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Mirjana Borisavljević

DOCTORAL DISSERTATIONS IN LOGIC
FROM VIRTUAL LIBRARY OF
THE FACULTY OF MATHEMATICS IN BELGRADE

Abstract. We present Doctoral dissertations in logic of Serbian scientists from the Virtual Library of the Faculty of Mathematics, University of Belgrade (http://elib.matf.bg.ac.yu:8080/virlib/).

Introduction

It is possible to distinguish two periods in the history of logic. The first period can be called Aristotle's era. The second period, the period of modern logic, was inaugurated by the works of George Boole and Gottlob Frege. The year 1879, when Frege's *Begriffsschrift* was published can be taken as the beginning of modern logic. In the 1870s Georg Kantor introduced *set theory*, which would become one of the major areas of logic. In the period between the two world wars the other major areas of logic, model theory, proof theory and recursive theory, were born. Gedel's completeness theorem (1930) can be considered as the beginning of *model theory*, the dominant area of logic in the second half of the 20th century. In the 1920s the constructivist foundational views of L.E.J. Brouwer (Luitzen Egbertus Jan Brouwer) formalize the major alternative logic, *intuitionistic logic*. Intuitionistic implication (which is different from material implication) and Hilbert's views about foundations of mathematics made the basic of *proof theory*. Finally, the 1930s saw the birth of *recursion theory*. At the end of the 20th century, very strong connections between logic and *theoretical computer science* were made. Through this connection logic became one of the most applied areas of mathematics.

The first Serb with modern logic as his area of research was Konstantin (Kosta) Branković (1814–1865). The first Doctoral dissertation in logic of a Serbian scientist was the one of Ljubomir Nedić (1858–1902): "Die Lehre von der Quantification des Prädicats in der neueren englischen Logik" (Leipzig 1885).

If our goal were to present modern logic in Serbian science and Serbian scientists in modern logic, then we would have a very extensive and complex task. It could be done in several ways. The presentation could consist of brief reviews of works of Serbian logicians. Đuro Kurepa is probably the best known Serbian logician. Kurepa gave valuable contribution to many areas of mathematics. He was especially interested in set theory, general topology, foundations of mathematics, number theory and algebra. His work includes themes on partially ordered sets, particularly trees, the continuum hypothesis, the principle of mathematical induction, cardinal functions in topology, the general theory of uniform and metric spaces, fixed point theorems, the so-called left factorial function, and some problems in matrix theory. Several important mathematical notions carry the name after him: Kurepa tree, Kurepa hypothesis, Kurepa line, Kurepa space, etc. We can say that in Serbia the research of modern

logic, as an area of mathematics and philosophy, begins with the founding of Seminar for Algebra and Logic. In 1965 Professors Slaviša Prešić and Aleksandar Kron, together with some other mathematicians and philosophers from the University of Belgrade, founded Seminar for Algebra and Logic at the Faculty of Mathematics. That seminar was later renamed into Seminar for Mathematical Logic and it has been a seminar of Mathematical Institute SANU since 1970. The development of logic in Serbia is closely connected with the work of that Seminar. So, the presentation of logic in Serbia can be a presentation of the history of the Seminar.

Here we present only a small part of many important results of Serbian logicians. Namely, we present brief reviews of Doctoral dissertations in logic from Virtual Library of the Faculty of Mathematics of the University of Belgrade (http://elib.matf.bg.ac.yu:8080-/virlib/). That library contains all Doctoral dissertations of Serbian logician who have received PhD degree in logic at the Faculty of Mathematics of the University of Belgrade. Moreover, it also contains several Doctoral dissertations of Serbian logicians, who have received PhD degree at Universities around of the world.

Reviews of Doctoral dissertations

Đuro Kurepa, Ensembles ordonnés et ramifiés

Keywords: set theory, partially ordered sets, linear ordering, trees
PhD Committee: Maurice Frechet (supervisor), Paul Montel, Arnaud Denjoy
Faculty: Université de Paris, Sorbonne; *Date*: 1935
Language: French

This dissertation is the first systematic study of trees and ramified partially ordered sets and of their close relationship to linear orderings. It was the source of many crucial notions and problems in this area as, for example, the notions of Aronszajn tree and Souslin tree. The problem whether inaccessible cardinals have the tree property i. e., whether they satisfy the analogue of Koning's infinity lemma is considered in this thesis for the first time. The thesis consists of Chapter I (the subchapters t1-t8), Chapter II (the subchapters t9-t12), and an appendix ("Complément"). In t8A11 trees are classified as "large", "étroit" and "ambigu" according to their heights and widths. In the Theorem 5bis the following property is presented: the very thin and tall trees ("étroit") always have cofinal branches i.e., chains intersecting every level. This result was a source of the problem whether the same fact is true about the class of slightly wider trees ("ambigu") i.e., the trees of height equal to some cardinal θ and whose levels are now only assumed to be of size less than θ. This is the problem known today as the problem whether θ has the tree property. In t10.2 the important notion σE is defined, where E is a linearly (or partially) ordered set. Namely, σE is a tree of all nonempty bounded and well-ordered subsets of E with the end-extension as the tree ordering. The problem whether inaccessible cardinals have the tree property appeared in t10.3. In t10.4 two following problems are mentioned: whether every Aronszajn tree is a subtree of σQ, and whether every two uniformly branching Aronszajn trees are isomorphic. A question related to previous one, whether there is a homogeneous Aronszajn tree is also mentioned. The property that every two countable infinitely branching trees of the same height are isomorphic is proved in t10.5 (Theorem 1). Appendix contains a proof that Souslin's problem is equivalent to the statement that every uncountable tree contains an uncountable chain or antichain i.e., that three are no Souslin trees.

Žarko Mijajlović, Prilog teoriji modela i Bulovih algebri
(A Contribution to Model Theory and Boolean Algebras)

Keywords: model theory, Boolean algebras, Lindenbaum algebras, type, elimination of quantifiers, model completion, atomless, absoluteness, Levy hierarchy, Dedekind number, Kurepa number
PhD Committee: Slaviša Prešić (supervisor), Djuro Kurepa, Branka Alimpić
Faculty: Faculty of Mathematics of the University of Belgrade; *Date*: April 1977
Language: Serbian; *References*: 43; *Pages*: VI+158

The dissertation consists of six chapters. Basic notions of model theory are given in Chapter 1. In Chapter 2, dual notions in categories of Boolean algebras and Stone spaces are studied with respect to natural contra-variant functor, and cardinal properties of the cellularity number of a Boolean algebra are proved. The property that every Boolean algebra is isomorphic to the Lindenbaum algebra B_1 of Σ_1 formulas of certain first-order complete theory is proved in Chapter 3. Stability number $ST(k)$ of a first-order theory T is studied, and it is shown that $ST(k) = Ku(k)$, where $Ku(k)$ is the Kurepa number and T is the theory of dense linear ordering without end-points, while the cardinality of the Stone space of $B_1(A)$ (A is a model of T) is equal to the Dedekind number of the ordering A. In Chapter 4 the Σ_n, Π_n ramifications of various notions in model theory are defined and studied. Preservation theorems for these types of formulas are proved and examples for including ordered structures and algebraic fields are given too. In Chapter 5 model completions and elimination of quantifiers are studied. It is proved that the classes of Boolean algebras and distributive lattices with the least and the greatest elements are Jonsson's classes, and an algebraic description of saturated models of submodel-complete theories is given. Chapter 6 studies the question: what model-theoretic properties are absolute in ZF in the sense introduced by Levy? It is shown that the properties of consistency, completeness, model-completeness and elimination of quantifiers of first-order theories are absolute.

Ratko Tošić, Prilog teoriji Bulovih algebri
(A Contribution to the Theory of Boolean Algebras)

Keywords: Boolean algebras, Boolean functions, search theory
PhD Committee: Đuro Kurepa, (supervisor), Slaviša Prešić, Koriolan Gilezan
Faculty: Faculty of Mathematics of the University of Belgrade; *Date*: April 1978
Language: Serbian; *References*: 71; *Pages*: 94

The dissertation consists of five chapters. In Chapter 1 definitions and well-known results from the theory of Boolean algebras and Boolean functions are given. In Chapter 2 some properties of Boolean functions, which preserve constants under finite Boolean algebras, are presented by using the component representation. Their consequences about the number of Boolean's functions are also given. The theorems which are the generalization of Scognamiglio theorem and Andreoli's theorem for Boolean functions with one variable are proved in Chapter 3. The following new notions are introduced for monotone logical functions: the profile, the level, homogeneous, the corresponding matrix, etc. Some properties of these functions are shown and some consequences about the number of homogeneous monotone logical functions are presented. In Chapter 4 the applications of monotone Boolean functions, in solving the problems of search theory (a branch of the theory of information), are presented. It is shown that the general problem of a type is, in fact, the problem of identifications of homogeneous monotone Boolean functions of the given profile by checking the value of

that function for combinations of values of variables. Optimal or almost optimal solutions for some profiles are shown. It is also shown that monotonic logical functions are natural instrument for the generalization of these problems. Some open problems are presented in Chapter 5.

Gradimir Vojvodić, Prilog proučavanju raznovrednosnog predikatskog računa (A Contribution to Research of Mixed-Valued Predicate Calculus)

Keywords: model theory, mixed-valued predicate calculi
PhD Committee: Slaviša Prešić (supervisor), Branka Alimpić, Svetozar Milić
Faculty: Faculty of Mathematics of the University of Belgrade; *Date*: January 1979
Language: Serbian; *References*: 40; *Pages*: 79

Results from model theory of mixed-valued predicate calculus are presented, by using results of the model theory of classical predicate calculus. The dissertation consists of four chapters. The basic notions, definitions and properties are given in the introductory chapter. The main theorems, which are proved in Chapter 1, are the following: the weak separable theorem for k-models (the characteristic theorem for mixed-valued predicate calculus with respect to other logics based of Post's algebras); the theorem about the connection between k-models of mixed-valued predicate calculus and models of classical predicate calculus; and the theorem of the difference between the theory of k-models of mixed-valued predicate calculus and theory of models of classical predicate calculus. By using results from the previous chapter, the theorems of mixed-valued predicate calculus (analogous to the well-known theorems from classical predicate calculus: Craig's interpolation lemma, Beth's theorem about definability and the II_ε-theorem) are proved in Chapter 2. Results concerning structures of mixed-valued equivalence relations and congruence relations (which is connected with the well-known results from universal algebras about the structure of equivalence relations and congruence relations) are presented in Chapter 3.

Stevo Todorčević, Rezultati i dokazi nezavisnosti u kombinatornoj teoriji skupova (Results and Proofs of Independence in Combinatorial Set Theory)

Keywords: combinatorial set theory, Aronszajn's trees, Cantor's tree, linearly ordered topological spaces, Continuum hypothesis
PhD Committee: Đuro Kurepa (supervisor), K.J. Devlin, Milosav Marjanović, Slaviša Prešić
Faculty: Faculty of Mathematics of the University of Belgrade; *Date*: 1979
Language: Serbian; *References*: 79; *Pages*: 157

The dissertation consists of four chapters and one appendix. The relation between trees and ordering types, especially the relation between tree-subtree and the type-subtype are considered in Chapter 1. By using Jensen's principle, Aronszajn's tree which does not contain any Aronszajn's subtree and Cantor's subtree are constructed. Moreover, it is shown that in the model ZFC+GCH each ω_2- Aronszajn's tree contains Aronszajn's and Cantor's subtree. In the first part of Chapter 2 the problem of the existence of Boolean algebras which have non-trivial automorphisms and endomorphisms are studied. It is shown that for each cardinal $\kappa, \kappa > \omega$, there are exactly 2^κ types of isomorphic Boolean algebras without non-trivial automorphisms. In the second part of that chapter the problem of isomorphism and automorphism of ω_1-trees is studied. It is shown that there are 2^{ω_1} types of isomorphic total rigid

Aronszajn's trees, so an Aronszajn tree does not have any nontrivial automorphism. Several problems of the partition relations of cardinal numbers are solved in Chapter 3. The appendix contains the proof of the property that in ZFC the σ-dense partial ordered set of power ω_1 does not exist. It is shown that in ZFC there is not any linearly ordered topological space with weight less than or equal to ω_1 which satisfies Kurepa's generalization of the notion of separable topological space. It is also shown that if $\neg\omega$ Kurepa's hypothesis+Martin's axiom+$\neg$Continuum hypothesis is assumed, then each perfect normal non-Archimedean space whose weight is ω_1 is measurable.

Zoran Marković, Model Theory for Intuitionistic Logic

Keywords: intuitionistic logic, Kripke structures, model theory
PhD Committee: Scott Weinstein (supervisor), Michael Friedman,
Faculty: University of Pennsylvania; *Date*: 1979
Language: English; *References*: 41; *Pages*: 77

The results from this dissertation are obtained by using notions and procedures which are well-known in the Kripke structures in the first place, together with some other constructions. They might provide insights about intuitionistic formal theories analogous to insights about classical logic provided by results of classical model theory. The dissertation consists of three chapters. The definitions concerning syntax of the first order intuitionistic logic, the definitions and theorems about the Kripke structures, Heyting algebras and saturated theories are given in Chapter 1. In the first part of the next chapter a few results about the connection between forcing and classical satisfaction relation are proved. In the second part of that chapter three alternatives of the antecedent of the omitting type theorem are presented, and an omitting types theorem is proved. It is important that there are many applications of that theorem. In Chapter 3 the following two kinds of products are considered: prime products of saturated theories and ultra products and reduced products of the Kripke structures. In the first part of that chapter the following property is proved: a simple analogue of the ultraproduct construction can be defined in terms of saturated theories. An important result from the second part of Chapter 3 is that the class of formulas preserved under reduced products is much broader than the class of formulas which are intuitionistically equivalent to Horn formulas.

Kosta Došen, Logical Constants an Essay in Proof Theory

Keywords: logical constant, sequent system, limits of logic, alternative logics
PhD Committee: Michael Dummett (supervisor), Dana Scott (secondary and temporary supervisor), Daniel Isaacson
Faculty: Litterae Humaniores in the University of Oxford; *Date*: October 1980
Language: English; *References*: 146; *Pages*: 325

A characterization is given in terms of invertible sequent rules of the logical constants of first-order classical, intuitionistic and relevant logic. The dissertation consists of ten chapters. Some preliminary considerations on languages and systems of provable sentences are presented in Chapter 1. In Chapter 2 structural systems are considered, and it is shown that in these systems a general form of the cut rule is not eliminable. In Chapter 3 the structural analysis, whose main ingredient is the notion of analytic rules, is defined. Analytic rules for the propositional constants of classical logic are considered in Chapter 4. In Chapter 5 the same analytic rules are used to analyze the propositional constants of intuitionistic logic and

of a relevant logic. As suggested by Gentzen's treatment of classical and intuitionistic logic, and as in substructural logics, when one passes from classical to nonclassical logics, the rules for the logical constants don't change; what changes is only the structural rules. This characterization yields a criterion for the demarcation of logic. Analytic rules for modal propositional, first-order quantifiers and identity are considered in Chapters 6-8. In Chapter 9, uniqueness of expressions, which is another ingredient of the notion structural analysis, is introduced. The notion of structurally alternative systems is introduced in Chapter 10. This notion together with the results of the thesis (which can be expressed with help of this notion) are compared with other works.

Slobodan Vujošević, Prilog teoriji Hejtingovih algebri (A Contribution to the Theory of Heyting Algebras)

Keywords: Heyting algebras, Boolean algebras, distributive nets, categories, functor, adjunction, monad, comonad
PhD Committee: Žarko Mijajlović (supervisor), Slaviša Prešić, Aleksandar Kron
Faculty: Faculty of Mathematics of the University of Belgrade; *Date*: 1982
Language: Serbian; *References*: 75; *Pages*: 106

The dissertation consists of three chapters. Heyting algebras are studied as an equality category in Chapter 1. The properties of filters and ideals of Heyting algebras are presented together with corresponding properties in distributive nets and Boolean algebras. Free, injective and projective Heyting algebras are presented and a theorem about the representation in algebras with closing is proved. Some properties of Heyting algebras, which are important for study of formal logics closely to intuitionistic logic, are also presented. Complete Heyting algebras are studied in Chapter 2. The family of complete Heyting algebras is obtained by repeating the construction of the algebra of J-operators. Some properties of this family, when the initial Heyting algebras is linear order, are studied. Moreover, the characterization of complete Heyting algebras which can be approximated by complete Boolean algebras is given. Duality of categories of topological spaces and complete Heyting algebras are studied in Chapter 3. Some adjunctions are defined, and for those adjunctions the actions of monad and comonad are studied. It is shown that the category of complete Heyting algebras is reflective in the categories of sets, distributive bounded nets and complete Heyting algebras. It is shown that complete Heyting algebras correspond to "deposited" spaces, and distributive bounded nets correspond to a restriction of Stone's spaces.

Aleksandar Jovanović, Prilog teoriji ultraproizvoda (A Contribution to the Theory of Ultraproducts)

Keywords: ultrafilter regularity, weak normality, cardinality, measure
PhD Committee: Žarko Mijajlović (supervisor), Slaviša Prešić, Zoran Marković
Faculty: Faculty of Mathematics of the University of Belgrade; *Date*: October 1982
Language: Serbian; *References*: 176; *Pages*: 146

The dissertation consists of five chapters. The Chapters 1 and 2 contain the known results from the model theory and the set theory which are used in the other chapters. A classification of the properties of filters is given in Chapter 3. Some connections between combinatoric properties are made and the theorems about existence are also given. Ultraproducts are studied in Chapter 4. The structure of ultraproducts is connected with the structure of ultrafilters and cardinality of ultraproducts. Moreover, some other problems are studied, as the 2-

cardinality problem. In the case of a measurable cardinal the connection with continuum problem is presented and several theorems of the cardinality of ultraproducts are proved. The problems about the real measure are studied in Chapter 5. The forcing is presented and by using results from Chapter 3 and Chapter 4 several properties are proved. The notion of norm of measure is introduced and some possible relations between additivity and norm of a measure are studied. Real large measurable cardinals are introduced analogously as the other large cardinals. The inspiration for this introduction were Solovay's results of equconsistency of the theory ZDF + "there is a measurable cardinal" and the theory ZDF+"there is a real measurable cardinal". The relative consistency of the real large measurable cardinals with respect to ZDF+"the corresponding large cardinal" is proved by a generalization of Solovay's forcing.

Milan Božić, Prilog semantici relevantnih logika
(A Contribution to Semantic of Relevant Logics)

Keywords: relevant logic, Kripke structures, modal logics
PhD Committee: Aleksandar Kron (supervisor), Slaviša Prešić, Žarko Mijajlović
Faculty: Faculty of Mathematics of the University of Belgrade; *Date*: 1983
Language: Serbian; *References*: 39; *Pages*: iii+106

The dissertation consists of five chapters. In the introductory chapter some relational-operational structures building of sets of formulas in calculi RA^+ and R^+ are presented. These structures are used in other chapters in making of the canonical frames of Kripke-type for semantic of positive fragments of relevant propositional calculi. In Chapter 1, the semantic of these positive fragments, which is a mixture of known Routley-Meyer type semantic and Maksimova's semantic, is presented. A new way for the semantic of negation in relevant logics is introduced in Chapter 2. It this way semantic completeness theorems for a large class of expansions of the logic R_{min} are proved. It is shown that Routley-Meyer's semantic for the logic R is a special case of that semantic. Relevant modal logics are studied in Chapter 3. A semantic of Kripke's type by which a completeness of a large class of modal logics whose basics are different relevant calculi with or without negation is introduced. A characterization of a large class modal i.e. Hintikka schemas, is given too. They contain almost all modal schemas characterized by formulas of the first order. Moreover, it is proved that the only known semantic for the calculus $R_\odot$ is a special case of the semantic given in this chapter. Semantics of relevant calculi which are not distributive are studied in Chapter 4. It is shown that semantics of non-distributive relevant logics radically change Kripke's semantic.

Miodrag Rašković, Logike sa merom u Lajbnicovom univerzumu
(Logics with Measure in Leibniz's Universe)

Keywords: Loeb measure, nonstandard analysis, probability logic, alternative set theory, functional equations, lifting theorems, measurable function, continuous function
PhD Committee: Žarko Mijajlović (supervisor), Slaviša Prešić, Dragoljub Aranđelović
Faculty: Faculty of Mathematics of the University of Belgrade; *Date*: 1983
Language: Serbian; *References*: 47; *Pages*: 89

The results from this dissertation contributed to the development of model theory for probability logic with values in $\{0,+\infty\}$. The thesis consists of three chapters. The basic notions and theorems from nonstandard analysis and the measure theory are given in Chapter 1. Also, by using the methods of nonstandard analysis, it is proved that if a function f, $f : R \rightarrow R$ is

Lebesgue measurable, a function $g : R^4 \to R$ is continuous and the equation $f(x+y) = g(f(x), f(y), x, y)$ holds, then the function f is also continuous. The logics $L_{\omega M}$, $L_{\omega_1 M}$, L_{AM} and L^s_{AM} are defined in Chapter 2. The main characteristic of these logics is that their models are σ finite. Some of the axioms of these logics are modifications of known axioms and some of them are new, as the axioms of σ-finiteness. The property of completeness, Barwise completeness and compactness for L_{AM} are proved. Moreover, the theorem of elementary equivalence, the theorem of Robinson's coexistence, several theorems of interpolation, upper Skolem–Löwenheim theorem and the theorem of normal form are proved. In Chapter 3 of the dissertation the Loeb measure is founded in the alternative set theory. The theorems which are analogous to some theorems from nonstandard analysis are proved and some limitations of the alternative set theory are presented. Finally, a new proof of the well-known Lusin's theorem is given.

Branislav Boričić, Prilog teoriji intermedijalnih iskaznih logika (A Contribution to the Theory of Intermediate Propositional Logics)

Keywords: normalization, cut elimination, superintuitionistic logics, intermediate logics, natural deduction, Kripke model, decidability, classical logic
PhD Committee: Slaviša Prešić (supervisor), Žarko Mijajlović, Kosta Došen
Faculty: Faculty of Mathematics of the University of Belgrade; *Date*: March 1984
Language: Serbian; *References*: 136; *Pages*: vii+154

The dissertation consists of four chapters. Chapter 1 contains a general framework for deductive systems and contains a sequence-conclusion natural deduction system for classical first order logic. A sequence NLC_n of intermediate propositional logics is considered in Chapter 2. It is shown that the sequence NLC_n contains three different systems only. These are the classical calculus NLC_1, Dummett's system NLC_2 and the logic NLC_3, an extension of the Heyting propositional logic by the axiom $(A \Rightarrow B) \vee (B \Rightarrow C) \vee (C \Rightarrow A)$. It is also shown that the logic NLC_3 is separable. In the sequel, the completeness of NLC_3 with respect to the corresponding Kripke type models having the property that $\forall x \forall y \forall z (xRy \vee yRz \vee zRx)$ is proved, as well as its decidability and the independence of logical connectives. It is shown that some subsystems of NLC_3 are separable and that the limits of the considered systems are the Heyting propositional calculus. The logic of the weak law of excluded middle, an extension of the Heyting logic by $\neg A \vee \neg \neg A$, is considered in Chapter 3. An embedding of classical logic into this logic is described and it is proved that this logic is the minimal one having this property. A Hilbert-type formulation of implication fragment of the Heyting propositional logic formalizing the deducibility relation, is presented in Chapter 4, enabling to define a decision procedure based on a kind of cut-elimination theorem.

Milan Grulović, Forsing u teoriji modela   (Forcing in Model Theory)

Keywords: model theory, infinite logic, forcing
PhD Committee: Žarko Mijajlović (supervisor), Slaviša Prešić, Svetozar Milić
Faculty: Faculty of Mathematics of the University of Belgrade; *Date*: 1984
Language: Serbian; *References*: 45; *Pages*: iv+73

The dissertation consists of five chapters. In the first part of Chapter 1 forcing relations for infinite logics are considered. It is shown that if in the case of infinite logic $L_{\lambda\mu}$ we want to

extend syntactic apparatus adequately and that forcing joining stays a deductive closed set which contains all logically valid formulas, then forcing joining has to be formulated by "weak" formulas. In the second part of this chapter a correction of the proof of the interpolation theorem for infinite logics is presented. The result from Chapter 2 is the following: it is shown that all important properties of Robinson's finite forcing are transmitted to n-finite forcing by corresponding "n-notions". Moreover, a construction of n-finite forcing joining by Henrard's approximation chains is presented. The main result of Chapter 3 is that for each theory T of a language L there is an extension T' defined in the corresponding extension L' such that $T^{f_n} = (T')^f \cap SENT(L)$. Relations between a theory T (the theory of dense linearly ordering with maximal and minimal elements, the theory of groups, the theory of Abelian groups, the theory of fields, full arithmetic, Peano's arithmetic) and its corresponding n-finite forcing joins are studied in Chapter 4. Also relations between n-finite forcing joins are studied. A connection between n-finite forcing and the type theory are studied in Chapter 5, and some generalizations of the known results are given.

Đorđe Vukomanović, Implikacija i mreže (ogled o implikaciji) (Implication and Nets (an experiment about implication))

Keywords: non-strict deductive implicative algebras, strict deductive implicative algebras, deductive nets
PhD Committee: Aleksandar Kron (supervisor), Slaviša Prešić, Žarko Mijajlović
Faculty: Faculty of Mathematics of the University of Belgrade; *Date*: 1985
Language: Serbian; *References*: 94; *Pages*: vi+150

The dissertation consists of three chapters. In Chapter 1 non-strict deductive implicative algebras are studied. Weak deductive, n-deductive and ω-deductive implicative algebras are introduced. Two kinds of complements, pseudo-complement and contraposition complement in deductive implicative algebras are defined, and the connection between these algebras and deductive implicative algebras with complement are presented. Certain properties of several implicative filters in implicative algebras and their connections with homomorphsms and congruences of these algebras are studied. In the last part of Chapter 1 the representation theorems for implicative algebras mentioned in the previous parts of the chapter are proved. The strict deductive implicative algebras and their properties, which are analogous to the properties algebras from the first chapter, are studied in Chapter 2. In the last part of that chapter the representation theorems for the strict implicative algebras are proved. In Chapter 3 deductive implicative algebras in the context of deductive (sub)nets are studied. Important notions of different forms of limited distribution and many interesting connections between these distributions and the properties of deductive nets are presented. It is shown that an implicative algebra can be isomorphicaly drowned into finite deductive subnet of sets or a net such that the implication are preserved.

Radosav Đorđević, Verovatnosne logike (Probability Logics)

Keywords: probability logics,
PhD Committee: Miodrag Rašković (supervisor), Žarko Mijajlović, Rade Živaljević
Faculty: Faculty of Science of University of Kragujevac; *Date*: 1991
Language: Serbian; *References*: 98; *Pages*: 53

The dissertation consists of six chapters. Chapter 1 contains the structures, in which the probability logics are realized, and the basic methods of nonstandard analysis which are used in

the other chapters. In Chapter 2 the syntax and semantics of the following probability logics are presented: the logic with the probability quantifiers L_{Ap}, the logic with the integral operators $L_{A\int}$, the logic with the operator of conditional expectation L_{AE} and adapted probability logic L_{ad}. Moreover, the certain important results about these logics are given. The problems of Barwise completeness, completeness, compactness, the existence of analytic and hyperfinite models for biprobability logics $L_{A\,P1\,P2}$, $L_{A\int_1\int_2}$ and L_{ad} in absolute continuous and singular cases are solved in Chapter 3. The many-probability logic $BC\{L_{APi}:i\in I\}$, $I\in A$ obtained by Boolean combinations of probability logics L_{AP} is introduced and some model-theoretical properties of that logic are given in Chapter 4. In Chapter 5 the probability logic $L^2_{AP}\forall$ of second order is introduced, which is motivated by Keisler's problems with $L_{AP}\forall$ and some topological logics. The problem of completeness for the logic $L^2_{AP}\forall$ is proved. In Chapter 6 cylinder probability algebras are introduced and some possibilities to solve problems for these algebras (which are characteristics of standard cylinder algebras, as the representation, axiomatization and decidability) are presented.

Predrag Tanović, Fundamental Order and the Number of Countable Models

The dissertation is a research about nonisolation properties of superstable types over finite domains in general. Two notions of nonisolations, the notion of eventual-strong (i.e., esn) and the notion of internal are introduced. The dissertation consists of three chapters. In Chapter 1 of the dissertation the techniques of the stability theory which are used in Chapter 2 and Chapter 3 are overviewed. In Chapter 2 of the dissertation NDFC theories are studied and the notions of dimension and U_α-rank through partial orders are developed. It is proved that if the theory T is strictly stable and the the order type of rationals cannot be embedded into the fundamental order of T and there is no strictly stable group interpretable in T^{eq}, then the theory T has continuum non-isomorphic countable models. It is noted that strongly non-isolated types can be present due to the dimensional discontinuity property. In Chapter 3 of the dissertation small superstable theories are studied. In the first part of that chapter the eventual-strong and internally nonisolated types are considered, and some properties were proved. The second part of Chapter 3 contains the proof of the following theorem: if the theory T is a complete, superstable theory, the generic type of every simple group definable in T^{eq} is orthogonal to all NENI types and $\sup\{U(p)\,|\,p\in S(T)\}\geq \omega^\omega$ holds, then the theory T has continuum non-isomorphic countable models.

Zoran Petrić, Jednakosti izvodjenja u kategorijalnoj teoriji dokaza (Equivalens in Proofs of Categorial Proof Theory)

Language: Serbian; *References*: 18; *Pages*: 98

In this dissertation methods of the proof theory are used to investigate coherence in some categories. Moreover, it is shown what the categorical notion of coherence means in the categorial proof theory. The thesis consists of three chapters. MacLane's results for monoidal categories and symmetric monoidal categories are extended in Chapter 1 of the dissertation to some other categories with multiplication: relevant categories, affine categories and symmetric monoidal categories. All the results are formulated in terms of natural transformations equipped with "grafs" (g-natural transformations). It is proved, as consequences of these results, that relevant categories, affine categories and symmetric monoidal categories have the coherence property. Moreover, using these results, some basic relations between the free categories of these classes of categories are presented in Chapter 2 of the dissertation. In Chapter 3, an extension of the notion of dinatural transfomation is introduced in order to give a criterion of preservation of dinaturality under composition. An example of an application is given by proving that all cartesian closed canonical categories transformations are dinatural. Finally, an alternative sequent system for a fragment of intuitionistic propositional logic is introduced as a device, and a cut-elimination procedure is established for this system.

Mirjana Borisavljević, Sekventi, prirodna dedukcija i multikategorije (Sequents, Natural Deduction and Multicategories)

The problem of the connection between Gentzen's natural deduction system ND and Gentzen's system of sequents G for intuitionistic predicate logic are studied in the dissertation. That connection is made by the introduction of the natural deduction systems N and N' and the system of sequents G'. The main characteristic of the systems N and N' is that their elimination rules of all connectives and quantifiers have the form as the elimination rules of $\vee$ and $\exists$ in the system ND. The dissertation consists of four chapters. The systems ND, N, N', G' and G are defined in Chapter 1. The language of multicategories is used to present derivations and to define reductions of derivations in these systems. Maps which connect derivations of these systems are defined in Chapter 2. In Chapter 3, by using the results from the previous chapters, a connection between the most important theorems of the systems ND and G, i.e., a connection between the normalization theorem of ND and the cut-elimination theorem of G, is made. The extended normal forms are studied in Chapter 4. The derivations of extended normal form and reductions which make these derivations are defined in the systems ND, N and N'. Moreover, in the systems G' and G the corresponding sequent derivations and their reductions are formed. In the system G the theorem which corresponds to the extended normalization theorem from ND (the atomization theorem) is presented.

Zoran Ognjanović, Neke verovatnosne logike i njihove primene u računarstvu (Some Probability Logics and their Applications in Computer Sciences)

Keywords: probability logic, uncertain reasoning, axiomatization, infinitary inference rules, completeness, decidability, linear programming, temporal logic, automated theorem proving
PhD Committee: Miodrag Rašković (supervisor), Žarko Mijajlović, Zoran Marković, Dragić Banković
Faculty: Faculty of Sciences of the University of Kragujevac; *Date*: September 1999
Language: Serbian; *References*: 123; *Pages*: 128

The dissertation consists of seven chapters and two appendices. Chapter 1 and the appendices contain known notions and properties from probability logics. In Chapter 2 some propositional probability logics are introduced and their languages, models, satisfiability relations, and (in)finitary axiomatic systems are given. Object languages are countable, formulas are finite, while only proofs are allowed to be infinite. The considered languages are obtained by adding unary probabilistic operators of the form P≥s. Decidability of the logics is proved. In Chapter 3 some first order probability logics are considered while in Chapter 4 new types of probability operators are introduced. The new operators are suitable for describing events in discrete sample spaces. It is shown that they are not definable in languages of probability logics that have been used so far. A propositional and a first-order logic for reasoning about discrete linear time and finitely additive probability are given in Chapter 5. Sound and complete infinitary axiomatizations for the logics are provided as well. In Chapter 6 a probabilistic extension of modal logic is studied and it is shown that those logics are closely related, but that modal necessity is a stronger notion than probability necessity. In Chapter 7 decidability of these logics is shown by reducing the corresponding satisfiability problem to the linear programming problem. Finally, two automated theorems provers based on that idea are described.

Milenko Mosurović, Složenost opisnih logika s modalnim operatorima (On the Complexity of Description Logics with Modal Operators)

Keywords: description logics, complexity, modal logics
PhD Committee: Žarko Mijajlović and Michael Zakharyashchev (supervisors), Kosta Došen, Slobodan Vujošević
Faculty: Faculty of Mathematics of the University of Belgrade; *Date*: June 2001
Language: Serbian; *References*: 32; *Pages*: V+93

The results of this dissertation are connected with papers of Wolter and Zakharyaschev in which various expressive and decidable description logics with epistemic, temporal, and dynamic operators are constructed, but the complexity of the satisfaction problem in these logics has remained unclear. The dissertation consists of seven chapters. Chapter 1 and 5 contain the notions and properties which are used in other chapters. In Chapter 2 a fragment of the logic **DIF**, which is called $\mathbf{D_1IF}$, is considered and new constructions and proofs for that logic are given. In Chapter 3 a fragment of the logic **DIO**, which is called $\mathbf{D_1IO}$, is studied. In Chapter 4 by using the results from Chapter 2 and Chapter 3 NEXPTIME algorithm, which tests whether a structure simple quasi-world or not, is constructed and it is used in Chapter 7. In Chapters 6 and 7 it is shown that the modal description logics of Wolter and Zakharyaschev based on arbitrary frames are NEXPTIME-complete, no matter whether the underlying description logic is **ALC, CI** or $\mathbf{C_1IQ}$. Moreover, it is shown that these logics based

on *S5*-frames (for knowledge), and *KD45*-frames (for beliefs) are also NEXPTIME-complete. Finally, the following property is shown: the description logics of Wolter and Zakharyaschev based on N-frames (for time) are EXPSPACE-complete, no matter whether the underlying description logic is $\mathbf{ALC_U}$, $\mathbf{CI_U}$ or $\mathbf{C_1IQ_U}$.

Nebojša Ikodinović, Neke verovatnosne i topološke logike
(Some Probability and Topological Logics)

Keywords: infinitary logics, completeness, topological class, conditional probability
PhD Committee: Radosav Đorđević (supervisor), Žarko Mijajlović, Miodrag Rašković, Zoran Marković, Zoran Ognjanović
Faculty: Faculty of Science of the University of Kragujevac; *Date*: March 2006
Language: Serbian; *References*: 52; *Pages*: 80

The dissertation is devoted to logics which are applicable in different areas of mathematics (such as topology and probability) and computer sciences (reasoning with uncertainty). Namely, some extensions of the classical logic, which are either model-theoretical or non-classical, are studied. The dissertation consists of three chapters: an introductory chapter and two main parts (Chapter 2 and Chapter 3). In the introductory chapter of the dissertation the well-known notions and properties from extensions of the first order logic and nonclassical logics are presented. Chapter 2 of the dissertation is related to logics for topological structures, particularly, topological class spaces (topologies on proper classes). One infinite logic with new quantifiers added is considered as the corresponding logic. Methods of constructing models, which can be useful for many others similar logics, are used to prove the completeness theorem. A number of probabilistic logics suitable for reasoning with uncertainty are investigated in Chapter 3. Especially, some ways of incorporation into the realm of logic conditional probability understood in different ways (in the sense of Kolmogorov or De Finetti) are given. For all these logics the corresponding axiomatizations are given and the completeness for each of them is proved. The decidability for all these logics is discussed too.

Aleksandar Perović, Neke primene formalne metode u teoriji skupova, teoriji modela, verovatnosnih logika i fazi logikama
(Some Applications of the Formal Method in Set Theory, Model Theory, Probability Logics and Fuzzy Logics)

Keywords: Lindenbaum algebra, Cohen forcing, dense embedding, completeness, qualitative probability, polynomial weight formula, interpretation method, elimination of quantifiers, compactness
PhD Committee: Aleksandar Jovanović (supervisor), Žarko Mijajlović, Miodrag Rašković, Zoran Ognjanović
Faculty: Faculty of Mathematics of the University of Belgrade; *Date*: May 2008
Language: Serbian; *References*: 52; *Pages*: 204

The interpretation method is a characteristic common for all results from this dissertation. The dissertation consists of five chapters and two appendices. A brief overview of the contents of the dissertation and the obtained results are presented in Chapter 1. The logical background and the well-known notions the basic notions, definitions and properties from forcing are given in the appendices of the dissertation. An elementary proof of equivalence between Cohen forcing and forcing with propositional Lindenbaum algebras is presented in Chapter 2.

Dense embedding and the interpretation method are used in that proof. A complete axiomatization of the notion of qualitative probability is presented in Chapter 3 of the dissertation. Probabilistic logic LPP_2 $LPP_2^{FR(n)}$ and LPP^S are extended with the qualitative probability operator $\underset{\sim}{\prec}$. Several formal techniques as infinite rules, elimination of quantifiers and interpretation method (implicitly), are used to prove the extended completeness theorem and decidability for these logics . In Chapter 4 of the dissertation a complete axiomatization of the logic with polynomial weight formulas is presented and the extended completeness theorem is proved. Applications of the interpretation method are given. By using that method the compactness theorem for the non-archimedean valued probabilistic logics is proved in Chapter 5.

Mirjana Isaković-Ilić, Neke supstrukturalne logike
(On some Substructural Logics)

Keywords: substructural logics, proof theory, cut elimination, decidability, tableaux
PhD Committee: Miodrag Kapetanović (supervisor), Milan Božić, Zoran Petrić
Faculty: Faculty of Mathematics of the University of Belgrade; *Date*: August 2008
Language: Serbian; *References*: 31; *Pages*: 169

In this dissertation cut elimination and decidability for several propositional substructural logics are studied. The dissertation consists of nine chapters. Chapters 2-6 make the first part of the dissertation. In that part sequent systems for substructural logics are formulated. In Chapters 3 and 4 for each of these systems an algebraic structure is given and completeness and consistency are proved. In Chapters 5 and 6 cut elimination and decidability for these systems are studied. It is well known that sequent system of classical Lambek logic, the system CL, is decidable and that cut is not an admissible rule in CL. Another sequent system for classical Lambek logic, the system CL*, is formulated, and the elimination of cut in CL* is proved. The system CL* does not posses the subformula property, so decidability for classical Lambek logic is not the direct consequence of the cut-elimination procedure in CL*. However, on the basis of the cut-elimination procedure in CL*, a procedure for deciding of whether a sequent is provable in CL* or not is given (i. e., a new (pure syntactic) proof of decidability for classical Lambek logic is given). The same result is shown for classical Lambek logic with weakening. In the second part of the thesis (chapters 7-9), by using that proof, an algorithm (based on the tableau method) is formulated for deciding whether or not a formula is the theorem in any of considered substructural logics.

Further work

Our presentation of doctoral dissertations in logic of Serbian scientists is not complete because the Virtual Library does not contain all doctoral dissertations in logic of Serbian scientists. Besides, there are dissertations in mathematical and philosophical areas which are very close to logic, or use results and methods of logic. Some of them are in the Virtual Library. Finally, there are Doctoral dissertations of mathematicians who gave important contribution to logic, but their dissertations belong to other areas of mathematics. Thus, our further work could be a presentation of all these dissertations. Here we will give only the list of doctoral dissertations which should belong to a more extensive further presentation. The doctoral dissertations below, marked with [*], are in the Virtual Library of the Faculty of Mathematics of the University of Belgrade.

The scientists who have received PhD degree
at the Faculty of Mathematics of the University of Belgrade:

~ Slaviša Prešić, Prilog teoriji algebarskih struktura (A Contribution to the Theory of Algebraic Structures), 1963*

~ Koriolan Gilezan, Neke generalizacije pseudo-bulovog programiranja (Some Generalizations of Pseudo-Boolean Programming), 1971*

~ Marica Prešić, Jedan iterativni postupak za odredjivanje k polinoma (An Iterative Procedure of Determining k Polynomial Zeros), 1972*

~ Nataša Božović, Nerešivi problemi u teoriji grupa (Unsolvable Problems in Group Theory), 1975*

~ Dragić Banković, Reproduktivna rešenja jednačina (Reproductive Solutions of Equations), 1980*

~ Petar Hotomski, Metode i pravila za mehaničko dokazivanje teorema u teorijama prvog reda matematičkom indukcijom (Methods and Rules for Mechanical Theorem Proving in the First Order Theory by Mathematical Induction), 1982*

~ Rade Živaljević, Deset etida o hiperkonačnom (Ten Etudes about Hyperfiniteness), 1983*

~ Danijel Romano, Konstruktivna algebra-algebarske strukture i prsten endomorfizama (Constructive Algebra-Algebraic Structures and the Ring of Endomorphisms), 1986*

~ Mirjana Šovljanski, Epistemičke i intuicionističke aritmetike (Epistemic and Intuitionistic Arithmetics), 1988*

~ Miodrag Kapetanović, Metod semantičkih tabloa (A Method of Semantic Tables), 1996*

~ Irena Pevac, Dokazivanje teorema prirodnim izvođenjem uz pomoć računara (Automated Theorem Proving by the Natural Deduction), 1997*

~ Žana Kovijanić, Kombinatorno-vjerovatnosni metod u problemima prebrojavanja K-značne logike (Combinatorial-Probability Method in Problems of Counting of the K-Valued Logic), 1999

~ Predrag Janičić, Ugradnja procedura odlučivanja u sisteme za automatsko rezonovanje (Building-in Decision Procedures into Systems for Automated Reasoning), 2001

~ Filip Marić, Formalizacija, implementacija i primene SAT rešavača (Formalization, Implementation and Applications of SAT Solvers), 2009*

The scientists who have received PhD degree
at The Faculty of Philosophy of the university of Belgrade:

~ Aleksandar Kron, Odnos polivalentnih logika i teorije verovatnoće (Relation between Polyvalent Logics and the Probability Theory), 1965

~ Svetlana Knjazeva, Filozofija Lavovsko-Varšavske škole (The Philosophy of Lviv-Warsaw's School), 1964

The scientists who have received PhD degree
at The Department of Mathematics and Informatics of the University of Novi Sad:

~ Silvia Gilezan, Tipovi sa presekom u računu lambda i logici (Intersection Types in Lambda calculus and Logic), 1993

~ Miloš Kurilić, Redukovani ideal-proizvod topoloskih prostora (Reduced ideal-product of topological spaces), 1994

~ Jovanka Pantović, Potpunost, relativna potpunost i funkcionalna potpunost (Completeness, Functional Completeness and Relative Completeness), 2000

~ Aleksandar Pavlović, Sequential Topologies on Boolean algebras, 2009

~ Boris Šobot, Games on Boolean algebras, 2009

The scientists who have received PhD degree at other Universities:

~ Aleksandar Ignjatović, Fragments of First and Second Order Arithmetic and Length of Proofs, University of California at Berkeley, USA, 1990

~ Željko Sokolović, Model Theory of Differential Fields, University of Notre Dame, Indiana, USA, 1992

~ Đorđe Čubrić, Results in Categorical Proof Theory, McGill University, Montreal, Canada, 1993

~ Boban Veličković, Contribution to Combinatorial Set Theory, University of Wisconsin, Madison, USA, 1986

~ Valentina Harizanov, Degree Spectrum of a Recursive Relation on a Recursive Structure, University of Wisconsin, Madison, USA, 1987

~ Žikica Perović, Cardinalities of Algebraic Structures Satisfying Completeness and Saturation Conditions, University of Minnesota, Minneapolis, USA, 1987

~ Zoran Spasojević, Gaps, Trees and Iterated Forcing, University of Wisconsin, Madison, USA, 1994

~ Ilijas Farah, Analytic Ideals and their Quotients, University of Toronto, Toronto, Canada, 1997

~ Nataša Rakić, Common Sense Time and Special Relativity, University of Amsterdam, Holland, 1997

~ Goran Nenadić, Mining for Terms and their Similarities from Domain-specific Literature, Informatics Research Institute, School of Computing, Science and Engineering, University of Salford, UK, 2003

~ **Silvia Likavec, Types for Object-Oriented and Functional Programming Languages**, University of Torino, Italy and Ecole Normale Superieure de Lyon, France, 2005

~ **Viktor Kunčak, Modular Data Structure Verification**, Electrical Engineering and Computer Science, MIT, USA, 2007

~ **Dragiša Žunić, Computing with Sequents and Diagrams in Classical Logic - Calculi *X, dX and ©X**, Ecole Normale Superieure de Lyon, France, 2007.

Acknowledgements

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(http://www.mi.sanu.ac.rs/~kosta/Logic.pdf)
[2] Mijajlović, Ž., *Đuro Kurepa (1907–1993),* Publ. Inst. Math. 57(71), Đuro Kurepa memorial volume 1995, 13–18
[3] Todorčević, S., *Theory of Partially Ordered Sets*, In: Ivić, A., Mamuzić, Z., Mijajlović, Ž., Todorčević, S., (eds.), Selected Papers of Đuro Kurepa, Matematički Institut SANU, Belgrade, 1996, 6–11
[4] Vukomanović, Đ., Seminar for Mathematical Logic (in Serbian), preprint
[5] http://elib.matf.bg.ac.yu:8080/virlib/

Mirjana Borisavljević

Addendum to the paper
"Doctoral Dissertations in Logic from Virtual Library of Faculty of Mathematics in Belgrade"

(1) The title of Vukomanovic's dissertation should be: **Implikacija i mreze (ogled o implikaciji) (Implication and Lattices (an essay about implication))**. In the review of this dissertation the notion *nets* should be substituted by the notion *lattices*.

(2) The following dissertations should be added to the list of dissertations given at the end of the paper:

The scientists who have received PhD degree at the Faculty of Mathematics of the University of Belgrade:

**Aleksandar Krapež, Prilog teoriji funkcionalnih jednačina na kvazigrupama
(A Contribution to the Theory of Functional Equations on Quasigroups),** 1980*

Sava Krstić, Kvadratni kvazigrupni identiteti (Square Identities of Quasigroups), 1985*

The scientists who have received PhD degree at The Faculty of Electronic Engineerig of the University of Niš:

**Živko Tošić, Analitičke reprezentacije m-valentnih logičkih funkcija u prstenu modulo m
(Analytical Representations of m-valued Logical Functions Over the Ring of Integers Modulo m),** 1972

**Milan R. Vukadinović, Jedan prilaz razvoju Bulovih funkcija u razlicitim bazama
(An Approach to Development of Boolean Functions in Different Bases),** 1982

The scientists who have received PhD degree at other Universities:

Mihajlo Marković, The Concept of Logic, University College London, UK, 1956

Miroslav Martinović, AnEvulation System for Parsing and Generation Algorithms Courant Institute of Mathematical Sciences, New York University, USA, 1992*

Miodrag Sokić, Ramsey Property of Posets and Related Structures, University of Toronto, Toronto, Canada, 2010