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THESES IN ALGEBRA DEFENDED AT SERBIAN UNIVERSITIES

Abstract. The theses in the field of algebra, defended at Serbian universities, which may be found in the digital archive of the Faculty of Mathematics in Belgrade are presented. A brief overview of all theses defended at Serbian universities is also given.

1. Introduction

The aim of this article is to give a short presentation of doctoral theses in the field of algebra, defended at one of the Serbian universities, which are available at the Virtual library of the Faculty of Mathematics in Belgrade. The presentation is organized according to subfields in order to help the reader find what interests him or her. Mathematics genealogy project has also been used in order to determine some further details (especially the advisor) concerning these theses.

Since not all of the theses are available in the electronic form, a brief overview of all theses in the form of several tables is given here. Again, Mathematics genealogy project was used in order to get necessary information.

Let us look at the subject areas first.

	Subject area	Cardinality
1.	Semigroups	13
2.	Quasigroups	6
3.	Groups	2
4.	Rings	2
5.	Fields	1
6.	Numbers	4
7.	Equations and algebraic geometry	5
8.	General algebraic structures	10
	Total	43

The next table shows the distribution of all theses through our university centers.

The author would like to thank the organizers of the conference emphHistory of logic for a splendid conference.

	Center	Cardinality
1.	Belgrade	27
2.	Novi Sad	12
3.	Niš	2
4.	Priština	2

The following table gives distribution according to the time period. One can see that the peak is during the 80s, with a big fall during the 90s but the previous decade also shows an improvement.

	Years	Cardinality
1.	60s	4
2.	70s	8
3.	80s	16
4.	90s	5
5.	2000s	10

The next table shows which subject areas were dominant in certain decades. We can see that semigroups were the most dominant area both in the number of theses and in the length of the period.

	Years	Area
1.	70s	Quasigroups
2.	80s	Semigroups
3.	90s	Semigroups
4.	2000s	General algebraic structures

Finally, we present the top centers. We can see that Novi Sad emerged in the previous decade as the center with the highest number of theses defended in the field of algebra.

	Years	Center
1.	70s	Belgrade
2.	80s	Belgrade
3.	90s	Belgrade
4.	2000s	Novi Sad

2. Semigroups

A contribution to the theory of regular semigroups, **Stojan Bogdanović**, University of Novi Sad, 1980 (advisor: Svetozar Milić). This thesis has 141 pages. It consists of a brief introduction, 4 chapters and the bibliography with 63 references.

Chapter one, *Elementary notions and characterizations of semigroups from some classes of regular semigroups*, contains mainly background material on those classes of semigroups which are treated in the rest of the thesis.

Chapter two, *Generalized ideals*, treats (m, n)-ideals, $(m, n)^*$-ideals and (m, n)-regular semigroups. At the end of this chapter π-semigroups and homogroups are characterized by the $(m, n)^*$-ideals.

Chapter three, *Weakly commutative semigroups*, deals primarily with weakly commutative semigroups. A semigroup S is weakly commutative if and only if

$$(\forall a \in S)(\forall b \in S)(\exists x \in S)(\exists y \in S)(\exists n \in \mathbb{N})\,(ab)^n = xa = by.$$

The notions of semiprimary semigroups, r-semigroups and r-semiprimary semigroups are introduced and various connections between these classes of semigroups have been established. Also, in this chapter one finds various results concerning Archimedean weakly commutative semigroups and regular weakly commutative semigroups. Nilpotent semigroups as well as semilattices of groups have also been characterized in this chapter.

Chapter four, (m,n)-*anti-inverse semigroups*, is devoted to the investigation of some generalizations of anti-inverse semigroups. Some results concerning decompositions of (m,n)-anti-inverse semigroups are given. Green relations have also been considered, as well as an algorithm for the determination which classes $S_{m,n}$ are subclasses of anti-inverse semigroups. At the end, some special cases of (m,n)-semigroups are given.

Some classes of semigroups, **Siniša Crvenković**, University of Novi Sad, 1981 (advisor: Svetozar Milić). This thesis has 102 pages. It consists of a brief introduction, four chapters and the bibliography with 44 references.

Chapter one, *Elementary notions and characterizations of semigroups from some classes of semigroups*, mainly contains some background material.

Chapter two, $(m,n)^*$-*anti-inverse semigroups*, contains some results concerning decompositions of $(m,n)^*$-anti-inverse semigroups, which form a subclass of the class of all completely regular semigroups. A semigroup S is$(m,n)^*$-anti-inverse semigroup if

$$(\forall x \in S)(\exists y \in S)(x^m = y^m \wedge yx = x^{m+1}y \wedge x^n = x).$$

The Green relations have also been discussed and some characterizations of semigroups from the class $S^*_{m,n}$ have been obtained.

Chapter three, *Bases classes of some classes of semigroups*, contains, among other things, an algorithm for the determination of the basis class of any class of $(m,n)^*$-anti-inverse semigroups. Some examples are also given as well as the discussion of the determination of bases classes for some more general classes of semigroups.

Chapter four, *Subalgebras of semilattices*, deals with subalgebras of semilattices. It contains a necessary and sufficient condition for an algebra to be a subalgebra of a semilattice.

A contribution to the theory of regular semigroups, **Dragica Krgović**, University of Belgrade, 1982 (advisor: Mario Petrich). This thesis has 75 pages. It consists of a brief introduction, 4 chapters and the bibliography with 50 references.

Chapter one, *Some known notions and results of semigroup theory*, contains background material.

Chapter two, *Some characterizations of regular, intraregular and* (m,n)*-regular semigroups*, gives some characterizations of the classes of semigroups listed in the title by using ideals and their appropriate generalizations. A semigroup S is (m,n)-regular if

$$(\forall a \in S)(\exists x \in S)a^m x a^n = a.$$

Chapter three, *0-minimal bi-ideals and completely* 0*-simple semigroups*, in particular, contains a characterization of completely 0-simple semigroups and completely 0-simple ideals by 0-minimal ideals.

Chapter four, *On the problem of bi-ideal extension*, contains some results concerning the problem of bi-ideal extensions: given a semigroup S and a semigroup with zero Q, find all semigroups V containing a bi-ideal S' such that $S' \cong S$ and $V/S' \cong Q$.

Characterizations of some classes of semigroups by using subsemigroups, **Todor Malinović**, University of Novi Sad, 1986. This thesis has 161 pages and it consists of a brief introduction, 4 chapters and bibliography with 68 references. It also contains an index at the end.

Chapter one, *Elementary notions and characterizations of semigroups from some classes of π-regular semigroups*, contains previously known material on semigroups in general and regular and π-regular semigroups in particular.

Chapter two, *Characterizations of some classes of semigroups*, begins with some results on maximal ideals in semigroups. Next, some generalizations of characterizations of semigroups by using π-regular ideals and periodic semigroups by using proper π-regular subsemigroups. The notion of the strict left (right) regular semigroup is introduced and this class of semigroups is described with the help of left (right) unitary ideals. Also, a generalization of some previous results concerning those semigroups whose all subsemigroups are t-archimedean is also presented in this chapter.

Congruences on π-regular semigroups, **Petar Protić**, University of Novi Sad, 1986. This thesis has 147 pages and it consists of a brief introduction, 6 chapters and bibliography with 53 references.

Chapter one, *Elementary notions and π-regular semigroups*, contains background material on semigroups in general and on regular and π-regular semigroups in particular. A semigroup S is π-regular if

$$(\forall a \in S)(\exists m \in \mathbb{N})(\exists x \in S)a^m x a^m = a^m.$$

Chapter two, *Congruences, homomorphisms, idempotents and inverses on π-regular semigroups*, starts with a generalization of the Lallement lemma and introduction of the notion of r-congruences and r-homomorphisms. It is proved that on π-regular semigroups there is a bijective correspondence between these two notions.

Chapter three, *Some congruences on strictly π-inverse semigroup*, is mainly devoted to the study of r-semigroups.

Chapter four, *Congruences which separates idempotents on a π-regular semigroup*, is devoted to the study of those congruences on π-regular semigrous which separate idempotents. Certain relations which generalize the well-known Green relations are extensively used here. Among other things, the description of the biggest and the smallest congruences which separate idempotents is given in this chapter.

Chapter five, *Group congruences and the lattice of group congruences on an r-semigroup*, is devoted to the study of group congruences on π-regular semigroups. It turns out that the group r-semisimple congruences on an r-semigroup form a lattice and a description of this lattice is given.

Chapter six, *Congruences equivalent on idempotents*, is, among other things, devoted to the following. If U is the smallest selfconjugate subsemigroup of a r-semigroup S, which contains the set of idempotents E, and τ a normal congruence on U, then the r-semisimple congruence on S which on the set E induced the same decomposition as τ is described.

Structure of some classes of regular semigroups, **Dragan Blagojević**, University of Belgrade, 1987. (advisor: Svetozar Milić) This thesis has 67 pages and it consists of a brief introduction, 6 chapters and bibliography with 61 references.

Chapter 0, *Some basic notions and results*, contains background material.

Chapter 1, *Unions of dihedral groups*, contains a detailed description of the unions of dihedral groups.

Chapter 2, *Anti-inverse semigroups*, builds on the results of previous investigations by other authors and gives the more precise characterization.

Chapter 3, *Some generalizations of anti-inverse semigroups*, completely solves the previous problems concerning some generalizations of anti-inverse semigroups investigated by Milić, Bogdanović and Crvenković.

Chapter 4, *Anti-inverse unions of dihedral groups*, is devoted to the search for anti-inverse unions of dihedral groups which are not Boolean. It has been shown that the smallest one contains 32 elements and this semigroup has been completely described.

Chapter 5, *Free regular orthocryptogroup*, gives the description of the free regular orthocryptogroup and its construction in terms of ordered triples.

Structural properties of extensions of some classes of semigroups, **Blagoje Stamenković**, University of Belgrade, 1989. (advisor: Branka Alimpić) This thesis has 122 pages and it consists of a brief introduction, 4 chapters and bibliography with 58 references.

Chapter one, *Introductory notions and results*, contains background material.

Chapter two, *Semigroups in which S^{n+1} is a completely simple semigroup*, begins with a comparison between two different notions of an n-inflation. The notion of a L_n-semigroup is also introduced here and some subclasses of these semigroups have been discussed.

Chapter three, *Semigroups in which S^{n+1} is a semilattice of right groups*, investigates those semigroups S such that S^{n+1} is a semillatice of right groups ($n \in \mathbb{N}$). S is a right group if

$$(\forall x \in S)(\forall a \in S)x \in aSx.$$

Chapter four, *Some congruences on $\mathcal{L}^*$-unipotent semigroups*, introduces the notion of a r-cancelative semigroup which need not be regular. The natural partial order is introduced on such semigroups. Also, the notion of a L^*-unipotent semigroups is introduced and their properties were investigated.

Chapter three, *(m,n)-two-sided (one-sided) clean semigroups*, starts with the notion of a (m,n)-two-sided (one-sided) clean semigroup. A subset B of a semigroup S is (m,n)-two-sided clean if for all $x_1,\ldots,x_m,y_1,\ldots,y_n$ one has

$$B \cap x_1 \cdots x_m S y_1 \cdots y_n = x_1 \cdots x_m B y_1 \cdots y_n.$$

A semigroup S is (m,n)-two-sided clean if every bi-ideal in S is two-sided clean subset of S. There are two main results in this chapter. Firstly, a characterization of (m,n)-two-sided clean semigroups by subsemigroups which are groups. Secondly, (m,n)-one-sided clean semigroups are characterized in terms of subsemigroups which are π-groups.

Chapter four, *Congruences on some π-regular semigroups*, starts with a characterization of inverse congruences on a π-orthodox semigroup. Later $\mathcal{L}$-unipotent congruences on a generalized strictly π-inverse semigroups are given. At the end, semilattice and group congruences on (m,n)-two-sided (one-sided) clean semigroups are discussed.

Medial semigroups and their generalizations, **Abdullah Zejnullahu**, University of Priština, 1989. (advisor: Tomaš Kepka) This thesis has 82 pages. It consists of 4 chapters and the bibliography with 36 references and a short biography of the author.

In chapter zero, *Introduction*, the well-known facts about semigroups and varieties of semigroups are stated. Chapter one, *Medial and semi-medial semigroups*, is devoted to the study of medial semigroups and their generalizations. A semigroup S is medial if for all x, a, b, y in S one has: $xaby = xbay$. A semigroup S is left (right) semi-medial if for all $a, b, c \in S$ one has: $aabc = abac$ ($abcc = acbc$). Some results concerning decompositions of (semi-)medial semigroups are given, as well as some discussion concerning some subclasses of semi-medial semigroups.

Chapter two, *Left distributive semigroups*, is devoted to the study of this class of semigroups. A semigroup S is left distributive if for all $x, y, z \in S$ one has: $xyz = xyxz$. One defines right distributive semigroups analogously. Various results concerning (sub)direct decompositions and free left distributive semigroups are given.

Chapter three, *Distributive semigroups*, is devoted to the study of distributive semigroups (those semigroups which are both left and right distributive). Results concerning decompositions, free distributive semigroups, etc. are given in this concluding chapter.

A contribution to the theory of translational hull of semigroups, **Sadri J. Shkodra**, University of Priština, 1990. This thesis has 72 pages. It consists of a brief introduction, three chapters and the bibliography with 23 references. At the end, the brief summary in both English and Albanian is given as well as a short biography of the author.

Chapter one, *Some known notions and results from the theory of semigroups and translational hull*, contains background material from the semigroup theory in general and the translational hull in particular. If S is a semigroup and $\lambda : S \to S$ is such that for all $x, y \in S$ one has: $\lambda(xy) = \lambda(x)y$, then λ is called a left translation of semigroup S. One defines right translation analogously and if λ is a left translation and ρ is a right translation, one calls the pair (λ, ρ) bitranslation. It is possible to define operation on the set of all bitranslations and with this operation that set $\Omega(S)$ becomes a semigroup and this semigroup is the translational hull of the semigroup S.

Chapter two, *Translational hull of a semigroup*, begins with an example which shows that the translational hull of a band need not be a band. The same example shows that if the semigroup is the union of groups, the translational hull need not be so. Later, the characterization of the center $C(\Omega(S))$ is given for a class of semigroups and this result is used to prove that for such semigroups $C(\Omega(S))$ embedds into $\Omega(C(S))$. A sufficient condition for $C(\Omega(S)) \cong \Omega(C(S))$ is also given.

Chapter three, *Representation of an inverse semigroup and an embedding of the translational hull of a semigroup into an inverse semigroup*, begins with the relation between the notion of ideal levels and Green relations. At the end of this chapter it is proved that the translational hull of a semigroup may be embedded into a group if the original semigroup may be embedded into a group.

3. Quasigroups

A contribution to the theory of quasigroups, **Svetozar Milić**, University of Belgrade, 1971. (advisor: Slaviša Prešić) This thesis has 70 pages and it consists of a brief introduction, 4 chapters and bibliography with 52 references.

Chapter one, *Some definitions, notation and known results from the theory of quasigroups*, is mostly reserved for the necessary notation and recollection of known results concerning quasigroups which are needed in the subsequent chapters.

Chapter two, *On a class of quasigroup operations of associative type*, contains discussion of various systems of quasigroups satisfying different algebraic laws of special type. For some of these cases it has been proved that these quasigroups are isotopic to some group. The method of proof may also be used for laws not necessarily of associative type.

Chapter three, *On modular systems of n-quasigroups*, is devoted to the discussion of the generalized (i, j)-modular systems of n-quasigroups. It has also been proved that the quasigroups satisfying all (i, j)-modular laws are of a very simple kind — they all come from some Abelian group.

Chapter four, *Some results from the theory of UD-groupoids with applications to quasigroups*, is mainly devoted to the investigation of generalized groupoids with division which satisfy a balanced algebraic law. Some of the results proved in this chapter are convenient for application for solving functional equation of general associativity. Some examples are also presented.

On a class of quasigroups, **Janez Ušan**, University of Belgrade, 1971. (advisor: Slaviša Prešić) This thesis has 77 pages and it consists of a brief introduction, 3 chapters and bibliography with 41 references.

Chapter one, *Background results*, needs no explanation.

Chapter two, *n-ary quasigroups and general n-ary associativity*, An n-ary quasigroup is a n-ary groupoid (Q, A), where A is a n-ary operation on Q, such that there exist all inverse operations of the operation A, i. e. the equations

$$A(a_1, \ldots, a_{i-1}, x_i, a_{i+1}, \ldots, a_n) = b,$$

have unique solution for every $a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_n, b \in Q$ and all i such that $1 \leq i \leq n$. The well-known Belousov theorem on four quasigroups is generalized to the case of ternary quasigroups and this generalization is the cornerstone result of this chapter. The generalization of this to the case of n-ary quasigroups is also presented in this chapter along with various related results.

Chapter three, *Associative in the whole systems of ternary quasigroups*, begins with necessary definitions leading to the central notion of this chapter — that of the system of ternary quasigroups associative in the whole. Various results are proven, among them some generalizations to the ternary case of some previously known theorems. Some methods of constructions of these systems have also been presented.

Isotopy of a class of quasigroups, **Branka Alimpić**, University of Belgrade, 1972. (advisor: Slaviša Prešić) This thesis has 71 pages and it consists of a brief introduction, 4 chapters and bibliography with 58 references.

Chapter one, *Some known results from the theory of quasigroups and GD-groupoids*, contains background results. The notion of n-ary GD-groupoid is introduced and several lemmas related to the homotopy of n-ary GD-groupoids have been proved.

Chapter two, *Balanced laws on binary quasigroups and GD-groupoids*, discusses arbitrary balanced laws $w_1 = w_2$. Some generalizations of previous results for quasigroups are given and some conditions under which these results may be extended to the case of GD-groupoids have been established.

Chapter three, *Some generalizations of the Dicker law on quasigroups*, discusses generalization of the Dicker law

$$A(A(x_1^n), y_1^n) = A(x_1, A(x_2, y_2^n), \ldots, A(x_n, y_2^n))$$

to the case of n-ary semigroups are discussed.

Chapter four, *One class of balanced laws on quasigroups of various lengths*, deals with a broad class of balanced laws on quasigroups. Quasigroups A, B, A_i, B_i connected by the law

$$A(A_1(x_1, \ldots, x_\alpha), \ldots, A_m(x_\beta, \ldots, x_p)) = B(B_1(y_1, \ldots, y_\gamma), \ldots, B_n(y_\delta, \ldots, y_p)).$$

The method for going from this law for quasigroups to the appropriate law for GD-groupoids is presented and this law for GD-groupoids have been analyzed.

Quasigroups and some classes of functional equations on them, **Zoran Stojaković**, University of Belgrade, 1974. (advisor: Svetozar Milić) This thesis has 99 pages and it consists of a brief introduction, 6 chapters and bibliography with 98 references.

Chapter one, *Some definitions, notation and known results from the theory of quasigroups*, contains the background material.

Chapter two, *General entropy on GD-groupoids with applications to quasigroups with different arity*, contains the proof of the following result: if G-quasigroups A_1, B_1 and GD-groupoids A_2, A_3, B_2, B_3 satisfy the law

$$A_1(A_2(x, y), A_3(u, v)) = B_1(B_2(x, u), B_3(y, v)),$$

then there exists an Abelian group $(S, +)$ which is a homotopic image of all A_i, B_i. From this one gets the general solution of the previous functional equation for GD-groupoids.

Chapter three, *Balanced laws on ternary quasigroups*, contains a description of ternary quasigroups which satisfy arbitrary balanced law of the first kind. The notion of derived binary operations from the given ternary ones is introduced and in the set of all derived binary operations an equivalence relation is introduced. It has been shown that all binary operations from the same class are isomorphic to the same loop.

Chapter four, *Balanced laws on ternary GD-groupoids*, contains the description of ternary GD-groupoids which satisfy a balanced law of the first kind. It has been shown that, by introducing some additional conditions, one can prove the results analogous to the one from the previous chapter.

Chapter five, *On generalized (i, j)-modular quasigroups*, contains a complete description of generalized (i, j)-modular quasigroups.

Chapter six, *Infinitary quasigroups*, contains a new notion — that of an infinitary quasigroup (based on infinitary operations). It has been shown that there exists countable and uncountable infinitary quasigroups and that there exist finite quasigroups of arbitrary order. Among other things, it has been proved that there exist infinitary loops as well.

A contribution to the theory of functional equations on quasigroups, **Aleksandar Krapež**, University of Belgrade, 1980. (advisor: Branka Alimpić) This thesis has 105 pages and it consists of a brief introduction (chapter one in the thesis), 5 chapters and bibliography with 54 references.

Chapter two, *Basic properties of quasigroups*, contains background material on quasigroups.

Chapter three, *Systems of balanced functional equations on quasigroups*, contains solutions of systems of balanced functional equations on quasigroups of various length. The process of achieving the most general form proceeds from an arbitrary irreducible E-system, then the generalized E-system and finally, the solution of the system of balanced functional equations in the most general form.

Chapter four, *Various examples of balanced systems of functional equations*, presents examples of applications of previous results.

Chapter five, *Strictly quadratic functional equations on quasigroups*, discusses strictly quadratic functional equations of binary quasigroups and the solution for a wide class of such equations is presented.

Chapter six, *Functional equation of generalized associativity on groupoids*, contains the solution of the generalized equation for associativity for groupoids and some applications of this result.

Quadratic quasigroup identities, **Sava Krtić**, University of Belgrade, 1985. (advisor: Branka Alimpić) This thesis has 105 pages and it consists of 13 sections and bibliography with 50 references.

Section one is the *Introduction*.

Section two, *Quadratic identities and quasi-identities*, contains discussion about quadratic identities and quasi-identities on quasigroups. Quasi-identity is a formula of the form $\phi_1 \wedge \ldots \wedge \phi_m \Rightarrow \phi_0$, where every ϕ_i is an identity.

Section three, *Systems of quasigroup relations and their graphs*, introduces the notion of a quasigroup relation. Namely, a ternary relation Q on the set S is a quasigroup relation if for all $a, b \in S$ there exist uniquely determined $x, y, z \in S$ such that $Q(x, a, b)$, $Q(a, y, b)$ and $Q(a, b, z)$. The correspondence $Q(x, y, z) \leftrightarrow A(x, y) = y$ establishes a bijection between quasigroup relations and quasigroup operations on the same set. This section also establishes a method of associating a graph to a system of identities. Some relations between these various notions have been established.

Section four, *Reidemeister and Thomsen condition vs. theorems on four and six quasigroups*, contains some known results which are presented in order for the presentation to be more complete. The Reidemeister condition is

$$x_1y_1 = x_2y_2 \wedge x_3y_1 = x_4y_2 \wedge x_3y_3x_4y_4 \Rightarrow x_1y_3 = x_2y_4$$

and the Thomsen condition is

$$x_1y_2 = x_2y_1 \wedge x_2y_3 = x_3y_2 \Rightarrow x_1y_3 = x_3y_1.$$

Section five, *Subjugated systems*, along with the next four sections, is dedicated to solving free systems. Relations between systems and their associated graphs is widely used.

Section six, *Factorization of cubic graphs*, discusses a particularly defined relation on cubic graphs. It completely deals with graphs.

Section seven, *Connected sum of systems*, uses the notion of connectedness for graphs to discuss related systems.

Section eight, *Solution of indecomposable systems*, introduces the notion of the principal solution and it describes the general solution of the system in terms of the principal one.

Section nine, *Solution of arbitrary systems*, extends the notion of principal solution to arbitrary systems and extends the previous results to arbitrary systems.

Section ten, *Quadratic quasigroup varieties*, is dedicated to discussion of the identities in which only one operation symbol appears. A question of Krapež related to the characterization of some identities was answered in this section.

Section eleven, *Closure with respect to isotopy*, gives an answer to the following question: *which quasigroup varieties are closed with respect to an isotopy* (in the quadratic case)?

Section twelve, *Malcev quasi-identities*, contains some results concerning Malcev and Lambek quasi-identities.

Section thirteen, *An appendix. On geometric terminology*, discusses the use of geometrical and topological notions in investigations related to the identities on quasigroups.

4. Groups

Unsolvable problems in group theory, **Nataša Božović**, University of Belgrade, 1975. (advisor: Slaviša Prešić) This thesis has 67 pages and it consists of a brief introduction, 4 chapters and bibliography with 69 references.

Chapter one, *Basic notions*, gives fundamental notions from group theory related to the question of solvability. It also contains definition of Markov properties. Namely, a property P is algebraic iff all isomorphic images of all groups which have property P also have property P. Algebraic property P is Markov property iff it is proper and there exists a group F which is not isomorphic to any subgroup H of group G which has property P. The other important notion is that of a universal group. Let $\mathcal{S}$ be the class of all groups which have a property S. A group $G \in \mathcal{S}$ is universal for $\mathcal{S}$ iff every group $F \in \mathcal{S}$ is isomorphic to a subgroup of group G.

Chapter two, *Determination of the class of Markov properties by using universal groups*, has as the main result necessary and sufficient condition for an algebraic property of finitely presented groups to be Markov property. Namely: an algebraic property P of finitely presented groups is Markov property if and only if no universal finitely presented group has property P. This chapter also contains a discussion about possible application of this result.

Chapter three, *Some undecidable problems for groups*, mainly concerns with unrecognizability properties of groups and methods for establishing them.

Chapter four, *Characteristics of universal groups and Markov properties*, deals with Markov properties and their complements and some characteristics of universal groups.

Semi-direct factorization of finite groups, **Radoš Bakić**, University of Belgrade, 2002. (advisor: Žarko Mijajlović) This thesis has 42 pages and it contains 79 references.

The aim of the thesis is that along the original results also gives an overview of existing criteria for semi-direct factorization.

After a brief introduction an overview of basic classes of groups follows.

The special care is devoted to the discussion of the notion of π-nilpotency. Suppose that H is a Hall subgroup of G. If H is normal in G and if π is the set of all prime divisors of the order of G, then it is said that G is π-nilpotent. The case when H is a p-group is particularly important. Several criteria for π-nilpotence is stated and one of them has been generalized in this thesis. Also, some other new results concerning p-nilpotence are proved.

Later, the notion of the absolute factor is introduced. A group G is an absolute factor if for every group H such that G is a normal subgroup in H, it is true that H has a semi-direct factorization with the normal subgroup G. Several results concerning absolute factor have been given.

The thesis continues with other criteria for semi-direct factorization and some new proofs of known theorems have been given.

Next part deals with formation theory and some previously known results have been generalized.

After some overview of representation theory and the discussion of the relation between semi-direct and direct factorization, there comes the part about complementation

in infinite groups. Many theorems are cited in this part as well as in the next part on generalized complementation.

The thesis concludes with some results concerning the Frattini subgroup (intersection of all maximal subgroups) in a given group.

5. Rings

On the rings of Hermitian type and modules over them, **Gojko Kalajdžić**, University of Belgrade, 1982. (advisor: Đuro Kurepa) This thesis has 141 pages and it consists of a brief introduction, 5 chapters and bibliography with 59 references.

Chapter one, *On Euclidean valuations of modules and rings*, introduces the notion of Σ-euclidean valuation $\Phi : M \to W$ of a right A-module M, where W is a well ordered set. Some other conditions on valuations are introduced and the rings satisfying some of these conditions investigated.

Chapter two, *Euclidean classes and Euclidean kernel of a module*, introduces, using an idea of Samuel, the notion of Σ-euclidean class and Σ-euclidean kernel of an arbitrary module over a ring. Several properties of this class have been established.

Chapter three, *Localization, products and direct sums of Euclidean rings*, is dedicated to the investigation whether the localization A_S is Euclidean as well as the discussion related to the "Euclidicity" of the direct product and sum of Euclidean rings.

Chapter four, *On the rings of Hermitian type*, introduces the notion of the Hermitian sequence over a given ring and also the notion of the (right, left) Hermitian ring with respect to some Hermitian sequence. Some results of Kaplansky have been generalized.

Chapter five, *Matrices over Euclidean rings*, is dedicated to the investigation of the rings of quadratic matrices over a given Euclidean ring. It is proved that, under some conditions, this ring is Γ-euclidean.

Some classes of regular rings, **Blagoje Cerović**, University of Novi Sad, 1982. (advisor: Svetozar Milić) This thesis has 86 pages and it consists of a brief introduction, 4 chapters and bibliography with 51 references.

Chapter one, *Some notions and results on anti-inverse semigroups and rings*, contains background material.

Chapter two, *On the class of rings such that for any element x: $x^n = x$*, the structure of the class of rings $\mathcal{R}_n$ given by

$$R \in \mathcal{R}_n \Leftrightarrow (\forall x \in R)(x^n = x)$$

has been investigated.

Chapter three, *The class of (m, n)-anti-inverse rings*, the class of rings $\mathcal{AR}_{m,n}(m, n)$ of anti-inverse rings, defined by

$$R \in \mathcal{AR}_{m,n}(m, n) \Leftrightarrow (\forall x \in R)(\exists y \in R)(x^m = y^m = (xy)^m \wedge x^n = x)$$

has been investigated.

Chapter four, *The class of anti-inverse rings*, contains the investigation of the class $\mathcal{AR}$ of anti-inverse rings defined by

$$R \in \mathcal{AR} \Leftrightarrow (\forall x \in R)(\exists y \in R)(xyx = y \wedge yxy = x).$$

Ringoid structures, **Veljko Vuković**, University of Priština, 1984. (advisor: Stojan Bogdanović) This thesis has 148 pages and it consists of a brief introduction, 5 chapters and bibliography with 66 references. In also contains an index.

Chapter one, *Afine product of groupoids*, introduced the notions of afine product, afine quasigroup etc. It contains several results connecting about these objects. For example, it contains a sufficient condition for affine product of two groups to be a quasigroup (group).

Chapter two, *Ringoid structure*, introduces this notion. Namely, if we have a set S with two binary operations $+$ and $\cdot$ such that $(S, +)$ is a group and $(S, \cdot)$ a groupoid (so we have no restrictions on the second operation), then the structure $(S, +, \cdot)$ is called a ringoid structure. This generalizes the notion of near-rings. Also, the notions of distributor, associator and commutator are introduced and some necessary and sufficient conditions under which distributor (and others) are ideals have been established.

Chapter three *Afine ringoid structure*, introduces this notion and establishes several results concerning afine ringoid structures associated to, for example, an ordered couple consisting of a ringoid structure and a group.

Chapter four, *Nilpotence, radicals and locality of ringoid structures*, introduces these notions as well as some related notions. The results concerning nilpotent and nill ideals and radicals presented in this chapter are generalizations of the appropriate results for near-rings.

Chapter five mainly deals with the notions of afine endomorphisms and afine semi-endomorphisms of a ringoid structure.

6. Fields

On transcendental extensions of differential fields, **Branko Malešević**, 2007. (advisor: Žarko Mijajlović) This thesis has 116 pages and it consists of a brief preface, 3 chapters and bibliography with 128 references.

Chapter one, *Algebraic field theory*, discusses algebraic field theory and transcendental field extensions. It contains an overview of known results which are necessary for the later discussion. Section 8, contains an algorithmic proof of quantifier elimination for the theory of algebraic fields of an arbitrary characteristic. Section 9 contains some improvements of the Lüroth theorem about certain simple transcendental extensions.

Chapter two, *Theory of differential fields*, begins with an overview of some results of Model theory. It then proceeds with basic notions of the theory of differential fields. Section 7 contains an algorithmic proof of quantifier elimination for the theory of differential fields of characteristic 0. Section 9 contains several new results: a differential characterization of extensions of a differential field which consists only of differential-algebraic elements; two extensions of the Lüroth theorem in the theory of differential fields.

Chapter three, *One method for proving differential transcendency*, begins with an overview of the methods of proof of differential transcendency. Section 2 contains a complete proof of the Hölder theorem which claims that the gamma function is differential transcendent. Section 3 discusses analytical properties of the gamma and some related functions. It also contains some new representations of the Kurepa function $K(z)$ and the alternating Kurepa function $A(z)$. Section 4 presents a new method for proving differential transcendency. Section 5 contains many examples of application of this method. For example the proof differential transcendency is given for Hadamard function, Barnes factorial function, Ramanujan-Dirichlet L series, all Dirichlet L series (therefore Riemann zeta function as well) and for various special functions defined by appropriate integrals.

7. General algebraic structures

A contribution to the theory of algebraic structures, **Slaviša Prešić**, University of Belgrade, 1963. (advisor: Tadija Pejović) This thesis has 38 pages and it consists of a brief introduction (chapter one), 3 chapters and bibliography with 18 references.

Chapter 2, *One method for constructing relations and operations*, is devoted to the methods of construction of smallest relations (of arbitrary length) containing a given one (which usually arises from various algebraic laws.

Chaper 3, *Some inequalities*, gives some estimates on the number of different finite algebras of the given type satisfying algebraic laws $w = u$ satisfying the condition that the same letters appear in w and u.

Chapter 4, *Automorphisms of universal algebras*, is devoted to the study of the relation between an algebra and its group of automorphisms. The main theorem is the following: if G is an arbitrary group and $n > 1$, then one can define an n-ary operation f on this group such that the group of automorphisms of (G, f) is exactly the group G.

Constructive algebra — algebraic structures and the ring of endomorphisms, **Daniel Romano**, University of Belgrade, 1985. (advisor: Milan Božić) This thesis has 103 pages and it consists of the introduction, 3 chapters and bibliography with 57 references.

Chapter one, *Preliminary notions*, mainly contains background material about constructive mathematics and the logic of constructive mathematics. Some previous results of the author have also been quoted.

Chapter two, *Algebraic structures*, contains discussion about fundamental algebraic structures (groups, rings, fields, modules) in constructive mathematics. Several new results are stated and proved in this chapter. It also contains comparisons between these results and the results in classical mathematics — some examples of the results from classical mathematics which cannot be extended to constructive mathematics have been given.

Chapter three, *The ring of endomorphisms*, continues investigation from the previous chapter and examines the group of endomorphicms as well as the ring of endomorphisms and the ideals in this ring. As before, the emphasize is put on the results from constructive mathematics that differ from the results from classical mathematics.

8. Numbers

A contribution to solving the Fermat problem, **Zoran Šami**, University of Belgrade, 1978. (advisor: Đuro Kurepa) This thesis has 126 pages and it consists of the introduction, 4 chapters and bibliography with 63 references.

Chapter one presents the results toward solving the Fermat problem which had been established up to the moment of the research for this thesis.

Chapter two contains some generalizations of the author's previous results. Namely, the author proved the following result. If at least one of the numbers $2p + 1$, $4p + 1$ is prime, then the equation $x^p + y^p = z^p$ has no solutions relatively prime to p. The main effort in this chapter is devoted to showing that instead of conditions on $2p + 1$, $4p + 1$ one can extend these results to the numbers of the form $2^m np + 1$ where $n = 1$ or n iz prime, $n \geq 5$, with some appropriate additional conditions.

Chapter three does not explicitly deals with the Fermat problem. This chapter contains some results needed for chapter 4. It introduces certain auxiliary sequences of numbers $x_{n,k}$ and sequences of polynomials $D_k(x)$ and contains certain results about these objects.

Chapter four contains applications of the sequence $D_k(x)$ to the Fermat problem and to the investigation of the Bernoulli numbers.

On spectra of algebraic integers, **Dragan Stankov**, University of Belgrade, 2002. (advisor: Žarko Mijajlović) This thesis has 91 pages and it consists of a brief preface, 6 chapters and bibliography with 42 references. It ends with some commentaries and an index of used notions.

Chapter one, *Introduction and basic settings*, is introductory in nature and contains definitions and properties of the Piseaux and Salem numbers as well as spectra.

Chapter two, *Families of dicrete spectra*, contains a result of the author about a family of algebraic numbers whose ± 1 spectra are discrete. It also contains a similar results about $0, -1$ spectra.

Chapter three, *Algorithm with a criterion for discreteness*, contains the author's algorithm for computation of spectra and determination whether a given spectrum is discrete. Some applications of this algorithm have also been presented.

Chapter four, *Space interpretation of a spectrum*, establishes a bijection between spectra and an n-dimensional vector space. By using eigenvalues of a certain matrix one can improve the previous algorithm.

Chapter five, *Spectra with initial elements*, gives a condition on initial values for the spectrum to be discrete.

Chapter six, *Elements of iterative theories*, contains the basics of the iterative theory. Some properties of the set of periodic points have been established. Some methods of fractal geometry also appear in this context.

9. Equations

Ill-conditioned systems of linear algebraic equations and their solutions, **Petar Madić**, University of Belgrade, 1965. (advisor: Konstantin Orlov) This thesis has 91 pages and it consists of a brief introduction (chapter one), 4 chapters and bibliography with 30 references.

Chapter two, *Properties of ill-conditioned systems of linear algebraic equations*, discusses properties of ill-conditioned systems. These systems of equations are characterized by the fact that some approximate solutions (solutions which satisfy the system pretty well) greatly differ from the correct solutions. Also, some small changes in absolute values of the coefficients may lead to great differences in solutions.

Chapter three, *Improvements of the solvability of systems*, introduces some methods for improvement of solvability of ill-conditioned systems. These methods include: using quasi-inverse of the matrix, using quasi-orhtogonal method etc.

Chapter four, *Solving ill-conditioned systems*, a method for solving this kind of systems of equations, developed by the author, is given. It mainly consists in the step-by-step computation of the determinants of the system in question. Detailed discussion of the method and the guide for its application is presented.

Chapter five, *The conclusion*, gives some final remarks about solving ill-conditioned systems.

References

[1] Virtual library of the Faculty of Mathematics, Belgrade, available at `http://elib.matf.bg.ac.rs:8080/virlib/`

[2] Mathematics genealogy project at `http://genealogy.math.ndsu.nodak.edu/`

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