Review on Computers and Software*, July 2008.
- [39] Miodrag Rašković, Zoran Marković, Zoran Ognjanović: A Logic with Approximate Conditional Probabilities that Can Model Default Reasoning. *International Journal of Approximate Reasoning*, 49(1), 2008.
- [40] Miodrag Rašković, Zoran Ognjanović, Zoran Marković: A Probabilistic Approach to Default Reasoning. Proceedings of *10th International Workshop on Non-Monotonic Reasoning*, 2004.
- [41] Sana Stojanović, Vesna Pavlović, Predrag Janičić: Automated Generation of Formal and Readable Proofs in Geometry Using Coherent Logic. Proceedings of *8th Workshop on Automated Deduction in Geometry*, 2010.
- [42] Milena Vujošević-Janičić: Ensuring Safe Usage of Buffers in Programming Language C. Proceedings of *3rd International Conference on Software and Data Technologies*, 2008.
- [43] Milena Vujošević-Janičić, Filip Marić, Dušan Tošić: Using Simplex Method in Verifying Software Safety. *Yugoslav Journal of Operations Research*, 19(1), 2009.

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