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# DIGITIZATION OF MATHEMATICAL TEXTBOOKS USED IN SERBIA IN THE PAST

**Abstract:** We describe six mathematical books digitized and available online in the Virtual Library of the National Digitization Centre (Virtual Library, http://elib.matf.bg.ac.yu:8080/virlib/). The first two books are the oldest printed mathematical textbooks, *Аритметика* (arithmetic) by Vasilije Damjanović printed in 1767 and *Численица* (old form in Serbian of the word arithmetic) by Jovan Došenović printed in 1809. These old books are elementary from the mathematical point of view, but, certainly, of interest to the history of the Serbian people, especially to studies of the Serbian language because they were written before Vuk Karadžić's reform of language and orthography. The other four books are gymnasium textbooks written by Rista Karljiković: *Тригонометрија, Алгебра, Аналитичка геометрија* and *Геометрија* (Trigonometry, Algebra, Analytic Geometry and Geometry). These excellent textbooks were used in Serbian gymnasiums and other secondary schools between the two World Wars (1922–1941). The digitization of these books is part of the project of forming electronic archives containing Serbian mathematical books printed in the past. In the article other reasons why these books are selected to be included in the Virtual Library are also given. The contents of these books are briefly presented together with some interesting points.

## 1. Introduction

The Virtual Library of the National Digitization Centre was founded with objective to form a collection as complete and representative as possible which contains retrodigitized books and other documents, preferentially of Serbian authors. Though in this library there are books from various fields, the main objective of the project is to preserve and present to the publicity, first of all, works in the field of mathematics and related sciences, such as mathematics, astronomy, mechanics, theoretical physics, mathematical geography and others in which the mathematical apparatus has a dominant role. In the present author's opinion in this literature an important place belongs to educational books, especially textbooks used in the teaching process in secondary schools and at the Belgrade University. There are many reasons to select this kind of books to be digitized and kept in computer archives. Of course, any word written or printed in the past, especially long ago, is part of cultural inheritance and this is a sufficient reason to qualify these works to be stored in such a way. Some more pragmatical reasons should be also given.

At first, the easy and simple access to these books via Internet enables historians of education and educators to study the education process in the past in Serbian schools. Any reader already by simple inspection of these books can make sure immediately that in this matter Serbia was not isolated, on the contrary the schooling system and education in Serbia, at least when mathematics and related sciences are the topic, followed the best ones in Europe. The reason may be looked for in the fact that the most educated Serbs from the XVIII and XIX centuries largely lived and were educated in the diaspora, above all in the Austrian Empire. It is also possible to make sure that the frequently present opinion that modern textbooks are better than those from the past is not correct. On the contrary, the themes in a majority of old textbooks were presented in an interesting and methodically correct way followed by a lot graphics and nice illustrations. The examples and problems were not closely connected to the topic, but they often illustrated applications in other fields, for instance in geodesy and astronomy if the topic concerned trigonometry. The sources of the problems were cited, also the names of mathematicians, authors of important theorems and often one can find detailed historical notes. In these books important themes were treated, today omitted from the teaching process in secondary schools; some of them are absent in the teaching process even at universities. Here only two such examples are to be mentioned: solving the cube equation and spherical trigonometry. All these topics were presented within not large books, usually about hundred pages, rarely by about ten pages more.

These books are of interest, especially those written long ago, because in them one can follow the development of the Serbian language, at least in that part concerning mathematics and related sciences. The oldest ones were written in an ancient Serbian and by using a script preceding Vuk Karadžić's reform. Here *Аритметика* by Vasilije Damjanović, the first mathematical book published in Serbian (Venice, 1767), and *Численица* by Jovan Došenović (Budapest, 1809), the textbook used at theBelgrade Grand School a few years after the First Serbian National Revolution. In somewhat more recent books one makes sure that the terminology changed and it is not always easy to recognise what a given term meant. This is, certainly, a special subject of studying, but it is enough to mention here that, for instance, the Serbian word for *index* by the time of the First World War had been identical to the word for *hand* of watches (clocks). There are other examples concerning *set, mapping, real number* etc; to add that *cosmography* often meant *astronomy*.

At last to mention the Cyrillic fonts used in printing these books. In the main text we have nice Cyrillic letters including all kinds of fonts, italic and bold, the size, from big fonts used in the titles to small fonts used in the petit. The Latin letters in which the mathematics was printed were consistent in the shape and size with the main text. All of this contributed to the esthetical value of the books from the graphical point of view and to an easy reading. It should be mentioned that lead plates - letter carriers used in printing before the Second World War have been remelted so that the printed material is the only source for these fonts. The digitized copies of these texts appear as a good source for the reconstruction of the old fonts and enriching the presentation of the Cyrillic script.

On this occasion our decision is to present six books. These are the two oldest mathematical books in Serbian, *Аритметика* by Damjanović and *Численица* by Došenović and the four mathematical textbooks written by Rista Karljiković used between the two World Wars.

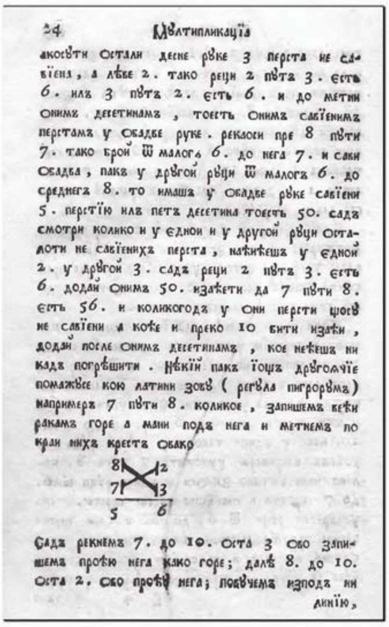
## 2. *Аритметика* by Damjanović and *Численица* by Došenović

About Vasilije Damjanović (1734–1792) not much is known. It is known that he was born in Sombor, that he was a Sombor Senator, that he lived in diasphora and that he wrote *Аритметика*, the oldest Serbian book in mathematics. The book has been digitized and in this way preserved as a cultural treasure of the XVIII century.

Vasilije Damjanović's *Аритметика* is the first mathematical book printed in Serbian in Venice in 1767. As far as known to the present author, the only copy of this old book is in the Library of Matica Srpska in Novi Sad. The book has 368 pages and consists of two parts. The first part has six chapters, the second one seven chapters. In the first part the elementary arithmetical operations with natural numbers are presented- addition, subtraction, multiplication and division, whereas in the second part one has the fraction calculus, proportion calculus, interest rate calculus and conversions of units and currencies. The book contains a large number of examples demonstrated and explained. Its purpose is practical, preferentially in trade. In the book various measuring units for goods used in the trade in that time and numerous kinds of currencies are presented.

Jovan Došenović (1781–1813) was born in Počitelj in Lika. He was educated in Latin schools in Zadar and Novi Sad. In Padova he studied philosophy and took PhD in it. As a book keeper he worked in Triest and later in Budapest. His *Численица* was printed in Budapest in 1809 and was dedicated to a Budapest merchant Ignjatije Stanković. More data from Došenović's biography can be found in K. Došen's article [12].

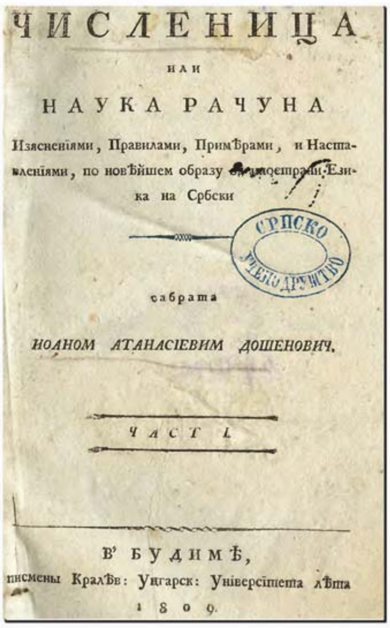

Новаѧ
Сербскаѧ
Арнѳметика
или
Простое наставленіе къ Хесапу
Изъ разныхъ Книгъ собрано, новыми примѣры кратчайшимъ образомъ изяснено, и къ употребленію Сербскіѧ учащіѧсѧ юности издано
отъ
Василіѧ Даміѧновича.
Въ Млеткахъ 1767.

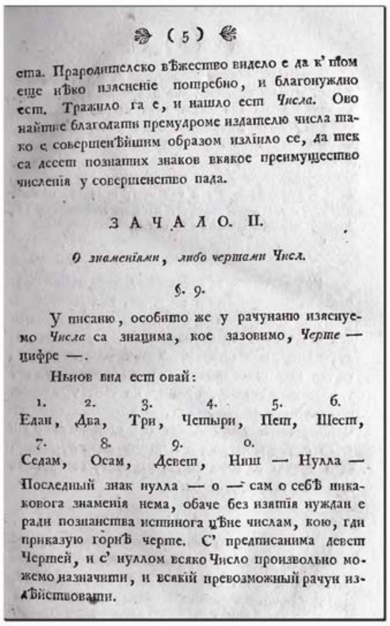

24 Мултипликаціа

акосути остали десне руке 3 перста не савієна, а лѣве 2. тако реци 2 пута 3. есть 6. или 3 пута 2. есть 6. и до метни оним десетинам, тоесть оним савієним перстам у обадве руке. рекосмо пре 8 пути 7. тако брой ѿ малогъ 6. до нега 7. и сави обадва, пак у другой руци ѿ малог 6. до среднег 8. то имаш у обадве руке савієни 5. перстію или пет десетина тоесть 50. сад смотри колико и у едной и у другой руци остало ти не савієних перста, найдеш у едной 2. у другой 3. сад реци 2 пута 3. есть 6. додай оним 50. изаћети да 7 пути 8. есть 56. и коликогод у они персти што су не савієни а коѥ и преко 10 бити изаћи, додай после оним десетинам, кое нећеш ни кад погрѣшити. Нѣкій пак іошъ другојачіє поможесе кою латини зову ( регула пигрорум ) напримеръ 7 пути 8. коликое, запишем веки ракам горе а мани под нега и метнем по крај них крест овако

8 2
7 3
5 6

Сад рекнем 7. до 10. оста 3 ово запишем прочю нега како горе; далѣ 8. до 10. оста 2. ово прочу нега; повучем изпод ни линію,


ЧИСЛЕНИЦА
или
НАУКА РАЧУНА
Изясненіями, Правилами, Примѣрами, и Наставленіями, по новѣйшем образу ... Езика на Србски
сабрата
ІОАНОМ АТАНАСІЕВИМ ДОШЕНОВИЧ.
ЧАСТ I.
В' БУДИМѢ,
писмены КралѢв: Унгарск: Універсітета лѣта
1809.


( 5 )

ста. Прародителско вѢжество видело е да к' том еще нѢко изясненіе потребно, и благонужно есть. Тражило га е, и нашло ест *Числа.* Ово найтие благодати премудроме издателю числа тако е совершенѢйшим образом излїило се, да тек са десет познатих знаков вкакое преимущество численія у совершенство пада.

З А Ч А Л О. II.

*О знаменіями, либо чертами Чисел.*

§. 9.

У писаню, особито же у рачунаню изяснуемо *Числа* са знацима, кое зазовимо, *Черте* — цифре —.

Ньиов вид ест овай:

1. Едан, 2. Два, 3. Три, 4. Четыри, 5. Пет, 6. Шест, 7. Седам, 8. Осам, 9. Девет, 0. Ниш — Нулла —

Последный знак нулла — о — сам о себѢ никаковога знаменія нема, обаче без изятія нуждан е ради познанства истинога цѢне числам, кою, гди приказую горнѢ черте. С' предписанима девет Чертей, и с' нуллом всяко Число произволно можемо назначити, и всякій превозможный рачун изДѢйствовати.

Jovan Došenović's *Численица* has 327 pages and consists of nine chapters. The first chapter is about natural numbers, the second and third ones about elementary arithmetical operations – addition, subtraction, multiplication and division – the fourth about operations with mixed units, the fifth about fractions, the sixth about proportion calculus, the seventh about conversions of units of length and weight, as well as about converting currencies, the eighth about calculus of successive proportions (верижни fractions) and the nineth about percents and interest rates. Like *Аритметика Численица* also contains a large number of examples demonstrated and explained. In it one finds a list of names and professions of its subscribers. It is seen that they were mostly merchants. It is interesting that this list contains the names of eleven pupils of the Grand School in Belgrade, just liberated from the Turks. In his *Численица* Došenović announced the second volume for educated Serbs which was never printed.

*Аритметика* from the XVIII century and *Численица* from the XIX one cannot be matched in their contents and level with later books which appeared in the early XX century, as is the case with, for instance, Karljiković's works. On the other hand these old books are important because in addition to the calculus of that time, one also gets insight into the development of the Serbian language and the information where mathematics was largely used then. A similar situation was met in the case of other, more developed, peoples who published such books earlier than the Serbs, some of them not much earlier than the Serbs.

A special point of interest is that these books are the proper place where one can see the origin of the symbol «times» (×) for multiplication. Namely, to avoid memorizing the complete multiplication table for one-digit numbers, such numbers exceeding five were multiplied as shown in the figure below. The multiplication procedure in a general case was like this. Let $x$, $y$ be one-digit numbers exceeding 5 and $a$, $b$ their complements to 10 (for instance, if $x = 7$, then $a = 3$). Furthermore, if $a \cdot b < 10$, then let it be $B = a \cdot b$. If $a \cdot b \geq 10$, let $B$ be the last digit of this product and $p$ the first digit (transfer). Finally, let it be $A = x - b$ ($=y - a$), i.e. $A = x - b + p$ if there is no transfer. Then it is $x \cdot y = AB$, $AB$ is a two-digit number with digits $A$ and $B$. For instance, if $x = 7$ and $y = 8$, then $a = 3$ and $b = 2$, $B = 2 \cdot 3 = 6$ and $A = x - b = 7 - 2 = y - a= 8 - 3 = 5$, thus $x \cdot y = AB = 56$.

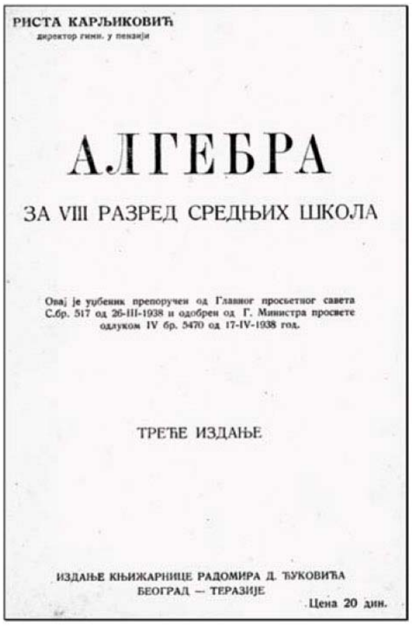
| | | | | | |
|---|---|---|---|---|---|
| $x$ | | $a$ | 7 | | 3 |
| | X | | | X | |
| $y$ | | $b$ | 8 | | 2 |
| $A$ | | $B$ | 5 | | 6 |

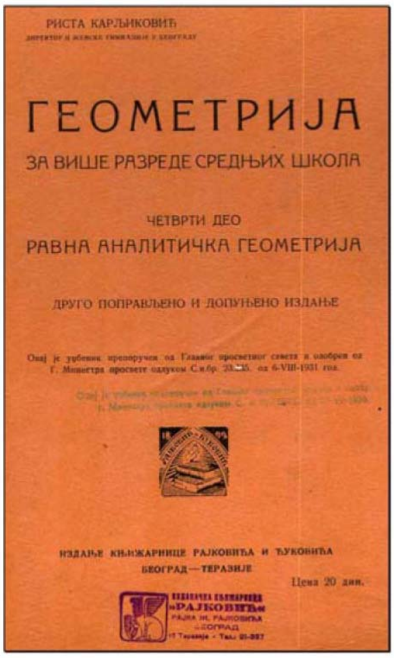
We see that «times» is here in fact cross subtraction. It is not difficult to verify the correctness of this procedure. In Došenović's book «times» already acquires its modern shape and size and it is written in the same line as the first factor whereas the second factor was still written under the first one.

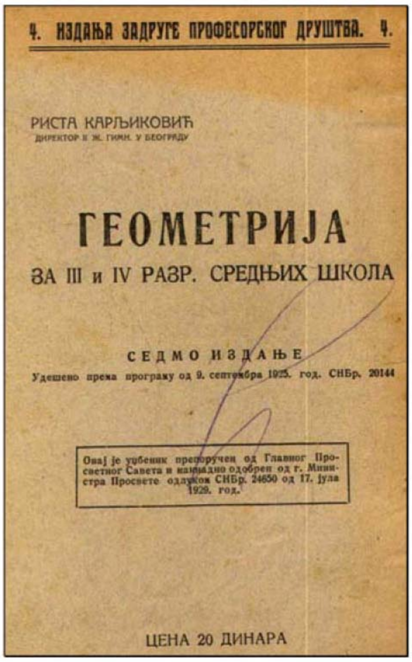
### 3. Rista Karljiković's Textbooks

About Rista Karljiković not much is known. It is known only that he was a gymnasium teacher of mathematics and Director of the II Female Gymnasium in Belgrade between the two World Wars. We have found that he wrote four gymnasium textbooks *Тригонометрија, Алгебра, Аналитичка геометрија* and *Геометрија*.

Karljiković's textbooks between the two World Wars were used not only in Serbia, but also in other parts of the Kingdom of Yugoslavia. The advanced chapters in these books were used in real gymnasiums which correspond to the scientific direction in gymnasiums in Serbia today. In his textbooks a really high level of mathematics teaching is seen. In the present author's opinion these books are the best representatives of mathematical textbooks in Serbia of the given period. In fact, his books show how the mathematics taught in Europe in the twenties and thirties of XX century looked like. By inspecting these books we can make sure that they are qualitative textbooks and that some of them, especially his *Тригонометрија*, can be of interest to be read and used in the teaching process also today.


**Тригонометрија** by Rista Karljiković was in use for higher forms of secondary schools. In the framework of the subject of geometry it was its third part. The book was recommended as a textbook by the Main Council of Education and approved by the Education Minister, Decision No 23235 of August 6, 1931. The book was published by the House of Rajković and Đurković in Belgrade. It is also known that the book had several editions and was in use till the beginning of the Second World War. The present author's decision to present this book was made by bearing in mind its, partly unusual, contents. Whereas in its first parts the subject is standard topics of trigonometry, the last one is devoted to spherical trigonometry. The book contains the following chapters: *Introduction, Goniometry, Plane Trigonometry and Spherical Trigonometry.*

It is interesting that Chapter of *Plane Trigonometry* contains the application of trigonometry to solving problems in stereometry, as well as that concerning problems of practical geometry and astronomy. This can be especially seen in Problems No 7–15. Let Problem No 13, p. 110, be selected. Its subject is the procedure of the distance determination Moon-to-Earth.

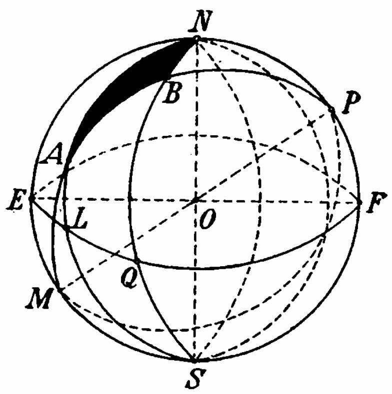
***Бројни пример.*** **Зенитни угао Месеца измерен у Берлину (северна географска ширина $\varphi = 52^0\,31'\,33''$) је $z = 32^0\,3'\,51''$, а зенитни угао месеца, измерен у истом тренутку на гребену Добре Наде (јужна географска ширина $\varphi' = 33^0\,56'\,3''$), је $z' = 55^0\,42'\,48''$; наћи одстојање месеца до центра наше Земље када је њен полупречник $R = 6370{,}308\ km$.**

**Note.** Aristarchus as early as in the III century BC found a geometric method for the purpose of determining the distance to the Moon, but the value obtained by him then was too crude. The first accurate distance determination for a celestial body using the Moon as the example was done by Lalande and La Caille in the XVIII century just measuring the coordinates of the Moon from the ends of a baseline connecting the Cape of Good Hope and Berlin. It is seen that namely this numerical example was used by Karljiković.

We want to pay attention to Chapter *Spherical Trigonometry* in which spherical triangles with their types and properties are described. Further on one gives the procedures how to find elements of a rectangular spherical triangle, also of a general spherical triangle, followed by applications in astronomy and geodesy. So on page 140 of *Тригонометрија* we find the following example:

**III. Сферна раздаљина између два места на земљи**

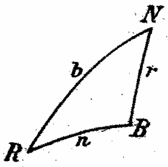
Да бисмо израчунали сферну раздаљину места $A$ и $B$ на земљи, треба да знамо географске ширине и географске дужине тих места. Ако су $\varphi_1$ и $\lambda_1$ географска ширина и дужина места $A$, $\varphi_2$ и $\lambda_2$ географска ширина и дужина места $B$, $N$ и $S$ полови земље, круг $NESF$ главни меридијан, а круг $ELQF$ екватор, кругови $NALS$ и $NBQS$ меридијани места $A$ и $B$, онда је: $\varphi_1 = AL, \lambda_1 = EL, \varphi_2 = BQ$ и $\lambda_2 = EQ$, тада је тражена сферна раздаљина $AB$ страна сферног троугла $ABN$ у коме знамо две стране $NA$ и $NB$ ($NA = 90^0 - \varphi_1$ и $NB = 90^0 - \varphi_2$) и захваћени угао $N = \lambda_2 - \lambda_1$. Задатак се, дакле, своди на пети случај решавања сферног троугла из претходног параграфа.



Сл. 70

In addition to the general solution Karljiković gives also the calculation example for solving a spherical triangle. The example concerns the distance determination in kilometers between Rome and Vienna. Here it is assumed that the geographic latitudes of Rome and Vienna are positive. Thus already in 1930, much earlier than this convention was adopted by the International Union (in 1977).

*Пример. Наћи сферну раздаљину између Рима и Беча, ако је геогр. ширина Рима* $\varphi_1 = 41^0\, 53'\, 54''$ *а Беча* $\varphi_2 = 48^0\, 12'\, 35''$, *геогр. дужина Рима* $\lambda_1 = 12^0\, 28'\, 48''$, *а Беча* $\lambda_2 = 16^0\, 22'\, 42''$, *рачунајући од гриничког меридијана.*




Сл. 71

Ако сферни пол $N$, Рим ($R$) и Беч ($B$) дају сферни троугао $NRB$, онда је страна $NR = 90^0 - \varphi_1 = 48^0\, 6'\, 16''$, страна $NB = 90^0 - \varphi_2 = 41^0\, 47'\, 25''$, а угао код $N = \lambda_2 - \lambda_1 = 3^0\, 53'\, 54''$.

Тада је:

$$tg\frac{B+R}{2} = \frac{\cos\frac{b-r}{2}}{\cos\frac{b+r}{2}}\, cotg\frac{N}{2} \text{ и } tg\frac{B-R}{2} = \frac{\sin\frac{b-r}{2}}{\sin\frac{b+r}{2}}\, cotg\frac{N}{2} \text{ или}$$

$$tg\frac{B+R}{2} = \frac{\cos 3^0\, 9'\, 25{,}5'' \cdot cotg\, 1^0\, 56'\, 57''}{\sin 44^0\, 56'\, 50{,}5''} \text{ и}$$

$$tg\frac{B-R}{2} = \frac{\sin 3^0\, 9'\, 25{,}5'' \cdot cotg\, 1^0\, 56'\, 57''}{\sin 45^0\, 56'\, 50{,}5''};$$

$$\log tg\frac{B+R}{2} = 1{,}48340 \text{ и } \log tg\frac{B-R}{2} = 0{,}79754;$$

$$\frac{B+R}{2} = 88^0\, 7'\, 6''; \frac{B-R}{2} = 80^0\, 56'\, 38''; \quad B + R = 176^0\, 14'\, 12'',$$

$$B - R = 161^0\, 53'\, 16''; \ B = 169^0\, 3'\, 44'', \ R = 7^0\, 10'\, 28''.$$

$$tg\frac{n}{2} = \frac{\sin\frac{B+R}{2}}{\sin\frac{B-R}{2}}\, tg\frac{b-r}{2} = \frac{\sin 88^0\, 7'\, 6'' \cdot tg\, 3^0\, 9'\, 25{,}5''}{\sin 80^0\, 56'\, 38''}; \ \log tg\frac{n}{2} =$$

$= 2{,}74682; \frac{n}{2} = 3^0\, 11'\, 42''; n = 6^0\, 23'\, 24''$. Дакле, сферна раздаљина Беч—Рим износи $6^0\, 23'\, 24''$, или у дужинској јединици:

$$RB = \frac{40000000 \cdot 6^0\, 23'\, 34''}{360^0} = \frac{40000000 \cdot 23004}{1\,296\,000} =$$

$= 710\,000\, m = 710\, km.$

In an analogous way in astronomy the spherical distance between any two celestial bodies on the celestial sphere is determined.

Karljiković in his *Тригонометрија* in addition to interesting examples gives a large number of problems. In the case of some of them, especially difficult ones, he cites the source and the year. These are largely the problems which were given at the mature exams in the Yugoslav Kingdom (Belgrade, Užice, Karlovac, Priština, Šibenik, etc) and France (Sorbonne, Caen, Marseille, etc). Some well-known theorems he cites after the author's name: cosine theorem – Carnot's theorem, the Gauss formulae – Mollweide's equations. It is curious that Heron's formula yielding the area of a triangle he refers to also as Brahmagupta's formula. In the book one finds words almost lost today, for instance *cosmography* and *goniometry.*

The spherical trigonometry presented in Karljiković's book was taught in real gymnasiums only which would correspond to the scientific direction in the modern gymnasiums. Today these topics are not taught in secondary schools.

**Алгебра** by Rista Karljiković was used as a textbook in the final form of secondary schools before the Second World War. It was adopted by the Main Education Council and approved by the Education Minister, Decision IV No 5470 of April 17, 1938. The textbook was published by the Đurković Publishing House in Belgrade. It had several editions and the third one was digitized. Karljiković's *Алгебра* contains the following chapters: *Theory of Derivatives and Notions of Maximum and Minimum, Fundaments of Integral Calculus, Complex Numbers, de Moivre's formula, Third-Power Equations* and

*Appendix.* The most interesting chapter in this book is *Complex Numbers, de Moivre's formula, Third-Power Equations* which was taught in real gymnasiums only. Here one presents the trigonometric form of a complex number and the operations with complex numbers. To find a power of a complex number he used de Moivre's formula and he gave the application of this formula to the case of calculating the functions of the *n*-th multiple of an angle by using its sine and cosine.

Из свега овога види се да једначина $x^3+px+q=0$ има један корен стваран а два имагинарна, ако је испуњен услов $27q^2+4p^3>0$. Образац:

$$x=\sqrt[3]{\frac{-27q+\sqrt{729q^2+108p^3}}{54}}+\sqrt[3]{\frac{-27q-\sqrt{729q^2+108p^3}}{54}}, \text{ или}$$

$$x=\sqrt[3]{\frac{-9q+\sqrt{81q^2+12p^3}}{18}}+\sqrt[3]{\frac{-9q-\sqrt{81q^2+12p^3}}{18}}, \text{ или}$$

$$x=\sqrt[3]{-\frac{q}{2}+\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}} \quad (11)$$

зове се ***Кардан-ов.*** Овај образац употребљава се при добивању корена оне једначине трећег степена облика $x^3+px+q=0$, која има имагинарне корене, за шта је потребан услов $\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3>0$ или $27q^2+4p^3>0$. Овај се образац не примењује при решавању једначине $x^3+px+q=0$, који има само

In this book Karljiković presents the procedure of solving a cube equation, a topic not learnt today in secondary schools. He cites various forms of third-power equations and their properties, types of roots (all three real or one real and a pair of a complex number and its conjugate) and the relationship between the roots and coefficients. The derivation of Cardano's formula is presented. This formula was used to obtain all the three roots of the third-power equation.

**Note.** Karljiković in *Appendix* says that the name of the formula after Cardano is an injustice since its real inventor is Tartaglia.

In the same chapter Karljiković presents a historical development of arithmetics and algebra, beginning with Phoenicians, Egyptians, Indians, Babylonians and Chaldeans which is unusual in more recent textbooks for secondary schools. So we can learn that Phoenicians created arithmetics, Egyptians geometry and Chaldeans and Babylonians astronomy. Further on Karljiković writes that the oldest arithmetical manuscripts found have their origin in Egypt and that they were written in the hieroglyph digits on papyrus between the XX and XVII centuries BC. He writes that the first elements of algebra are met by Euclid in his work *The Elements* and that the first work in algebra was published in the IV century AD by Diophant of Alexandria (325–409). Further on Karljiković presents the development of algebra in the Middle Ages and cites that the Arab and Indian arithmetics and algebra were brought to Western Europe in the XII century by Italian Leonardo of Pisa, whereas the creator of modern algebra is François Viète (1540–1603) who introduced algebraic symbolism which contributed to a significant simplification of mathematical operations and put up the level of algebra. Also, he presents the contributions of other mathematicians of that time saying that

Descartes (1596–1650) is the creator of analytic geometry and that he improved the application of algebra in geometry. He describes the contributions of later mathematicians saying that the theory of infinitesimal calculus was created in the XVII century by Leibniz and Newton, and that in the XVIII and XIX centuries there is a cluster of famous mathematicians: Euler, D'Alembert, Bézout, Laplace, Lagrange, Monge, Carnot, Argand?, Moore?, Gauss, Cauchy and Jacobi.

Karljiković at the end of the book gives a short review of introducing mathematical symbols. In this way we learn that the digits of nowadays originate from India and that they reached Europe through Arabs and for this reason are referred to as Arab digits. Then the symbols for addition and subtraction were introduced by Leonardo of Pisa, to appear for the first time in a printed form in Germany in 1489, the equality symbol was introduced by Recorde in 1552, the inequality symbols, greater and less (>, <), were introduced by Harriot in 1600 and the symbol “different from”(≠) by Christofen?

**Аналитичка геометрија** by Rista Karljiković was used as a textbook for higher forms of secondary schools. It was recommended as a textbook by the Main Education Council and approved by the Education Minister, Decision 23325 of August 6, 1931. This book was published by the House of Rajković and Đurković in Belgrade. It had several editions, the second one, corrected and extended, was digitized .

The book has the following chapters: *Introduction, Point, Straight Line, Circle, Ellipse, Hyperbola, Parabola* and *Tangents and Perpendicular Lines to Curves* .

Some sections in the book have an asterisk because they were foreseen for pupils of real gymnasiums only; examples are: Polar Equation of an Ellipse, Polar Equation of a Hyperbola, Polar Equation of a Parabola, Poles and Polar Lines of Curves, General Form of Equations of Second-Order Curves.

**Геометрија** by Karljiković was used as a textbook in lower forms of secondary schools before the Second World War. It was recommended as a textbook by the Main Education Council and approved by the Education Minister, Decision 24650 of July 17, 1929. This book was published by the Printing House of Jovanović in Belgrade. This textbook had many editions, the seventh one was digitized . It covers standard topics from geometry for the age of pupils.

Karljiković's books were written nicely from the methodical point of view and the material in them is introduced gradually. Karljiković gives a lot of treated examples so that any handbook containing problems which would follow the main book is not needed. A special accent the present author wants to stress on the simplicity in the derivation of fundamental theorems of spherical trigonometry in his *Тригонометрија*, also the procedure of obtaining the solution of cube equation in *Алгебра*. This part of mathematics was available in any case to the pupils of real gymnasiums of that time. Printing errors in his books are almost absent. Bearing in mind the methodical way of presentation, the contents, examples and problems the present author would recommend these books also to the pupils, teachers in secondary schools, even university students, of our time.

## 5. Conclusion

The oldest Serbian mathematical books *Аритметика* by Damjanović from the XVIII century and *Численица* by Došenović from the XIX one have been digitized in the framework of the Virtual Library of the National Digitization Centre and in this way preserved as an important cultural inheritance. Though in their contents elementary, these old books are, certainly, of importance and of interest to the history of the Serbian people, especially to the studies of the Serbian language because they were written before Vuk Karadžić’s reform of language and orthography. In them one can also find how and in what way elementary mathematics was learnt in the XVIII century and in the early XIX one, not only in Serbia, but also in other European countries. Of course, these first books cannot be matched in their contents and level with later Serbian books which appeared in the early XX century. Among them four textbooks by gymnasium teacher Rista Karljiković used between the two World Wars have a special place – *Тригонометрија, Алгебра, Аналитичка геометрија* and *Геометрија*. They are methodically nicely composed textbooks for secondary schools with many examples treated completely and explanations. With regard to their contents, methodical way of presentation, examples and problems they can be recommended to the pupils, teachers of secondary schools, even university students, of our time. In addition to Damjanović's *Аритметика* and Došenović's *Численица*. these four books by Karljiković have been also digitized. All the six digitized copies are available on the Internet site of the Faculty of Mathematics of Belgrade University in the framework of the Virtual Library.[1]

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