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LIFE AND WORK OF ĐURO KUREPA

Abstract. Professor Đuro Kurepa (1907–1993) belongs to a narrow circle of the most known and most significant Serbian mathematicians. In addition to important contributions to mathematics, he left behind an indelible impact on the development of modern mathematics in the former Yugoslavia. His name is well known in the world of mathematics and particularly his works in set-theory and general topology. His results are in almost every modern book of set theory. This paper presents the life and work of Đuro Kurepa.

Life of Đuro Kurepa

Professor Đuro Kurepa belongs to a narrow circle of the most important Serbian mathematicians. In addition to important contributions to mathematics, he left behind an indelible impact on the development of modern mathematics in the former Yugoslavia. His name is well known in the broad international mathematical circles, while his results are in almost every contemporary set theoretical book.

It is possible that some important details from Kurepa's biography are omitted here. An excuse for this may not lie in oblivion, but in his very rich life and professional biography. First of all, professor Kurepa had a very large and diverse scientific opus. He had traveled to many countries and visited the major mathematical centers, some of them more than several times. In his country, he was one of the main initiators and organizers of scientific, professional and pedagogical work in mathematics. He had a rich correspondence with significant figures from around the world. Therefore, it is simply impossible to expect one author to present a complete work and life of such a person, particularly in a paper of a limited extent.

Đuro Kurepa was born on August 16, 1907 in Majske Poljane near Glina in Serbian Krajina as the fourteenth and last child of Rade (1870–1909) and Anđelija (1869–1950, born Mraković) Kurepa. According to his family legend, Kurepa's ancestors on his father's side settled in the Krajina from Montenegro, Durmitor Mountain. However, there is no reliable data supporting this. Let's mention that Kurepa's Christian name was Đurađ. He changed the name to Đuro during the Second World War. He was especially close to his brothers Milan and Vasilije and his nephew Božidar, Milan's son. Božidar was persecuted as a communist by the police in Zagreb during the Second World War. Kurepa was hiding and helping Božidar, but Božidar was eventually found and killed by Ustashas. Đuro Kurepa was not the only offspring of science in his family. Milan's son, Svetozar Kurepa, was one of the leading postwar Zagreb mathematicians. Milan Kurepa, Vasilije's son, was the prominent Belgrade physicist and member of SANU.

Kurepa attended elementary and secondary school in Majske Poljane, Glina and Križevci. He simultaneously studied and graduated from agricultural high school and Gymnasium (high school), as a private student. Kurepa graduated in theoretical Mathematics and Physics at the Faculty of Philosophy, University of Zagreb in 1931, the following study groups: a) Theoretical Mathematics, b) Rational Mechanics and Theoretical Physics, c) Physics. Among the local professors who introduced young Kurepa to science was Vladimir Varićak (1865–1942), prominent mathematician of Serbian origin, a member of the Academies in Zagreb and Belgrade. Upon departure to Paris, professor Kurepa had correspondence with Varićak, and later often mentioned him as an example of a good mathematician and professor.

Đuro Kurepa married twice. The first time was with the professor Ružica Martinis (born in Vienna in 1903, died in Zagreb, 1965). With her he had one child, Božena, who tragically died in a Zagreb maternity in 1945. The second marriage was with prof. Nada Jagić (born in Novocherkask in 1919, died in Belgrade in 1973). Mention that Nada was the great-granddaughter of Vatroslav Jagić (1838–1923), the prominent Slavist and university professor in Odessa, Berlin, St. Petersburg and Vienna, the member of the many world and Slavic academies. Đuro and Nada had four children: Snježana, Radomir, Nada and Yuri-Archimedes. At the time of writing this article, Nada and Yuri live in Belgrade, Radomir in Croatia and Snježana in South Africa.

The period from 1932–1935 Kurepa spent in Paris at *Faculte des Sciences* and *College de France.* He received state PhD defending doctoral dissertation *Ensembles ordonnés et ramifies* in 1935 at Sorbonne before the dissertation committee whose members were Paul Montel (Chairman), Maurice Fréchet (mentor), and Arnaud Denjoy. Then, he specialized and self improved in some of the best institutions in the world. First, in the autumn of 1937 in Warsaw with Professor W. Sierpiński, one of the leading specialist in set theory of that time and immediately afterward for several months in Paris. In the second half of 1950, he visited a university in Boston and mathematical departments of universities in Chicago, Berkeley and Los Angeles. In the winter semester of 1959 he was the guest of Institute of Advanced Studies in Princeton. Incidentally, Kurepa became a polyglot. He knew and actively used several foreign and classical languages: French, English, German, Russian, Spanish, Italian, Esperanto, Latin and Ancient Greek. In younger days he used stenography. In the beginning, he wrote scientific papers in French, but later used mainly English, although he wrote several papers in German, Italian and Russian. More than often the participants in international meetings asked Kurepa questions in their native languages. Kurepa would immediately, to general surprise, answered and discussed with each of the interviewees on their languages. He was a great speaker and has held appropriate speeches on such meetings in three or four languages without notes or written papers.

Kurepa's first job was at the University of Zagreb in 1931, where he worked one year as an assistant professor of mathematics until the departure for Paris where he enrolled in doctoral studies. Upon returning to Zagreb in 1937, he worked in high school shortly, then at the Faculty of Philosophy, Zagreb University, where he was elected assistant-professor in 1938, associate professor in 1945 and full professor in 1946. In Zagreb he taught until 1965, when he moved to Belgrade. In fact, that year he was invited for a full professor at the Faculty of Science, where he remained until the end of his working life. At the same time he was invited for the professorship at the University of Boulder in the U.S.A., but Kurepa gave priority to Belgrade.

In Belgrade, he finds teachers: Konstantin Orlov, who was elected that year full professor, associate professors Borivoj Rašajki, Slobodan Aljančić, and Vojin Dajović and assistant professors Zagorka Šnajder, Dragomir Lopandić, Milorad Bertolino, Milosav Marjanović, Slaviša Prešić and Zoran Ivković. Kurepa and Orlov were the oldest and only mathematicians with PhD at the Department of Mathematics earned before the Second World War.

That year was in so many ways critical for the development of mathematics, scientific and teaching work at the Department of Mathematics of the faculty. Somewhere about that time there was a shift of scientific generations. For various reasons the older generation of professors were no longer active at the Faculty. Jovan Karamata went abroad as far back as 1951, Miloš Radojčić also in 1959, Professor Tadija Pejović went to retirement in 1963, and Dragoljub Marković, professor of algebra, suddenly died just in that 1965. The young teachers were most responsible for introducing new mathematical disciplines in undergraduate and graduate studies, such as: mathematical logic, modern algebra, topology, functional analysis, differential geometry, probability theory and other areas. Several scientific seminars were established, most of which have been lasting until today. In the beginning, the role of the seminars was to introduce students to modern developments in these areas. Seminars shortly become a commonplace of scientific work and gathering place of mathematicians of similar affinities, not only within the Faculty and in Belgrade, but throughout Serbia and other places in Yugoslavia. New books with modern amenities were published. Let's mention the first book of this kind, "*Introduction to functional analysis*" by professor Aljančić, printed in 1963. This was our first book in that area which announced these changes. The book had an enormous significance for the development of the newly introduced graduate studies, and beyond, for the development of the entire field of mathematical analysis here. The world-renowned mathematicians visit Belgrade, let's mention, for example A. Tarski, P.S. Alexander and P. Erdős. An international symposiums and conferences that have an echo in the world were organized. It was professor Dajović's, Kurepa's personal friend, great merit that Mathematical High School was established in 1966, which brings together young talents in mathematics. The young scientific workers and assistants usually went abroad for specialization, not only in traditional centers in Russia and France, but in U.S.A., Germany and England as well. The society recognized that graduate mathematicians are not just high school teachers, or scientists of a heavy and closed science, but also experts who can contribute much to the industry and other areas of common interest. One must admit that at that time the state aids the development of such events, primarily by opening new jobs at the University, scholarships, significant scientific research funding and equipping libraries. So, during these changes Kurepa comes to Belgrade. Professor Kurepa as a very strong mathematical spirit, a great authority and great connoisseur of modern mathematical disciplines helped this development of mathematical sciences in Serbia. In many of these events he was directly involved, or was their organizer. Here's an example. In 1968 the first of five international symposia in topology and its application in whose work participated leading topologists from Europe, U.S.A. and other countries was held. Members of the Department were organizers of these symposiums together with topologists from other Yugoslav centers. Professor Kurepa was the chief organizer and three times chairman of the Organizing Committee.

Professor Kurepa's influence on the development of mathematical science in the former Yugoslavia was immense. As a professor at the University of Zagreb, he introduced a new mathematical discipline, mainly related to the foundations of mathematics and set theory. Zagreb mathematician Kajetan Šeper supports the claim: *"Professor Kurepa was not only a professional mathematician, but a true scientist, philosopher and humanist in the true sense of the word. He was a pioneer and founder in the field of mathematical logic and the establishment in Croatia and of modern mathematical theories in Croatia and Yugoslavia. He was the catalyst, initiator and carrier of mathematical sciences in us".* [1]

On another occasion[2] leading mathematician from Zagreb Sibe Mardešić said: *"With his work and influence at the University of Zagreb, in particular introducing modern aspects of mathematics, Kurepa gave a great contribution to our society".*

Although he arrived in Belgrade when he was at a relatively late age, his influence on Belgrade mathematics can be described almost in the same words. Professor Kurepa exhibited the latest results from various mathematical disciplines through regular and post-graduate courses, seminars and lectures at the Faculty of Science and Mathematical Institute. Themes of his lectures included: the problem of infinity and the continuum, the issues of independence from the cardinal and ordinal arithmetic, ordered sets and general topology. But Kurepa was interested in other topics as well. He gave valuable contributions to the development of the following disciplines at the University of Belgrade: analysis, algebra, number theory, as well as emerging computer science disciplines in Serbia, like computing and linear programming. The university courses he taught show his mathematical universality: algebra, algebraic structures, differential and integral calculus, complex and real analysis, differential equations (ordinary and partial), topology and the set theory. He led several seminars and special courses on post-graduate studies, mainly from the set theory and general topology. From other university activities, it should be mentioned that he was a member of the Board of source and then part-time professor of mathematics at the Faculty of Organizational Sciences (1971) and at the Multi-disciplinary center of Belgrade University. At the age of 70, in 1977 he retired and became *professor emeritus*.

Professor Kurepa had contacts with Belgrade mathematicians long before his arrival in Belgrade. Publishing his dissertation *in extenso* in *Publications Mathématiques de l'Universite de Belgrade*, 4 (1935), 1–138, Kurepa made first contact with the Belgrade mathematical environment. At the beginning of 1950s, these contacts became deeper and more frequent. Thus, in the article[3] of Professor Zlatko Mamuzić we can see that Kurepa had already been invited in 1952 to visit the University of Belgrade. On that occasion, he gave a lecture in the matrix calculus and also held a seminar with topics from set theory, topology and algebra. Participants of the seminar were our then young, later known mathematicians: Časlav Stanojević, Mirko Stojaković, Simon Ćetković and others. By participating in the work of these workshops, mathematicians got ideas for their works, while post-graduates received their themes for masters and doctoral dissertations. A large number of these themes were formulated or initiated by Kurepa himself. These works include almost all of the doctoral dissertation of the older generation of topologists and many algebraists from all (previous) Yugoslavia: Svetozar Kurepa, Zlatko Mamuzić, Sibe Mardešić, Pavle Papić, Viktor Sedmak, Ljubo Martić, Dragiša Mitrović, and then several years later of the mathematicians: Ljubomir Ćirić, Rade Dacić, Milosav Marjanović, Veselin Perić, Milan Popadić, Ernest Stipanić, Mirko

[1] Symposium: Set theory. Foundations of mathematics, International Symposium dedicated to Đuro Kurepa, Beograd 1977.

[2] *Professor Đuro Kurepa,* Glasnik Matematicki Vol. [28](48), 1993, No.2, pp. 333–343.

[3] *On the occasion of 35 years of the book "Set theory" by professor Đuro Kurepa* Historical writings on mathematics and mechanics, the history of mathematical and mechanical sciences, book 2, Mathematical Institute, Belgrade, 1989.

Stojaković, Pavle Miličić. Professor Kurepa was at the head of almost a hundred of theses and dissertations, 42 times of which for PhD candidates only, the younger generation academics among others, including Stevo Todorčević and Alexander Ivić. Among our other mathematicians on whom Professor Kurepa influenced as a mentor, a member of the doctoral committee or otherwise, we should mention: Miroslav Ašić, Đordje Dugošija, Nataša Božović, Aleksandar Jovanović, Dušan Ćirić, Milena Jelić, Gojko Kalajdzić, Ljubiša Kočinac, Žarko Mijajlović, Nada Miličić, Mila Mršević, Marica Prešic, Zoran Šami, Ratko Tošić. Many of these mathematicians continued and further developed Kurepa's work, especially Stevo Todorčević. In addition, he was a member of the Commission for a written evaluation of three doctoral dissertations defended in India.

Professor Kurepa was in contact with many mathematicians of the highest rank in the world. As a result, some of them visited Belgrade during the sixties and seventies: A. Tarski, P.S. Alexandrov, P. Erdős, M. Krasner, N.A. Shanin, T. Jech, K. Devlin and others. Professor Kurepa was especially proud of his meeting with Nikola Tesla, the great scientist and engineer of Serbian origin. Kurepa's respect for Tesla was very special and he often mentioned his name as a role model of a great scientist and a man. He wrote about Tesla and held occasional speeches on memorial gatherings dedicated to this great person. Kurepa met other significant persons from 20th century science world, such as Albert Einstein and Kurt Gödel. Those were not courteous meetings; on the contrary, Kurepa in these circumstances discussed various professional issues. For example, it is known that Gödel was interested in cosmological models, so he established such a model himself that is now known under his name. Since Gödel's main work was based on the foundations of mathematics, on one occasion our colleague Dragi Radojević, who was theoretical mechanist, gave a lecture at the Seminar for Mathematical Logic on this Gödel's work in theoretical mechanics. On that occasion we asked Professor Kurepa whether he knew about this Gödel theory. He replied that he had discussed with Einstein precisely on that subject and Einstein himself had had no favorable opinion on that model.

The writer of this text had the opportunity and luck to listen (and take) professor Kurepa's fundamental studies on regular courses, two courses on mathematical analysis and a course on algebra, and on post-graduate studies the course in set theory. His lectures were interesting and picturesque. Sometimes, following his mathematical thought he was able to go too far for the audience to understand. School board was not cleaned during the time of his lectures, but he sought the minimum of free space to write another formula. He was rigorous but fair on his exams. Students were always answering publicly and before the board. He used to get angry because students would wipe something off the board rather than their ignorance. On the other hand, a good response would be praised and rewarded with nice grade, while the suggestion that subsequently followed would often turn into a little lecture. Professor Kurepa was highly respected and appreciated by students but also a little feared of. The professor would sometimes ask a question, perfectly natural for himself, but very inconvenient for a student. He would preserve for decades students' written work, as well as the complete correspondence and other documents. It happened once that the results that Kurepa had announced in his letter were abused i.e. appropriated by recipients without mentioning Kurepa's name. Since then, Kurepa kept a copy of each sent letter. His very spacious cabinet at faculty and working rooms at home were overcrowded with books, letters, journals and various other papers. However, it seems that there was order in that mess and he managed to orientate. If someone known to Kurepa asked him for a rare book or old print, in a few days such colleague would have that document in his hands, although signing the reverse paper. Sometime in the mid-eighties, Professor Kurepa was cleaning his cabinet because it was supposed to give the site back to university. He duly returned to his former students, now his colleagues in their forties, their work papers. So, after nearly twenty years, I got back my seminar paper that I wrote as a student in the second year of undergraduate studies.

Similar manners characterized participation of Professor Kurepa on professional seminars and meetings. If at the end of a lecture his discussion or comment were missing, it usually meant that the presented results were not interesting. That mainly was his fiercest criticism. He only had comments for good results. Because of that, the speaker was always keen to hear the end of the discussion of Professor Kurepa, because it usually carried certain valuation results on the very topic. Professor's Kurepa behavior towards his colleagues was that of a true gentleman as well as a certain distance. One could never hear a bad comment from him about other people, but a good scientific work of his colleagues would be publicly praised, remembered, and represented in the foreign conferences and seminars during his frequent trips. Some of the colleagues tried sometimes to acquaint Kurepa with a current affair on the faculty. His response was reduced to a short answer like: "*Really? I did not know!*" or "*How uncomfortable*" and he would simply be gone. Simply put, Professor Kurepa was mainly interested in mathematical side of his colleagues and associates. However, that did not mean he never helped his colleagues, especially his younger associates. It was his merit that a few young mathematicians were employed as assistants at the faculty. He followed and encouraged them in the scientific work, while their more valuable results were included and cited in his papers and books.

Professor Kurepa showed particular interest in the problems of teaching at all levels. He actively participated in the creation of new curricula, and his numerous textbooks in mathematics for elementary and high school (about 50 titles) left a lasting trace. *Školska knjiga (School book)* from Zagreb only issued 23 titles, where the author or coauthor was Kurepa. In all of his books at least one chapter had the set-theoretical character in accordance with contemporary trends. From these books the most important ones are extensive university textbooks: *Set theory*, Školska knjiga (School book), Zagreb, 1951, and the two-volumes *Higher Algebra*, which was published by the same publishing house in 1965. *Higher Algebra* was re-released by the Belgrade Institute for textbooks in 1969, slightly amended, especially with the results of local mathematicians.

In his lectures and in the public and written exposures of mathematical characters in general, special attention was dedicated to concepts from the foundations of mathematics. He stressed that in mathematics all is reduced to the notion of function or set. Unlike most of our other theoretical mathematicians, he was inclined to mathematical logic, to connecting mathematics and other sciences and general connections between mathematics and natural phenomena. He speaks about that in a place: "*Polyvalent structure has this aspect also, that can be expressed in the following forms: Every phenomenon is a generator of mathematical or logical structure obtained from the transition from special to general. One of the fundamental goals is to find where the data structure appears and what degree of accuracy reflects the phenomenon*". He pointed out the important role of quantifiers *every* and *exists,* or, as he used to call them "kolikovniks" (a non-existing word in Serbian, meaning in English probably as "how many"). In this sense he paraphrases famous Hilbert's saying about the role of Cantor's set theory: "*The new science of quantifiers is a real mathematical paradise*". On the other hand, his general attitude about the position of mathematics on one occasion Kurepa described as: "*Aei ta phainomena mathematikeyetai – The Phenomena are always creating mathematics – Pojave uvek matematikuju*".

In the meantime, as a visiting professor he stays at Columbia University in New York (Summer School 1959) and at the University in Boulder, Colorado, 1960. Despite university lectures Kurepa has also successfully organized the scientific work of the Faculty and Mathematical Institute SANU, and was very active in administrative affairs. Since 1943 to 1965 he was the head of the Mathematical Institute of Faculty Science, University of Zagreb, then the head of the Institute of Mathematics of the Faculty of Sciences in Belgrade. For a decade, in the period 1970 – 1980, he was the head of the Department of Mathematics of the Mathematics Institute SANU. At the time the most important scientific and professional activities of the Belgrade mathematicians were conducted within this Department of Mathematics Institute. Short time he was also acting Director of Institute of Mathematics (1968–69) in Belgrade. His other duties have included the following functions: Chairman of the Council of Faculty of Sciences in Belgrade, President of the Scientific Council of the Mathematical Institute of SANU (1966–68), president of the Union of mathematical institutes of Yugoslavia, President of the Serbian Board of Education (1968–72).

Professor Kurepa was a member of the editorial board of several prominent scientific journals: Zagreb's *Mathematical-physical and astronomical gazette* (1945 – 65), Swiss *L' Enseignement Mathematique* (1952–62), since 1973. Belgrade *Publications de l'Institut Mathématique*, and *Mathematical journal*, German *Zeitschrijt fiir mathematische Logik und Grundlagen der Mathematik*, Indian *Pure Mathematics Manuscripts* since 1982, and Sarajevo's *Mathematical papers ANU BiH* since 1985. In 1971 he founded international mathematical journal *Mathematica Balkanica* and then many years was its chief editor, and later honorary editor. Unfortunately, for financial reasons editorial office was moved in the mid-eighties to Sofia and now the journal is edited and published there. Besides these, Professor Kurepa stresses his membership in the editorial the following professional journals: *Nature*, Zagreb (1945–53) and Belgrade's *Teaching of mathematics and physics*, and *Mathematics teaching*. In Kurepa's autobiographical note from 1989, we find the fact that until then he wrote 975 reviews! He published over 600 scientific and professional papers and other notes. Alone or as a co-author he published 33 books and textbooks in Serbo-Croatian, many of which have been re-edited, 14 in Macedonian, and four Italian and one in French. Besides all that, he was the associate of the famous journals, which publish reviews on the scientific works in mathematics: American *Mathematical Reviews* and German *Zentarblatt fur Mathematik*.

He wrote articles and encyclopedic entries describing numerous old and modern, compatriot and foreign mathematicians and scientists of other disciplines as well: several times of Nikola Tesla and Vladimir Varićak, then on Vladimir Dvorniković, Mihailo Petrović, Joseph Plemelj, Elie Cartan, George Boole, Luitzen Egbert Brouwer, Georg Cantor, Gerolamo Cardano, Augustin Cauchy, Julius Dedekind, Paul Lengvin, René Descartes, Harald Bohr, Émile Borel, Jacques Hadamard, his mentor Maurice Fréchet, Waclaw Sierpinski, Bernard Bolzano and others.

Professor Kurepa won numerous awards and prizes. Thus, he is the laureate of the highest award of the former Yugoslavia, AVNOJ award (1976). In addition to this, he also won the Medal of work with red flag (1965) and the Medal of merit for the people with golden star (1979). He was a member of the American-Canadian Tesla Memorial Society (1982), and winner of charter Bernard Bolzano in Prague in 1981, and Marin Drinov shells of Bulgarian Academy of Sciences in Sofia in 1987. He was elected a full member of SASA (Serbian Academy of Sciences and Arts) in 1988, and was a member of the Academy of Sciences of Bosnia and Herzegovina since 1984, and associate member of JAZU (Yugoslav Academy of Sciences and Arts) since 1952. We see that SASA was delaying his election as a member, only to be elected in his 81 year. However, the Academy must have had its own reasons, most probably non-scientific, for such an attitude. Nevertheless, Kurepa's SASA election was also an election for a full member of the Academy, which rarely happened. He was also a member and one of the founders of the Scientific Society of Serbia in 1969.

In addition to membership in these institutions he was a member of many other compatriot and foreign professional associations, and those often had a high function. So, he was a member, founder and president of the Society of the Croatian mathematicians and physicists, vice president of the Croatian Natural History Society (1947–51), a member of mathematician societies of Serbia, France (since 1936), USA (since 1950), West Germany (since 1983), Association for Symbolic Logic (since 1952), Leibnitz Gesellschaft in Hanover (from 1976). Then he was the corresponding member of the World federation of scientific workers (since 1981), President of *Mathematician, physicist and astronomer societies of Yugoslavia* (1955–60), President of the Balkan mathematical union, co-president of the same (the 1984th), president of the Yugoslav Commission for Mathematics (1970–80), the representative of Yugoslavia at the main assemblies of the International Mathematical Union (Rome, 1953, The Hague 1954, St. Andrew 1958, Sweden in 1962, the Soviet Union in 1966, France, 1970, Vancouver 1974 and Helsinki in 1978). He was also the vice president of the International Mathematical Education Commission (convened by three in four years 1952–62).

Two international symposia were organized as dedicated to the work of professor Kurepa in science. The first was held in late August 1977 in Belgrade on the occasion of the seventy years of life of Đuro Kurepa entitled "*Set Theory. Foundations of Mathematics*". The meeting was organized by the Mathematical Institute of SASA with the assistance of the International Union of mathematicians and a few Serbian state institutions. In the works he considered important he loved to play a decisive role. So, the same happened this time, i.e. the President of the Organizing Committee was Đuro Kurepa, regardless the fact that the meeting was dedicated to him. Scientific Committee meeting consisted of the imminent mathematicians: Paul Cohen, Yuri Yershov, Georg Kreisel and Peter Vopenka. Also, among the participants were a very distinguished foreign mathematicians in the field of set theory and foundations of mathematics: his personal friend, Michael Krasner, B.S. Steckin, T. Jech, N.A. Sanin, G. Kreisel, M. Loi, J.R. Burgess and K. Namba. Although the meeting had a relatively small number of participants, about thirty, notable participation had a group of mathematicians from Zagreb (M. Mihaljinec, K. Šeper and D. Rosenzweig).[4] Also, in Belgrade, in the premises of Faculty of Science, a second, two-day symposium was held in late May 1966, entitled "*International Mathematical Symposium dedicated to the memory of Đuro Kurepa*". This meeting was jointly organized by Serbian Scientific Society, Faculty of Mathematics, Mathematical Institute SANU and the Union of mathematical societies of Yugoslavia. President of the Organizing Committee was Stevo Todorčević, Kurepa's most successful disciple and follower of his work. Beside a large number of compatriot participants, the meeting was attended by a fifteen mathematicians from abroad. Among others, the meeting attended Menahem Magidor, one of the world's leading experts in the field of set theory and Rector of the Hebrew University in Jerusalem, Stylianos Negrepontis of Athens, Theodoros Wallis, then president of Greek mathematical society, Nuel Belnap and Marion Scheepers from the United States, Vladimir Shark from Ukraine, Santish Joshi from India and others. Over seventy scientific papers were presented at this symposium. Selected scientific papers from this meeting were printed in a special issue of Scientific Society of Serbia journal, Scientific Review. Also, one edition of the journal Publications de L'Institut

[4] *Proceedings of the Symposium "Set Theory, Foundations of Mathematics.* Proceedings N.S. book. 2(10), Belgrade 1977. Math. Inst. SANU.

Mathématique[5] was dedicated to Đuro Kurepa. It contained works from great world mathematicians Paul Erdős and Saharon Shelah, among others, who published and dedicated their works to Kurepa. Next year, already, Mathematical Institute SANU published the book *Selected Works of Đuro Kurepa* containing Kurepa's selected works. The book was edited by A. Ivić, Z. Mamuzić, Ž. Mijajlović (editor-in-chief) and S.Todorčević.

On various aspects of the scientific activity of professor Kurepa a number of our authors wrote: Z. Mamuzić, S. Mardešić, R. Papić, R. Dacić, E. Stipanić, V. Stanković, S. Todorčević, A. Ivić, M. Grulović, Lj. Kočinac and Ž. Mijajlovic. For a detailed description and analysis of Kurepa's scientific work, the reader should particularly pay attention to author's contributions in the book *Selected works of Đuro Kurepa*.

Professor Kurepa was very active in his old age also. He used to visit the library of the Institute of Mathematics very often, always carrying an ancient, but legendary bag full of some papers he jealously kept. Sometimes, at a seminar he would surprise us by showing and reading some of those papers. For example, there was the original doctoral dissertation manuscript by the famous mathematician Luitzen Egbertus Jan Brouwer, or correspondence with the leading Polish mathematician Kuratovski. He never missed the meetings of the Department of Mathematics and meetings of the Seminar for mathematical logic on Fridays, or the sessions of the Editorial Board of Publications. He used to take notes of all the speakers on seminars without exception. Both in spirit and in enthusiasm for work, he was much fresher than many of his much younger colleagues. He continued to write scientific papers, several of which appeared only after his death. True, those were mainly works with retrospective nature, but they still had a mathematical weight and recognizable Kurepa's spirit and style.

As a man of the universal spirit to whom the limits of any kind were not close, the disintegration of the former Yugoslavia in the early nineties was something he could hardly bear. On that occasion, he wrote a protest letter to the former U.S. President, George Bush. In contrast to the interlocutor, his comments on the subject were not intense. Sometimes, with a sad and serious face he would scantily spoke about that. Mentioning Yugoslavia and his kindred he would say, "*You know, these events are a great evil. What is happening to the Serbs in Krajina and Croatia is unhuman and really terrifying*". This is probably the hardest grade and criticisms about any subject I ever heard from Professor Kurepa.

Until the very end, Kurepa was very vital and in a good health. Professor Đuro Kurepa died suddenly on November 2, 1993. The year 1993 and especially its fall were very gloomy and unfortunate for our people and our country. This fact, as well as the unexpected death of Kurepa, was probably the reason why this event is colored by a certain amount of unnecessary controversy. Daily press wrote a lot on the subject, unfortunately often prevailing towards different speculations in relation to the great scientific work of Đuro Kurepa.

Scientific work of Đuro Kurepa

Several general characteristics define scientific work of Đuro Kurepa. First of all, Kurepa has a very large scientific opus. He published over 200 scientific papers[6] and more than 700 other

[5] 57(71), 1995.

[6] It is difficult to specify the exact number of Kurepa's scientific papers. A number of scientific works has the general character and by contemporary standards they did not contain new scientific results but they expressed the views of the author on mathematics as well as on the other scientific disciplines. Kurepa himself never extracted the works of "*pure*" scientific character, but listed all the writings in his bibliography compilations. On the other hand, in a memorial article by Z. Mamuzic we find that Kurepa published at least 212 scientific works, while S. Mardesic in his article states the number of 170 scientific works.

writings: books, articles and reviews. The first scientific paper was published as early as in 1933 in the French magazine C.R. Acad. Sci. Paris 197, while the last of his work was printed in 1993. He published scientific papers in journals worldwide, and some of them were printed in the most famous mathematical journals, for example: *Matematische Annalen, Izvestiya Akademii Nauk SSSR, Acta Mathematica, Comptes Rendus de l 'Academie des Sciences, Bulletin de la Societe Mathematique de France, Zeitschrift für mathematische Logik und Grundlagen der Mathematik, Journal of Symbolic Logic, Pacific Journal of Mathematics*. Many of his discussions after publishing were translated into English, French, Italian and other languages. Kurepa also wrote several hundred reviews for paper journals *Mathematical Reviews* and *Zentralblatt für Mathematik*.

Another characteristic of Kurepa's scientific papers is that he did not have co-authors except a few exceptions. And if he had any, those were not important works in relation to the total scientific work of Kurepa. In the work *On the summation of fundamental Fractions*, published in the *Bulletin of French mathematical society* in 1958, the co-author is D. Đoković. In the work of didactics of mathematics, presented at a symposium in Italy in 1964, the co-author was Kurepa's wife, Nada Kurepa. He also wrote two review articles, together with Bogumir Schon, on numbers and real functions for encyclopedic release in several volumes *Grundzüge der Mathematik*, published by Vandenhoeck Ruprecht, Gettingen, 1962. These books were also published in English in 1974 by MIT Press, Cambridge, Mass. The English edition also has an article about ordinal numbers with A. Aymansom.

One part of Kurepa's work was characterized by quite complicated notation. He often imposed new terms and labels which made further reading difficult, sometimes even blurring the basic idea of the article. This feature of Kurepa's works was more pronounced in later works. Sometimes he would rush to publish, not developing thoroughly newly introduced ideas. For example, Kurepa already in 1934 introduced the concept of pseudo-distance space, but never formulated its major property. Eleven years later, Fréchet defines *espaces ecartises* similarly as Kurepa, formulates its properties and associates them with uniform spaces, without citing Kurepa. As we shall see, Kurepa complained to Fréchet about this. Fréchet apologized to Kurepa during his visit to Belgrade, but still noted that Kurepa gave only the naked definition of these spaces[7].

Unlike most other mathematicians, Kurepa never hid his ideas. On the contrary, he exhibited them both in the works and in lectures in the form of hypotheses and open problems. Of course, other scientists used these ideas, solved problems, affirmed and refuted hypotheses, or more frequently, proved their independence, i.e. these were new postulates. On the other hand, Kurepa was endowed with a very strong intuition. He knew, unlike the most mathematicians, to sense a good problem and to predict a theorem. For example, Kurepa already in 1935 expressed the belief that Suslin's problem and other set theory problems cannot be solved with the final answer just on the basis of standard axioms of set theory. He also claimed the same for the CH (continuum hypothesis), namely that it can be $2^{\aleph_0} = \lambda$ for any cardinal number λ which satisfies the condition of the König lemma, i.e. the condition $cf(\lambda) > \aleph_0$. It was only after thirty years that these predictions of Kurepa were confirmed, for example Easton in 1970 proved Kurepa's hypothesis on CH.

The main scientific results of Đuro Kurepa are in set theory, general topology and partly in the theory of numbers. In set theory they can be further classified as works in the theory of partially ordered sets, cardinal functions in topology, infinite combinatorics

[7] See memorial article by Zlatko Mamuzić from 1994 devoted to Đuro Kurepa.

(partition calculus in set theory) and axiomatic set theory. However, the most important Kurepa's works are related to the *trees*, a special class of partially ordered sets.

Trees, partially ordered sets in which each lower cone is a well ordered set, can be considered a natural generalization of the concept of ordinal number. This is a special type of ramified sets which Kurepa introduced in his most important work, doctoral dissertation *Ensembles ordonnés et ramifiés* . It is believed that this work is the first systematic study on set trees, ordered structures of central interest in contemporary set theory. In the thesis and in a few works which were immediately published , Kurepa introduces fundamental concepts of the theory of infinite trees: Aronszajn trees (named after Nachman Aronszajn), trees of uncountable cardinality in which each chain and each layer is the most enumerable, Suslin's trees, i.e. Aronszajn trees in which each anti-chain is the most enumerable and Kurepa's trees. A Kurepa tree is a tree (T, <, 0) of height ω_1, each of its levels is at most countable, and has $\aleph_2$ branches. So, Kurepa in Suslin's and Kurepa's tree found two extreme terms, in the first, an uncountable tree with no uncountable branches and no uncountable levels, while Kurepa's trees have a maximum of such brunches, at least. In the thesis he presents Aroszajn's proof of the existence Aroszajn's trees and immediately raises the question of the existence of Suslin's and Kurepa's tree based on the standard axioms of set theory. He proves many important and interesting features concerning these structures. Probably the most famous is the following equivalence, which refers to the famous Suslin's hypothesis SH:

$$\text{SH} \Leftrightarrow \textit{Suslin's tree does not exist.}$$

Here SH denotes the hypothesis that there is no Suslin's continuum, i.e. a linearly ordered set L of countable cellularity (the set in which each family of disjoint intervals is the most countable), but which does not have countable dense subset.

Lebesgue, in his article from 1905, implicitly identified analytical functions with Bair's functions. In his proof, he used the argument that is "*simple and short, but wrong*". Wrong step in the proof was hidden in seemingly trivial lemma that the projection of Borel's set is also Borel's set. Ten years later, Suslin, then very young and very talented Luzin's student, discovered the mistake. Thus Suslin finds a new class of real line subsets, the class of analytical sets, which arise as projections of Borel's sets. At the same time, he proved that there are analytical sets that are not Borel's. Thus, the descriptive theory has been created, one of the most interesting and deepest disciplines in the set theory. But Suslin soon died (1919), and the formulation of Suslin's hypotheses appears after his death[12]. Suslin's hypothesis will have a central place in the development of the theories of infinite trees, and it is where Kurepa's works will be of essential importance. Starting from 1935, Kurepa was trying to solve SH. He failed, simply because of the fact that it was impossible to do that at that time. The means of the classical set theory which was developed by Cantor, and Zamelo, Fraenkel, Hausdorff, König, Tarski and others, did not allow to do so. However, Kurepa was the first

[8] The thesis is fully published in Publ. Math. Univ. Belgrade, 4,1–148. It is also reprinted in Selected Papers of Đuro Kurepa.

[9] See, for example, K. Kunen, Set Theory, North-Holland, Amsterdam, 1983, pp. 69. Also by Akihiro Kanamori in *The higher infinity - Large cardinals in set theory from their beginnings*, Springer Verlag, Berlin, 1994, pp. 72.

[10] *Ensembles lineaires et une classe de tableaux ramifiés*, Publ. Math. Univ. Belgrade, 6, 129–160, and *A propos d 'une generalisation de la notion d' ensamles IES ordonnés*, Acta Mathematica 75 (1942), 139–150.

[11] Although this important concept from the set theory bears his name, Aroszajn never specifically dealt with set theory, nor he published his proof by himself. At that time (1934–5) he was Kurepa's friend on studies in Paris. Later, he was quite well known analyst.

[12] "*Probleme 3*", Fund. Math., 1, 223, 1920.

mathematician who understood the importance of trees in the set theory. Later, several mathematicians discovered the characteristics of these partially ordered sets. For example, Miller in 1943 and Sierpinski in 1948 independently re-discovered the above equivalence of Suslin's hypothesis which Kurepa already discovered in 1935.

Using infinite tree, Kurepa has found examples of topological spaces with important and unusual properties. One example of this kind is in connection with Suslin's line. Namely, he proved[13] that the topological square of Suslin's continuum has uncountable cellularity, while it has a countable cellularity itself.

Kurepa did not settle the question of the existence of Suslin's tree, nor Kurepa's tree as well. The postulate which claims the existence of Kurepa's tree has been named after Kurepa's hypothesis, which is spelled shortly – KH. A complete solution of these problems has not obtained until the beginning of the seventies of XX century, with the appearance of Cohen's forcing, the main mean of modern set theory. Solovay, Tennenbaum and Jensen proved that SH is independent of ZFC + CH (Zermelo–Fraenkel's set theory plus the continuum hypothesis), while Devlin in 1978 proved that each of the following theories

$$ZFC \pm CH \pm SH \pm KH$$

is[14] consistent.[15] These facts show that the nature of the postulates SH and KH differs from the other ZFC axioms. Hence it is clear why these structures have so important place in various constructions of the set theory, topology, model theory and infinite combinatorics.

Kurepa's work in the field of the cardinal functions in topology is mainly motivated by his attempt to solve Suslin's problem. For this purpose, he introduces one of the most important cardinal function, *c(E)*, as a cardinal numbers supremum of the families of open disjoint subsets of the space *E*. By today's standards also, Kurepa has the best results which refer to this function. In works from 1962 he proved one of the deepest results pertaining to this function:

$$c(\Pi_{\alpha\in A} X_\alpha) \le \exp(\sup_{\alpha\in A}(c(X_\alpha)))$$

Kurepa's proof was the first example of using the method of partitions calculus in the proofs of cardinal inequalities. Since then, this method becomes the standard means of proof in this area. In the partitions calculus, Kurepa's approach was different from Erdös', a mathematician who founded a school in this discipline. While starting point of Erdös' and his associates were Ramsey's known theorem and its various generalizations, Kurepa started from the results of Hasudorff and Urysohn related to the size of linearly ordered sets. Namely, Kurepa's goal was to consider the same problem on partially ordered sets and other binary relations. Kurepa's fundamental result in this area is the inequality (1937, 1939) for a partially ordered set *E*:

$$pE \le (2p_s E)^{p_0 E}$$

where *s* = anti-chain, *o* = well ordered or dually well ordered set, *p* = puissance. That is, pE is the cardinal number of *E*, p_sE is the cardinal number of the maximal anti-chain of *E*, while p_oE is the cardinal number of the maximal well-ordered, or dually well-ordered subset of *E*.

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[13] *La condition de Suslin et une propriete characteristique des nombres reels,* C.R. Acad. Sci. Paris **231**, (1950), 1113–1114

[14] See Todorcic's article *Trees and linearly ordered sets* in: K.Kunen and J.E. Vaughan (eds), Set-Theoretic Topology, North-Holand, Amsterdam, 1985, pp. 235–294.

[15] Here, for sentence φ, $\pm\varphi$ marks both φ, or negation of φ.

Kurepa's works in the field of axiomatic set theory mostly originate from the fifties and mainly relates to the principles of maximum associated with certain set-theoretical binary relations. The most famous example of this type is Kurepa's principle associated with set relation of non-comparability $(x \not\subset y$ and $y \not\subset x)$. It has been proved that Kurepa's principle can serve as an elegant supplement to the principle of ordering, that each set can be linearly ordered. Namely, Kurepa proved that in the ZF system, Kurepa's principle together with the principle of ordering is equivalent to the axiom of choice.

In topology, a part of Kurepa's research was related to the distance functions that are not numeric. In this context, Kurepa introduced new areas that are known today as Kurepa's pseudo-metric spaces. As mentioned, in the middle thirties Kurepa was on doctoral studies in Paris, and in that time, understandably, was influenced by the French mathematical school. Studying the works of his teacher and mentor Fréchet, Kurepa approached in the completely new method to the concept of space. He defined a pseudo-distant spaces (Espaces pseudo-distancies)[16], thus generalizing Fréchet's class *D*. In this approach, the value of the distance function instead in the set of positive real numbers are in a linearly ordered set, while the triangle condition at distance is replaced with a relation on the ordered sets. Later Fréchet himself came to the same spaces, which testifies of the naturalness of this generalization, and since then this class of space has been known as Kurepa–Fréchet spaces. In the beginning Fréchet did not know about these works of Kurepa, but during his visit to Belgrade after the Second World War, he learned about Kurepa's results. Later, many mathematicians began investigating these areas, among others Z. Mamuzic, R. Rapić, A. Appert, J. Colmez and V.G. Boltjanski. Mention that one of Kurepa's last works was related to this topic[17]. One could say that in this generalization, on these days the much explored concept of fuzzy (fuzzy) sets was anticipated. In any case, the phrase "*sets with smeared structure*" already appears in his book *Set theory* from 1951.

Professor Kurepa had an exceptional ability to feel good and important mathematical problem, and sophisticated construction, particularly in relation to ordered sets. On this occasion, we cannot mention many examples of this kind, but a problem in number theory deserves special attention, given that a greater number of compatriot and foreign mathematicians dealt with it. In 1971, at a meeting of mathematicians in Ohrid, Kurepa formulated the following problem. First, he defined an arithmetic function, $!n$, that he called "*left factorial*", as the factorials sum of the first $n-1$ natural numbers:

$$!n = 0! + 1! + 2! + \cdots + (n-1)!$$

Then $!n$-hypothesis states as follows:

The greatest common divisor of numbers !n and n! is 2.

This hypothesis has many interesting equivalent formulations and has been discussed by many mathematicians, among others, contributions were given by L. Carlitz, Wagstaff, W. Keller, and Yugoslav mathematicians Šami, Žizovic, Stankovic, Gogić, Ivić, Mijajlović, and others. This hypothesis is cited in the book by R. Guy *Unsolved problems in number theory*, Springer-Verlag, 1981, issue V44. The book is actually a collection of the most important open problems of number theory with descriptions of their solution attempts. Kurepa was the only former Yugoslav mathematician who has been the author of some problems recorded in this book. The hypothesis was tested by computer methods and tested for $n < 8.000.000$.

[16] *Tableaux ramifies d'ensembles, Espaces pseudo-distancies,* C.R. 1938, Paris (1934), 1563–1565.
[17] *General Ecart,* Simp. Filomat '92, Nis October 8–10, Zb. Rad. Fil. Fak. Nis, Ser. Mat. 6;2(1992), 373–379

Kurepa announced the solution, but the solution was never published. There is a place on the Internet since 1994[18], where more than several mathematicians (Kevin Brown, Helen Marie, Kurt Foster and others) discussed various aspects of this problem. In 1991 I received a letter from R.Guy in which he informs me that R.Bond from England allegedly solved this problem, but the solution has not come up yet.

Beside set theory, general topology, foundations of mathematics and number theory, Kurepa was interested for other areas of mathematics also. His works include topics from algebra, primarily from matrix theory, mathematical induction, then numerical mathematics, computer science and the theory of fixed points. He wrote a number of different papers, mostly in Serbian, which expressed his views of mathematics, history and methodology of mathematics and works appropriate to certain events, situations and new trends and discoveries in mathematics. Here are some examples titles of which illustrate this activityof his: *Cogito ergo sum. Celebration of the 350-birth anniversary of Descartes*, Student newspaper, 1946, *How to calculate sin (18o) using a five-pointed star*, Mathematical-physical and astronomical journal, 1946, *Electronic brains – the newest computer machines*, Nature, 1952, *Programming and a problem of Mihailo Petrović*, Mathematical journal, 1968, *Mathematics – entire life*, Educational review, 1976, *Around Bolzano's approach to real numbers*, Plenary lecture organized on Karl's University in Prague on the occasion of 200 years of the birth of Bernard Bolzano, 1981, Published on *Czeshoslovak mathematical journal*, 1982 (32).

Kurepa also published a monograph – a university textbook in the set theory in the early fifties that represented significant work not only for our region. The fact that this book is in many university libraries in the world is the best proof. The author of these lines has found this book in mathematical libraries of the prestigious Hebrew University in Jerusalem, the University of Madison, as well as in the London Library (British Library). As a rule, that was the only work on mathematics in Serbian and one of the few books of Yugoslav authors. He also wrote an extensive two-volume textbook from algebra that has been used by generations of students of mathematics. Professor Kurepa was a true ambassador of the Yugoslav science. He lectured at many universities in Europe, America and Asia, among others, in Warsaw, Paris, Moscow, Jerusalem, Istanbul, Cambridge, Boston, Chicago, Berkeley, Princeton and Beijing. On one occasion Kurepa said: "*I have held lectures on each of the nineteen universities of the (former) Yugoslavia, then in almost all European countries, also in Canada, Cuba, Israel and Iraq. I also held at least ten lectures in each of the following countries: France, Italy, Germany, the Soviet Union and the United States*". He participated in dozens of international mathematical symposia, and for many of them he was the main organizer (for example, the International Conference on topology, Herceg Novi 1968, Budva–Becici 1972, Belgrade 1977).

Đuro Kurepa is often cited in the international mathematical literature, especially in the field of set theory and general topology. Of all his works, certainly the most cited is his PhD thesis, but a large number of other papers have also been cited often. Citation index (CI), is taken by science today as one of the main measures of the importance of the contribution of some scientists. Although Kurepa is one of our most cited mathematicians, he belongs to a rare group of scientists for which this index is not so important. In fact, many mathematical concepts bear Kurepa's name: Kurepa's hypothesis, weak Kurepa's hypothesis, Kurepa's tree, Kurepa's lines, Kurepa's continuum, Kurepa's hypothesis of ramification, Left-factorial hypothesis, and several other functions. Kurepa also introduced other definitions such as

[18] Newsgroups: sci.math, Subject: Number Theory Problem

terms ramified set, Aronszajn's and Suslin's trees. In addition, he proved many of their properties. Today, those are all adopted and standard parts of the set theory corpus and general topology. Mathematicians in their writings discuss, prove and deepen the theorems which refer to those properties. And, when using some widespread concepts without mentioning their sources, in this case, Kurepa's works in which such concepts have sprung are often omitted from references. It is simply assumed that the reader is already familiar with them. However, in review articles, monographs and textbooks where the origin of the concept and ideas is important, Kurepa's works are listed. Let's look at one more example. In reference monograph from the general topology *Handbook of Set-theoretical topology*[19] in six out of 24 authors' contributions to the bibliography, 16 Kurepa's different works are listed. In the texts themselves, his results have been cited or included in whole at least 40 times. The first cited bibliographic unit is from 1935, while the last is from 1968. Concepts related to Kurepa's name explicitly are stated in the index nine times. According to the authors themselves, in many cases Kurepa's works were the first results in certain areas of the theory of partially ordered sets, infinite combinatorics and general topology. Let's mention that other mathematicians from this area in this monograph have also been cited: S. Todorčević, R. Papić and S. Mardešić. Article *Trees and linearly ordered sets* by S. Todorčević in this monograph provide a very complete and detailed overview of Kurepa's contribution to this field. Despite everything, Kurepa was still tortured by questions of authority and he thought that he had not always been sufficiently cited. In that respect, most of his objections went to mathematicians of Poland, especially K.Kuratovski with whom he had correspondence on the subject. The fact is that other mathematicians came up independently with certain results, the same or similar to Kurepa's. However, their results were published after Kurepa's, often the whole decade or more after. It, however, happened that other authors uncritically took and cited these, later published works, without mentioning Kurepa's name. On the other hand, some authors disparaged Kurepa's results by the sheer fact that they used at the same time or emphasized only the results of their fellow-countrymen. For example, J. Roitman on one place in his article *Basic S and L* from the already mentioned monograph in set topology says that Kurepa in one corollary proves that Suslin's line is an example of one L-space. Immediately after Roitman writes that "*This fact was widely and independently known in the United States about the same time*". But he does not confirm with a name or a reference this statement of his. All other results that are still cited are from the sixties or later. In the attached references the oldest work is Kurepa's thesis from 1935, the next oldest bibliographic unit is the work by Marry Ellen Rudin from 1955. We must admit that it takes really a lot of imagination to correlate 1935 and 1955 in "*about the same time*". In this regard, papers by S. Todorčević, like already mentioned article *Trees and linearly ordered sets*, then the contributions of editors in the book "*Selected works of Duro Kurepa*", and correct citing of the foreign, very respected mathematicians such as K. Kunen and K. Devlin, have contributed to determine the exact place of Kurepa's work in mathematics.

In the former Yugoslavia, Kurepa contributed very much to the introduction of modern views in mathematics and influenced significantly on the development on many of our mathematicians. It still cannot be said that Kurepa in Zagreb or Belgrade left the school in those areas where he had the best and most important results. From the older generation of mathematicians, probably the most important followers, primarily in general topology, were: R. Papić and Z. Mamuzić. From the middle-aged generation of mathematicians in this field works Ljubiša Kočinac, the professor at the University of Niš.

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[19] Editors K. Kunen and J.E. Vaughan, North Holland, 1984, p. 1273

Until the appearance of S. Todorčević on the mathematical scene in the early eighties, apart from Kurepa, other Yugoslav mathematicians had had no major results in the set theory. One could say without hesitation that the world mathematicians, especially experts from set theory, far better understood the importance of Kurepa's work in this field. By realizing a new kind of mathematical objects and linking with old and unsolved problems and his hypothesis, Kurepa for the most part set the path for the development of certain set theory disciplines, especially the theory of partly ordered sets. Many modern scientists, including the big names in this field, such as R. Solovay, R. Jensen, S. Shelach, accepted Kurepa's work, studied and brought to perfection the theory that Kurepa started.

Professor Kurepa was very appreciated as a mathematician and respected as a scientific authority in our country. On the other hand, a number of our influential mathematicians showed a certain animosity, directed more towards the subject Kurepa dealt with, than towards Kurepa himself. Several reasons can be mentioned which support this fact. The first would be that very few understood that area. Another reason is well-known to theoretical mathematicians, and that is that pure mathematics has an end in itself, close to philosophy. Modern set theory as part of foundation of mathematics touches with that end. Of course, simpler and from the standpoint of working mathematics, useful part of set theory is generally accepted. These are the language and framework that this theory gives. Subtle issues of consistency and the problem of infinity, which are discussed nowadays through the hypothesis on large cardinal numbers, remain mainly outside the interest of mathematicians specialized in other areas of mathematics. Here we immediately encounter the following reason, and that is the relation of mathematicians to the idea of infinity in mathematics. Leibniz's idea of infinitely small size is by nature closer to the analysts. Cantor's set theory introduces actual infinity in mathematics, trans-finite numbers and transfinite induction as a method of proof and construction of new mathematical objects. Hence appears the natural question of what it means in mathematics to a certain object types to exist. Is it necessary to construct that object or is it enough to find a proof of its existence from the accepted axioms? By the sheer presentation of these problems, finite and constructive image of mathematics collapses. Some mathematicians recessed in their theories remain indifferent to all these questions. Others, smaller part of mathematicians, reject infinity of Cantor's type. The third, with caution in their meta-mathematics, pragmatically accept parts of this theory, mainly what is useful for the mathematics they are dealing with. Kurepa belongs to the fourth, perhaps the narrowest circle of mathematicians, where various set-theoretical structures are based on transfinite (ordinal and cardinal) numbers that are also legitimate like classical structures of natural and real numbers. In one place in his book *Set theory*, while talking about well-ordered sets, Kurepa writes: "*Any such set*[20] *is the holder of a certain number so we come to the ordinal numbers, one of the most beautiful and bravest heritages of Cantor's set theory. In particular, the set of enumerable ordinal numbers refers to an infinite series and its understanding is as natural as conception of a set C of all the real numbers"*.

The mentioned attitude of influential mathematicians has not remained without consequences. A significant group of young mathematicians educated in Belgrade under the influence of professor Kurepa or in his spirit, under the leadership of professor Kurepa's students is now located in other countries. The following mathematicians successfully work and are doing research in the field of set theory and general topology there: Stevo Todorčević and his PhD Ilijas Farah (in Canada, i.e. USA), Žikica Perović (USA) and Boban Veličković (Paris). The name of Zoran Spasojević should be joined this group, who was fully educated in

[20] Well ordered

USA (on the University of Madison under the direction of K. Kunen), but in recent years made significant contacts with the mentioned group.

Given the size and complexity, it was not possible to present a complete scientific opus of professor Kurepa here. We can only conclude that Professor Đuro Kurepa has great merit for the development of set theory and mathematics in general. This is witnessed by the fact that his name is part of several basic concepts of modern set theory and topology: Kurepa tree, Kurepa hypothesis, Kurepa line, Kurepa space, Kurepa continuum. Only a great mathematician deserves such an honor.

Items in Virtual Library related to Đuro Kurepa

1. Ž. Mijajlović, *Đuro Kurepa 1907–1993* (in Serbian), in: *Works and Life of Serbian Scientists,* vol. **2**, SASA, 2002.
2. Đ. Kurepa, *Ensembles ordonnés et ramifiés*, doctoral dissertation, Paris, 1935.
3. Đ. Kurepa, *Teorija skupova* (the book Set theory), Đ. Kurepa, Zagreb, 1951.
4. Đ. Kurepa, *Viša algebra – knjiga prva* (the book Higher algebra, **1**), Belgrade, 1969.
5. Đ. Kurepa, *Viša algebra – knjiga druga* (the book Higher algebra, **2**), Belgrade, 1969.
6. *Selected Papers of Đuro Kurepa*, editors: A. Ivić, Z. Mamuzić, Ž. Mijajlović, S. Todorčević, Math. Inst. SASA, Belgrade, 1996.

Note: The electronic version of this paper contains the scientific bibliography of Đuro Kurepa.

Жарко Мијајловић

ЖИВОТ И ДЕЛО ЂУРА КУРЕПЕ

Ђуро Курепа (1907–1993) припада најужем кругу најзначајнијих српских математичара. Поред великог научног опуса, за собом је оставио неизбрисив утицај на развој савремене математике у претходној Југославији. Његово имее добро је препознато у светској математици, нарочито по радовима из теорије скупова и опште топологије. Његови резултати налазе се у скоро свакој савременој књизи из теорије скупова. У овом рукопису представљени су живот и дело Ђура Курепе.

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