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DIGITISATION OF TEXTBOOK *НЕБЕСКА МЕХАНИКА* BY MILUTIN MILANKOVIĆ

Abstract. We give a description of the book *Небеска механика (Celestial Mechanics)*, written by Milutin Milanković, a renowned Serbian scientist . This book was the first university textbook of celestial mechanics at the Belgrade University. In it motion of celestial bodies was for the first time treated by applying vector analysis. A digitised copy of the book has become accessible through the Virtual Library of the Faculty of Mathematics at http://elib.matf.bg.ac.rs.

1. Introduction

Milutin Milanković in this book uses the most up-to-date mathematical apparatus of that time. About this in *Предговор* (*Foreword*) the author says: *The science of motion of celestial bodies is treated by using the modern tools of vector analysis. This first, consistent, application of vector calculus to the problems of classical celestial mechanics shows all advantages compared to the tools used by this science up to now. To me it has been also useful in creating the theory of motion of the terrestrial poles built by Belgrade scholars of applied mathematics.*

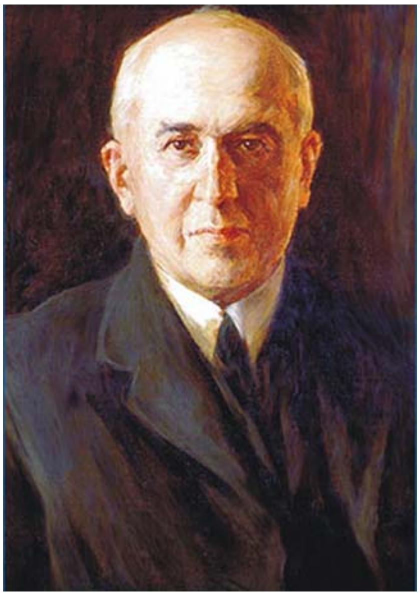
The university textbook *Небеска механика* (*Celestial Mechanics*) by Milutin Milanković introduces a reader in a very interesting way in the very complex problems of motion of celestial bodies giving them clear mathematical explanations and solutions. Although written, in Serbian, as long ago as in 1935, as the first textbook of celestial mechanics, it has remained, till nowadays, the only one. Using this textbook many generations of students have been learning.

2. Milutin Milanković's biography

Milutin Milanković was born on May 28, 1879 in Dalj (Slavonia, nowadays part of Republic of Croatia). The real gymnasium he finished in Osijek, to become a student of civil engineering at the Technische Hochschule in Vienna. There he took the first degree in 1903 and the PhD one in 1904. He was the first Serb who acquired the PhD title in engineering.

Following an invitation Milutin Milanković came from Vienna in 1909, after the University of Belgrade had been founded in 1905, to teach applied mathematics at the Faculty of Philosophy of this University. His coming was a merit of Bogdan Gavrilović and Mihailo Petrović Alas who both taught at the University of Belgrade. As a staff member Milanković was at the Belgrade University till his retirement in 1955. He taught applied mathematics (theoretical physics, mechanics and astronomy). He was the first in starting lectures in celestial mechanics. Among other subjects he taught history of astronomy and theory of relativity.

Milanković was a full member, and near the end of his life also Vice President of the Serbian Academy of Sciences, member of Deutsche Akademie der Naturforscher Leopoldina in Halle and corresponding member of a number of other academies and scientific societies in the world.



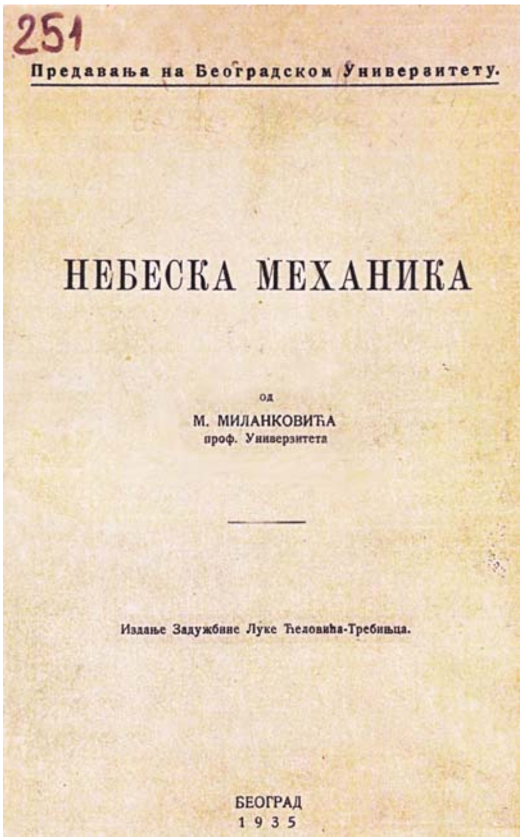
Milanković died on December 12, 1958 in Belgrade. His activity resulted in about ten books and more than hundred papers in mathematics, celestial mechanics, astronomy and geophysics.

Milutin Milanković gave to the world science community his well known theory of glacial ages. The results achieved by him taking into account the complex secular computation of perturbations in planet motions were published in *Théorie mathématique des phénomènes thermiques produits par la radiation solaire* in 1922; the publishers were Yugoslav Academy of Sciences and Arts in Zagreb and publishing house Gauthier Villars, Paris. Due to these results he became well known in the world scientific community, so that great German climatologist W. Köppen invited him to cooperate in building a great work *Handbuch der Klimatologie*. For this purpose a part, which was published in 1930, entitled *Mathematische Klimalehre und Astronomische Theorie der Klimaschwankungen* was written by Milanković. Here his theory of planet heating based on insolation was extended with a special reference to the Earth. By applying this theory to the run of glacial ages it was shown that Milanković had given an excellent model of Earth, i. e. that he had created a good and mathematically exact theory of terrestrial climate. This work was translated into Russian in 1939.

Already as a well known scientist he started the cooperation on creation of work *Handbuch der Geophysik* prepared by B. Gutenberg. For this work Milanković wrote four sections where he returned to an old and very difficult problem: that of motion of the terrestrial poles. In these contributions of his Milanković created a theory of motion of terrestrial poles and succession of glacial ages.

His main work is *Kanon der Erdbestrahlung und seine Anwendung auf das Eiszeitenproblem* the printing of which was finished shortly before the beginning of the Second World War. It should be mentioned that the printed sheets were damaged by a bomb which hit the printing office on April 6, 1941 during the Nazi air raid of Belgrade. Fortunately the original typography of the book was preserved. By using it the book was reprinted during the war on a sufficiently low-quality paper.

During the four war years Milanković could send abroad just a few copies. So this important scientific work was printed at the most unfavorable moment, in war time. Due to this the scientific world community learnt about Milanković's theory too late. As a consequence the world recognition arrived only after that. This work appears as a synthesis of his many earlier works which concern his research activity within boundary fields between many natural sciences and mathematics.



Milanković was successful in the field of calendar reform. During a Congress devoted to the calendar question organised by the Orthodox Churches in Constantinople in 1923 he proposed an improvement of the Gregorian time reckoning. The proposal contained in Milanković's calendar-reform project was accepted by the Orthodox Christian Community, but some Churches present at the Congress (for instance, the Serbian one) have not started its application.

After great Serbian scientist Milutin Milanković a minor planet, a lunar crater and a Mars crater have been named. In Belgrade one street and one gymnasium have been named after him.

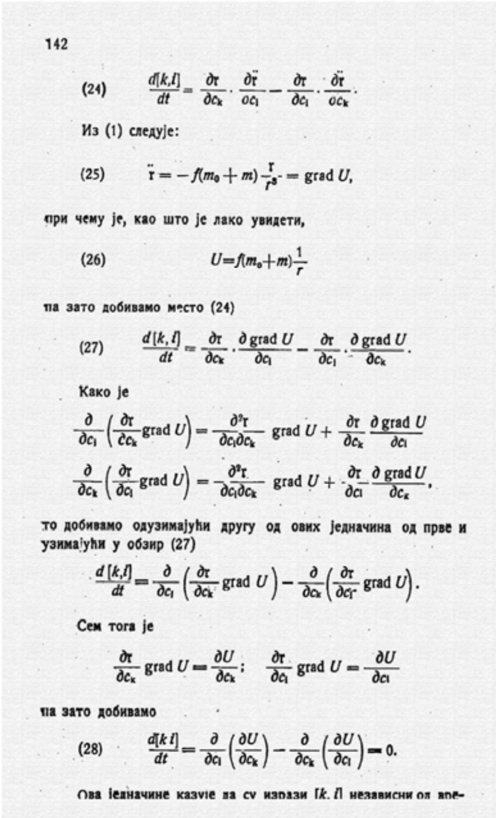
2.The Textbook

Milanković wrote two versions of his university textbook *Небеска механика.* The book was published due to the sponsorship of "Luka Ćelović-Trebinjac" in Belgrade in 1935. The base for this book was the lectures delivered by Milanković at the Belgrade University within the framework of subject Celestial Mechanics, Chair of Applied Mathematics. The main characteristic of this textbook is that in it vector calculus is introduced and used systematically in solving problems of celestial mechanics. Professor Milanković used the vector-scalar system of elements of planetary orbits. For this reason connecting his name with this system is fully justified.

Небеска механика was written on 333 pages with 22 figures in its text. The book contains two parts presented in 15 chapters. The first part *Транслаторно кретање небеских тела* (*Translatory Motion of Celestial Bodies*) contains nine chapters; the second one *Ротационо кретање небеских тела* (*Rotational Motion of Celestial Bodies*) is written on 130 pages containing six chapters. At the end of both parts there is a list of references in foreign languages.

2.1 First Part – *Translatory Motion of Celestial Bodies.* In the first chapter Milanković in an illustrative and very interesting way presented the historical development of the science concerning the motion of celestial bodies from the Chaldeans and Egyptians, through Greeks and Alexandrians, then through the Middle Ages, Copernicus, Galileo and Kepler till Newton. Concisely, chronologically and very illustratively he presented the Ptolemaic geocentric system of the world. Afterwards he explains Galileo's discoveries and Kepler's laws of planetary motion towards Newton's gravitation law. The first chapter offers to the reader an excellently clear and illustrative picture about the discoveries and development of astronomical science through centuries.

In the other chapters of this part Milanković in studying translatory motion of celestial bodies meets a number of problems: at first two-body problem, after this general ***n***-body problem, three-body problem, asteroid-motion problem, perturbation force, variation-constant method in equations of motion of celestial bodies, as well as perturbation calculation for motion of celestial bodies. At the beginning of each chapter he clearly formulated the problem which was the subject. Besides, he gave definitions, mathematical expressions, theorems with proofs, as well as methods necessary in solving these problems. The notions were introduced gradually and systematically, from simple towards the most complicated ones. In solving exceptionally complicated problems of translatory motion of celestial bodies he used, as in other cases, the vector-scalar system of elements of planetary orbits.



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$$(24)\qquad \frac{d[k,l]}{dt}=\frac{\partial \mathfrak{r}}{\partial c_k}\cdot\frac{\partial \ddot{\mathfrak{r}}}{\partial c_l}-\frac{\partial \mathfrak{r}}{\partial c_l}\cdot\frac{\partial \ddot{\mathfrak{r}}}{\partial c_k}.$$

Из (1) следује:

$$(25)\qquad \ddot{\mathfrak{r}}=-f(m_0+m)\frac{\mathfrak{r}}{r^3}=\text{grad}\,U,$$

при чему је, као што је лако увидети,

$$(26)\qquad U=f(m_0+m)\frac{1}{r}$$

па зато добивамо место (24)

$$(27)\qquad \frac{d[k,l]}{dt}=\frac{\partial \mathfrak{r}}{\partial c_k}\cdot\frac{\partial\,\text{grad}\,U}{\partial c_l}-\frac{\partial \mathfrak{r}}{\partial c_l}\cdot\frac{\partial\,\text{grad}\,U}{\partial c_k}.$$

Како је

$$\frac{\partial}{\partial c_l}\left(\frac{\partial \mathfrak{r}}{\partial c_k}\text{grad}\,U\right)=\frac{\partial^2 \mathfrak{r}}{\partial c_l \partial c_k}\,\text{grad}\,U+\frac{\partial \mathfrak{r}}{\partial c_k}\frac{\partial\,\text{grad}\,U}{\partial c_l}$$

$$\frac{\partial}{\partial c_k}\left(\frac{\partial \mathfrak{r}}{\partial c_l}\text{grad}\,U\right)=\frac{\partial^2 \mathfrak{r}}{\partial c_l \partial c_k}\,\text{grad}\,U+\frac{\partial \mathfrak{r}}{\partial c_l}\frac{\partial\,\text{grad}\,U}{\partial c_k},$$

то добивамо одузимајући другу од ових једначина од прве и узимајући у обзир (27)

$$\frac{d[k,l]}{dt}=\frac{\partial}{\partial c_l}\left(\frac{\partial \mathfrak{r}}{\partial c_k}\text{grad}\,U\right)-\frac{\partial}{\partial c_k}\left(\frac{\partial \mathfrak{r}}{\partial c_l}\text{grad}\,U\right).$$

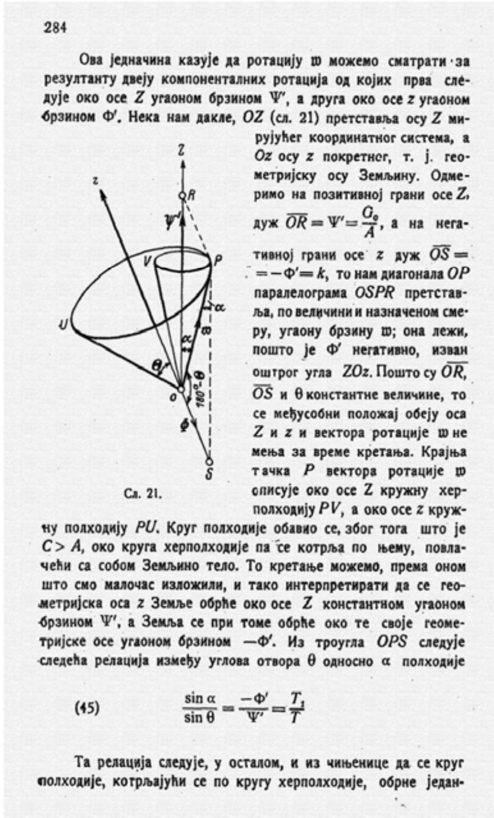
Сем тога је

$$\frac{\partial \mathfrak{r}}{\partial c_k}\,\text{grad}\,U=\frac{\partial U}{\partial c_k};\quad \frac{\partial \mathfrak{r}}{\partial c_l}\,\text{grad}\,U=\frac{\partial U}{\partial c_l}$$

па зато добивамо

$$(28)\qquad \frac{d[k,l]}{dt}=\frac{\partial}{\partial c_l}\left(\frac{\partial U}{\partial c_k}\right)-\frac{\partial}{\partial c_k}\left(\frac{\partial U}{\partial c_l}\right)=0.$$

Ове једначине казују да су изрази [k, l] независни од вре-



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Ова једначина казује да ротацију $\mathfrak{w}$ можемо сматрати за резултанту двеју компоненталних ротација од којих прва следује око осе Z угаоном брзином Ψ', а друга око осе z угаоном брзином Φ'. Нека нам дакле, OZ (сл. 21) претставља осу Z мирујућег координатног система, а Oz осу z покретног, т. ј. геометријску осу Земљину. Одмеримо на позитивној грани осе Z, дуж $\overline{OR}=\Psi'=\frac{G_0}{A}$, а на негативној грани осе z дуж $\overline{OS}=-\Phi'=k$, то нам дијагонала OP паралелограма $OSPR$ претставља, по величини и назначеном смеру, угаону брзину $\mathfrak{w}$; она лежи, пошто је Φ' негативно, изван оштрог угла ZOz. Пошто су $\overline{OR}$, $\overline{OS}$ и θ константне величине, то се међусобни положај обеју оса Z и z и вектора ротације $\mathfrak{w}$ не мења за време кретања. Крајња тачка P вектора ротације $\mathfrak{w}$ описује око осе Z кружну херполходију PV, а око осе z кружну полходију PU. Круг полходије обавио се, због тога што је $C>A$, око круга херполходије па се котрља по њему, повлачећи са собом Земљино тело. То кретање можемо, према оном што смо малочас изложили, и тако интерпретирати да се геометријска оса z Земље обрће око осе Z константном угаоном брзином Ψ', а Земља се при томе обрће око те своје геометријске осе угаоном брзином $-\Phi'$. Из троугла OPS следује следећа релација између углова отвора θ односно α полходије

Сл. 21.

$$(45)\qquad \frac{\sin\alpha}{\sin\theta}=\frac{-\Phi'}{\Psi'}=\frac{T_1}{T}$$

Та релација следује, у осталом, и из чињенице да се круг полходије, котрљајући се по кругу херполходије, обрне један-

Note. On page 142, Milanković's inference of mathematical formulas involving Langrage's brackets by use of vector analysis is displayed. On page 284, the cones of polhode and herpolhode are depicted. In fact, this picture represents the mechanism of moving of the Earth's poles.

2.2 Second Part – Rotational Motion of Celestial Bodies. At the beginning of this part, in Chapter 10, Milanković presented the theorems and formulae of rational mechanics which are necessary in studying rotational motions of celestial bodies. Among them are the impulse theorem, the one concerning the centre-of-mass motion and the Eulerian equations, followed by descriptions of Euler's angles, of polhode and herpolhode, as well as other formulae.

In the next chapter he described the rotation of a fluid body, presented Appell's theorem, equilibrium conditions and Clairaut' theorem. In the other chapters he mainly focused on the planet Earth. He described the precession of the Earth's axis, astronomical nutation of the Earth's axis, free nutation of the Earth and secular motion of the terrestrial poles. The Earth as a whole he considered as a fluid body, which in the case of short-duration forces behaves as a solid body, but under an influence behaves as an elastic body. Milanković possessed an extraordinary ability of describing natural phenomena by using the language of mathematics. For instance, using vector analysis he made an excellent mathematical model of the Earth which served him in creating a theory of secular motion of the terrestrial poles. In Section 69 he derived the equation of secular trajectory of a terrestrial pole and also the equation of pole motion along this trajectory. He found that the secular pole trajectory depends only on the configuration of the terrestrial outer shell and the instantaneous pole position on it, more precisely on geometry of the Earth mass. On this basis he could calculate the secular pole trajectory. In the next section (**70**) he gives a short presentation of another theory concerning the same problem that of Anton Bilimovitch, also a representative of the Belgrade school of applied mathematics.

Milanković's results have been confirmed by the International Latitude Service on the basis of measurements. The results obtained on the basis of his theory of secular motion of the terrestrial poles are concordant with those based on the measurements organised by the Service. For instance, both Milanković's model and the measurements show that this motion is very slow and its value does not exceed 160 mm per year. Such a value for the motion of the terrestrial poles has enabled us to explain, for instance, the formation of big coal reserves on the Spitsbergen Islands, which could not form at the present latitude of these islands. Also, based on Milanković's model one can determine the space orientation of the terrestrial axis depending on the polar motion. The mechanism of the phenomenon is given in Fig. 21.

The number of figures given in the textbook is small. In most cases instead of a figure Milanković gave its description, but exceptionally well, clearly and illustratively. So, a figure arose before reader's eyes.

The list of references shows that Milanković used a lot of references in foreign languages, both books and the most recent scientific results of his time. He also cites works of Ptolemy, Copernicus, Galileo, Kepler, Newton, as well as of Laplace, Poincaré and other great names of world science.

3. Conclusion

Milanković in his textbook *Небеска механика* presented systematically the use of vector calculus in solving problems of classical celestial mechanics. This is an important application of vector analysis in solving the complex problems of motion of celestial bodies. By using these modern tools the mathematical expressions and differential equations describing the translatory and rotational motions of celestial bodies were written in a clear and elegant way. Vector analysis was used by Milanković also in creating the theory of secular motion of the terrestrial poles presented in Chapter 15, the last one of this book. The theory of secular motion of the terrestrial poles is an important contribution of Milanković to the world science.

This textbook was used by many generations of students of the Belgrade University for the purpose of learning celestial mechanics. It is still used. Time has not made this textbook out-of-date, so it has had many editions. It is still the only textbook of celestial mechanics written in the Serbian language.

To meet the needs of students of the Faculty of Mathematics Dr S. Šegan, Reader at the Department of Astronomy, has prepared an edition of this textbook [3]. In the Virtual Library of the Faculty of Mathematics [5] there are digital copies of ten books written by Milanković, among them are all books mentioned here.

References

[1] М. Миланковић, *Небеска Механика*, Издање Задужбине Луке Ћеловића-Требињца, Штампарија Графички институт, Београд, 1935.

[2] М. Миланковић, *Реформа Јулијанског календара*, Српска Краљевска академија наука и уметности, Посебна иѕдања, књига XLVII, Науке природне и математичке, књига II, Штампарија Свети Сава, Београд, 1923.

[3] М. Миланковић, *Небеска механика*, едитор С. Шеган, Vesta Company, Београд, 1995.

[4] Ј. Симовљевић, *Астрономија*, Тридесет година Природно-математичког факултета Универзитета у Београду 1947–1977, Београд, 1980, 165–195.
[5] Ж. Мијајловић, Виртуелна библиотека: http://elib.matf.bg.ac.rs

Сажетак. Представљамо књигу *Небеска механика*, српског научника светског гласа Милутина Миланковића. Књига је била први универзитетски уџбеник из Небеске механике на Београдском универзитету. У њој су по први пут обрађена кретања небеских тела применом векторске анализе. Дигитализован примерак књиге постављен је у Виртелну библиотеку Математичког факултета на адреси http://elib.matf.bg.ac.rs.

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