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WEIGHTED APPROXIMATION RESULTS FOR LUPAŞ-JAIN OPERATORS VIA SUMMABILITY METHODS

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Abstract. A central issue of the present paper is to consider the Korovkin-type approximation properties of the Lupaş-Jain operators using A-statistical convergence and Abel convergence, which are well-known summability methods. We provide an instance of a sequence of positive linear operators to which the weighted Korovkin-type theorem does not apply, but the A-statistical approximation theorem does. Our results are one-way more substantial than some approximation results given in [Başcanbaz-Tunca et al., *On Lupaş-Jain Operators*, Stud. Univ. Babeş-Bolyai Math., **63(4)** (2018), 525–537].

1. Introduction

Lupaş-Jain operators are novel operators composed of Lupaş operators and Jain operators. (To compose, we need some techniques including the Lagrange inversion formula, for details refer to [2].) Lupaş-Jain operators were first introduced by Patel and Mishra [14]. Then, Başcanbaz-Tunca et al. [2] proposed the slightly different generalization of the operators

$$L_n^{\gamma}(f;x) = \sum_{k=0}^{\infty} \frac{nx\,(nx+1+k\gamma)_{k-1}}{2^k k!} 2^{-(nx+k\gamma)} f\left(\frac{k}{n}\right), \quad x \in (0,\infty) \tag{1}$$

and $L_n^{\gamma} f(0) = f(0)$ for real valued functions f on $[0,\infty)$, where $0 \le \gamma < 1$, γ may depend only on the positive integers n. The authors examined the approximation properties and the shape-preserving properties, as well. Some papers related to the Lupaş-Jain operators (see e.g., [3, 15]) have appeared so far in the literature of approximation theory.

Statistical convergence is one of the crucial topics that plays a key role in many branches of mathematics. The first idea of statistical convergence emerged in the first edition (Warsaw, 1935) of the celebrated monograph [20] of Zygmund. As stated

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in [5], the monograph by Burgin and Duman, the concept of statistical convergence was formally introduced by Steinhaus [18] and Fast [8] and later reintroduced by Schoenberg [17]. The historical background of statistical convergence, basic concepts and results can be found in [5].

We first quickly mention a few concepts and then begin to present the results. If the natural density of the set $E := \{k \in \mathbb{N} : |x_k - L| \geq \varepsilon\}$ is zero, i.e., then we say that the sequence (x_k) is statistically convergent to L (see, e.g. [8–10]). Replacing the Cesaro matrix $(C, 1)$ by a nonnegative regular matrix $A = (a_{nk})$, Freedman and Sember [9] extended the notion of natural density to the A-density for a subset E of positive integers. Let $A = (a_{nk})$ be a nonnegative regular summability matrix. We say that the sequence $x = (x_k)$ is A-statistically convergent to the number L if for each $\varepsilon > 0$, $\lim_n \sum_{k:|x_k - L| \geq \varepsilon} a_{nk} = 0$. In this case, we write $st_A - \lim x = L$ (see [9, 13]).

Recall that an infinite matrix $A = (a_{nk})$ is said to be regular if the sequence $Ax := ((Ax)_n) = (\sum_{k=1}^{\infty} a_{nk} x_k)$, exists (i.e., the series on the right hand side is convergent for each n) and $\lim (Ax)_n = \lim x_n$ for each convergent sequence $x = (x_n)$. A characterization of regularity of the matrix $A = (a_{nk})$ may be found in [4].

The summability theory has many applications in approximation theory for positive linear operators, whenever the ordinary limit does not exist. The statistical form of Korovkin's theorem was studied, firstly, by Gadjiev and Orhan in [12] and the A-statistical results on this theorem were given in [1, 6]. Other similar studies can be found in the literature (see [1, 6, 7, 12, 19]).

Now, we shall recall weight functions and weighted spaces considered also in [11]. A real valued function ϱ is called a weight function if it is continuous on $\mathbb{R}$ and

$$\lim_{x \to \infty} \varrho(x) = \infty, \ \varrho(x) \geq 1. \tag{2}$$

Related to ϱ, we take the space

$$B_\varrho(\mathbb{R}^+) := \left\{ f : \mathbb{R}^+ \to \mathbb{R} : |f(x)| \leq M_f \varrho(x), x \in \mathbb{R}^+ \right\},$$

where M_f is a constant depending on f. Moreover, we denote, as usual, by $C_\varrho(\mathbb{R}^+)$, the following subspace of $B_\varrho(\mathbb{R}^+)$

$$C_\varrho(\mathbb{R}^+) := \left\{ f \in B_\varrho(\mathbb{R}^+) : f \text{ is continuous on } \mathbb{R}^+ \right\}.$$

The spaces $B_\varrho(\mathbb{R}^+)$ and $C_\varrho(\mathbb{R}^+)$ are Banach spaces with the norm

$$\|f\|_\varrho := \sup_{x \in \mathbb{R}^+} \frac{|f(x)|}{\varrho(x)}.$$

Let ϱ_1 and ϱ_2 be two weight functions satisfying (2). Suppose also that

$$\lim_{|x| \to \infty} \frac{\varrho_1(x)}{\varrho_2(x)} = 0. \tag{3}$$

If T is a positive linear operators from $C_{\varrho_1}(\mathbb{R}^+)$ into $B_{\varrho_2}(\mathbb{R}^+)$, then the operators norm $\|T\|_{C_{\varrho_1} \to B_{\varrho_2}}$ is given by $\|T\|_{C_{\varrho_1} \to B_{\varrho_2}} := \sup_{\|f\|_{\varrho_1} = 1} \|Tf\|_{\varrho_2}$.

We recall the weighted Korovkin-type theorem (for further reading see [1]).

THEOREM 1.1 ([11, 12]). *Let ϱ_1 and ϱ_2 be two weight functions satisfying* (3). *Assume* *that* (T_n) *is a sequence of positive linear operators acting on* C_{ϱ_1} *and is into* B_{ϱ_2}. *If* $\lim_{n\to\infty} \|T_n(F_i) - F_i\|_{\varrho_1} = 0$, *where*

$$F_i(x) = \frac{x^i \varrho_1(x)}{1+x^2}, \quad i = 0, 1, 2,$$

then, for all $f \in C_{\varrho_1}$, *we have* $\lim_{n\to\infty} \|T_n(f) - f\|_{\varrho_2} = 0$.

Let us recall the Abel convergence which is the second summability method used in this paper: If the series $\sum_{n=1}^{\infty} x_n \alpha^n$ converges for all $\alpha \in (0,1)$ and

$$\lim_{\alpha\to 1^-} (1-\alpha) \sum_{n=1}^{\infty} x_n \alpha^n = L,$$

then we say that the sequence $x = (x_n)$ is Abel convergent to L [16].

Note that any convergent sequence is Abel convergent to the same value but not conversely [4, 16]. Some Korovkin-type approximation theorems related to this method can be found in [19].

The main goals of this paper are to obtain the Korovkin-type statistical approximation properties of the Lupaş-Jain operators using summability techniques. Lastly, we emphasize that our results are stronger than the classical results.

2. A-statistical approximation by Lupaş-Jain operators

In the following, we obtain an one-way result involving A-statistical approximation process of Lupaş-Jain operators to the identity operator. But first, we shall provide a basic lemma necessary to prove the main results of the paper.

LEMMA 2.1 ([2]). *Let* $e_i(t) := t^i$, $i = 0, 1, 2$. *For the Lupaş-Jain operators* L_n^γ, *we have*

$$L_n^\gamma(e_0; x) = 1, \quad L_n^\gamma(e_1; x) = \frac{x}{1-\gamma}, \quad L_n^\gamma(e_2; x) = \frac{x^2}{(1-\gamma)^2} + \frac{2x}{n(1-\gamma)^3}.$$

Now, we state the weighted Korovkin-type theorem for A-statistical convergence which was proposed in [7] (see also, [1, 6, 12]) as follows.

THEOREM 2.2 ([7]). *Let* $A = (a_{jn})$ *be a nonnegative regular summability matrix and let* (T_n) *be a sequence of positive linear operators acting from* $C_{\varrho_1}(\mathbb{R}^+)$ *into* $B_{\varrho_2}(\mathbb{R}^+)$. *If*

$$st_A - \lim_{n\to\infty} \|T_n(e_i) - e_i\|_{\varrho_1} = 0, \quad \text{for } i = 0, 1, 2 \tag{4}$$

we have

$$st_A - \lim_{n\to\infty} \|T_n(f) - f\|_{\varrho_2} = 0, \quad f \in C_{\varrho_1}(\mathbb{R}^+)$$

where $\varrho_1(x) = 1 + x^2$ *and* $\varrho_2(x) = 1 + x^{2\lambda}, \lambda > 1$.

Before presenting the main theorem, we mention the following remark to show that our operators act from $C_{\varrho_1}(\mathbb{R}^+)$ to $B_{\varrho_2}(\mathbb{R}^+)$.

Remark 2.3. Lupaş-Jain operators L_n^{γ} act from $C_{\varrho_1}(\mathbb{R}^+)$ to $B_{\varrho_2}(\mathbb{R}^+)$ where $\varrho_1(x) = 1+x^2$ and $\varrho_2(x) = 1+x^{2\lambda}, \lambda > 1$.

Proof. Let $f \in C_{\varrho_1}(\mathbb{R}^+)$. Then there exists a constant M_f such that $|f(t)| \leq M_f \varrho_1(t)$. From Lemma 2.1, it follows that

$$\begin{aligned}
|L_n^{\gamma}(f;x)| &\leq L_n^{\gamma}(|f|;x) \leq M_f L_n^{\gamma}(\varrho_1;x) = M_f(1+\frac{x^2}{(1-\gamma)^2}+\frac{2x}{n(1-\gamma)^3}) \\
&= \frac{M_f}{(1-\gamma)^3}((1-\gamma)^3 + x^2(1-\gamma) + \frac{2x}{n}) \leq \frac{M_f}{(1-\gamma)^3}(1+x^2+\frac{2x}{n}).
\end{aligned}$$

So we obtain

$$\frac{|L_n^{\gamma}(f;x)|}{1+x^{2\lambda}} \leq \frac{M_f}{(1-\gamma)^3}\frac{(1+x^2+\frac{2x}{n})}{1+x^{2\lambda}} \leq \frac{M_f}{(1-\gamma)^3}(2+\frac{2}{n}),$$

which yields that $|L_n^{\gamma}(f;x)| \leq K_f\left(1+x^{2\lambda}\right)$, where $K_f := \frac{M_f}{(1-\gamma)^3}(2+\frac{2}{n})$. □

To get an approximation result, we make an adjustment to the parameter γ by taking it as a sequence such that $\gamma := (\gamma_n)$, $0 \leq \gamma_n < 1$. We are now ready to establish our first main theorem.

Theorem 2.4. *Let $A = (a_{jn})$ be a nonnegative regular summability matrix and let (γ_n)be a sequence such that $0 \leq \gamma_n < 1$ for all $n \in \mathbb{N}$ and $st_A - \lim_{n\to\infty}\gamma_n = 0$. Then for each $f \in C_{\varrho_1}(\mathbb{R}^+)$ we have $st_A - \lim_{n\to\infty}\|L_n^{\gamma_n}(f) - f\|_{\varrho_2} = 0$, where $\varrho_1(x) = 1+x^2$ and $\varrho_2(x) = 1+x^{2\lambda}, \lambda > 1$.*

Proof. To prove the statement in the above theorem, it is sufficient to show that the operators $L_n^{\gamma_n}$ verify the conditions given in (4). From Lemma 2.1, it is clear that

$$st_A - \lim_{n\to\infty}\|L_n^{\gamma_n}(e_0) - e_0\|_{\varrho_1} = 0.$$

We can easily see that

$$\|L_n^{\gamma_n}(e_1) - e_1\|_{\varrho_1} = \left\|\frac{x}{1-\gamma_n} - x\right\|_{\varrho_1} = \left\|\frac{x\gamma_n}{1-\gamma_n}\right\|_{\varrho_1} \leq \frac{\gamma_n}{1-\gamma_n},$$

hence, we get

$$st_A - \lim_{n\to\infty}\|L_n^{\gamma_n}(e_1) - e_1\|_{\varrho_1} = 0.$$

We also have,

$$\begin{aligned}
\|L_n^{\gamma_n}(e_2) - e_2\|_{\varrho_1} &= \sup_{x\in\mathbb{R}^+}\left|\frac{1}{1+x^2}\left(\frac{x^2}{(1-\gamma_n)^2} + \frac{2x}{n(1-\gamma_n)^3} - x^2\right)\right| \\
&= \sup_{x\in\mathbb{R}^+}\left|\frac{x^2}{1+x^2}\frac{2\gamma_n - \gamma_n^2}{(1-\gamma_n)^2} + \frac{2x}{1+x^2}\frac{1}{n(1-\gamma_n)^3}\right| \qquad (5) \\
&\leq \frac{2\gamma_n - \gamma_n^2}{(1-\gamma_n)^2} + \frac{1}{n(1-\gamma_n)^3}.
\end{aligned}$$

Given $\varepsilon > 0$, define the following sets:

$$M := \left\{ n : \left\| L_n^{\gamma_n}(e_2) - e_2 \right\|_{\varrho_1} \geq \varepsilon \right\},$$

$$M_1 := \left\{ n : \frac{2\gamma_n - \gamma_n^2}{(1-\gamma_n)^2} \geq \frac{\varepsilon}{2} \right\}, \qquad M_2 := \left\{ n : \frac{1}{n(1-\gamma_n)^3} \geq \frac{\varepsilon}{2} \right\}.$$

Then, by (5) we immediately get $M \subseteq M_1 \cup M_2$. Hence, for all $j \in \mathbb{N}$, we get

$$\sum_{n \in M} a_{jn} \leq \sum_{n \in M_1} a_{jn} + \sum_{n \in M_2} a_{jn}. \tag{6}$$

Since $st_A - \lim_{n\to\infty} \gamma_n = 0$, letting $j \to \infty$ in (6), we obtain

$$st_A - \lim_{n\to\infty} \left\| L_n^{\gamma_n}(e_2) - e_2 \right\|_{\varrho_1} = 0,$$

which completes the proof. □

3. Abel convergence of the operators

In this section, using the Abel convergence of positive linear operators on weighted spaces (see, [1, 19]), we give a Korovkin-type approximation theorem for Lupaş-Jain operators.

Now, we recall some facts and notations stated in [19].

Let (T_n) be a sequence of positive linear operators from C_{ϱ_1} into B_{ϱ_2} such that

$$\sup_{\alpha \in (0,1)} \sum_{n=0}^{\infty} \left\| T_n(\varrho_1) \right\|_{\varrho_1} \alpha^n < \infty, \tag{7}$$

for all $\alpha \in (0,1)$, then for all $f \in C_{\varrho_1}$ the series $\sum_{n=0}^{\infty} T_n(f(t);x)\alpha^n$ converges. Hence the operators U_α defined by

$$U_\alpha(f;x) := (1-\alpha) \sum_{n=0}^{\infty} T_n(f(t);x)\alpha^n$$

is a positive linear operators from C_{ϱ_1} into B_{ϱ_2} which is bounded for all $\alpha \in (0,1)$. Thus

$$\left\| U_\alpha \right\|_{C_{\varrho_1} \to B_{\varrho_2}} = \sup_{\|f\|_{\varrho_1}=1} \left\| U_\alpha \right\|_{\varrho_2} \leq \sup_{x \in \mathbb{R}} \frac{(1-\alpha)\left| \sum\limits_{n=0}^{\infty} T_n(\varrho_1(t);x)\alpha^n \right|}{\varrho_2(x)} = \left\| U_\alpha \varrho_1 \right\|_{\varrho_2}$$

for all $\alpha \in (0,1)$.

Theorem 3.1 ([1, 19]). *Let (T_n) be a sequence of positive linear operators from C_{ϱ_1} into B_{ϱ_2} such that* (3) *and* (7) *hold. If*

$$\lim_{\alpha \to 1^-} \left\| U_\alpha F_i - F_i \right\|_{\varrho_1} = 0, \qquad i = 0, 1, 2. \tag{8}$$

then for all $f \in C_{\varrho_1}$ $\lim_{\alpha\to 1^-} \left\| U_\alpha f - f \right\|_{\varrho_2} = 0$, where $F_i(x) = \frac{x^i \varrho_1(x)}{1+x^2}$, $i = 0, 1, 2$.

Presently, we prove a Korovkin-type approximation theorem for the Lupaş-Jain operators by means of the Abel method which is a sequence-to-function transformation and which extends the ordinary convergence. Note that we choose $\gamma := (\gamma_n)$ such that the operators $L_n^{\gamma_n}$ satisfy the condition (7). Observe that in view of $\gamma_n = \frac{1}{2}$ for each positive integers n, then we get that $L_n^{1/2}$ satisfy the condition (7).

THEOREM 3.2. *Let (γ_n) be a sequence such that $0 \leq \gamma_n < 1$ for all $n \in \mathbb{N}$ and $\left(\frac{1}{1-\gamma_n}\right)$, $\left(\frac{1}{(1-\gamma_n)^2}\right)$ and $\left(\frac{1}{n(1-\gamma_n)^3}\right)$ be Abel null sequences. Then for all $f \in C_{\varrho_1}$ we have*

$$\lim_{\alpha \to 1^-} \left\| (1-\alpha) \sum_{n=0}^{\infty} L_n^{\gamma_n} f \alpha^n - f \right\|_{\varrho_2} = 0,$$

where $\varrho_1(x) = 1 + x^2$ and $\varrho_2(x) = 1 + x^{2\lambda}, \lambda > 1$.

Proof. Thanks to Theorem 3.1, we only need to show that three conditions in (8) hold for the operators $L_n^{\gamma_n}$.

For $i = 0$, it is clear that

$$\lim_{\alpha \to 1^-} \left\| (1-\alpha) \sum_{n=1}^{\infty} L_n^{\gamma_n} e_0 \alpha^n - e_0 \right\|_{\varrho_1} = 0.$$

For $i = 1$, we can write

$$0 \leq \lim_{\alpha \to 1^-} \left\| (1-\alpha) \sum_{n=1}^{\infty} L_n^{\gamma_n} e_1 \alpha^n - e_1 \right\|_{\varrho_1} = \lim_{\alpha \to 1^-} (1-\alpha) \sup_{x \in \mathbb{R}^+} \frac{1}{1+x^2} \left| \sum_{n=1}^{\infty} \frac{x}{1-\gamma_n} \alpha^n - x \right|$$

$$\leq \lim_{\alpha \to 1^-} (1-\alpha) \left| \sum_{n=1}^{\infty} \frac{\alpha^n}{1-\gamma_n} - 1 \right| \leq \lim_{\alpha \to 1^-} (1-\alpha) \sum_{n=1}^{\infty} \frac{\alpha^n}{1-\gamma_n} + \lim_{\alpha \to 1^-} (1-\alpha)$$

and since the sequence$\left(\frac{1}{1-\gamma_n}\right)$ is Abel null, we have

$$\lim_{\alpha \to 1^-} \left\| (1-\alpha) \sum_{n=1}^{\infty} L_n^{\gamma_n} e_1 \alpha^n - e_1 \right\|_{\varrho_1} = 0.$$

Finally, for $i = 2$, it follows that

$$\begin{aligned} 0 &\leq \lim_{\alpha \to 1^-} \left\| (1-\alpha) \sum_{n=1}^{\infty} L_n^{\gamma_n} e_2 \alpha^n - e_2 \right\|_{\varrho_1} \\ &\leq \lim_{\alpha \to 1^-} (1-\alpha) \sup_{x \in \mathbb{R}^+} \frac{1}{1+x^2} \left| \sum_{n=1}^{\infty} \left(\frac{x^2}{(1-\gamma_n)^2} + \frac{2x}{n(1-\gamma_n)^3} \right) \alpha^n - x^2 \right| \\ &\leq \lim_{\alpha \to 1^-} (1-\alpha) \sup_{x \in \mathbb{R}^+} \frac{x^2}{1+x^2} \sum_{n=1}^{\infty} \frac{\alpha^n}{(1-\gamma_n)^2} \\ &\quad + \lim_{\alpha \to 1^-} (1-\alpha) \sup_{x \in \mathbb{R}^+} \frac{2x}{1+x^2} \sum_{n=1}^{\infty} \frac{\alpha^n}{n(1-\gamma_n)^3} + \lim_{\alpha \to 1^-} (1-\alpha) \sup_{x \in \mathbb{R}^+} \frac{x^2}{1+x^2}. \end{aligned}$$

Using the hypotheses, we conclude that

$$\lim_{\alpha\to 1^-}\left\|(1-\alpha)\sum_{n=1}^{\infty}L_n^{\gamma_n}e_2\alpha^n-e_2\right\|_{\varrho_1}=0,$$

as desired. □

4. Remarks, numerical and graphical results

Finally, we point out the significance of the study. We assert our idea with two instances.

When (γ_n) does not converge to zero as $n\to\infty$, it may be useful to consider Theorem 2.4. Now, let's give an example that there is such a sequence (γ_n).

Example 4.1. Let (γ_n) be the sequence defined by

$$\gamma_n=\begin{cases}1-\frac{1}{n+1}, & n=m^2 \quad (m\in\mathbb{N})\\ \frac{1}{n+1}, & n\neq m^2.\end{cases}$$

It can be shown that the sequence (γ_n) is not convergent but statistically convergent, i.e., C_1-statistically convergent.

Example 4.2. Suppose that (u_n) is an A-statistically null sequence but not convergent. As known if $A=(a_{nk})$ be a nonnegative regular summability matrix such that $\lim_{n\to\infty}\max_k a_{nk}=0$, then A-statistical convergence is stronger than convergence [13]. Without loss of generality we may assume that (u_n) is a non-negative; otherwise we would replace (u_n) by $(|u_n|)$. Now, we define $P_n^{\gamma_n}$ positive linear operators mapping $C_{\varrho_1}(\mathbb{R}^+)$ into $B_{\varrho_2}(\mathbb{R}^+)$ by $P_n^{\gamma_n}(f;x)=(1+u_n)L_n^{\gamma_n}(f;x)$, where $L_n^{\gamma_n}$ is the Lupaş-Jain operators. Observe that $(u_nL_n^{\gamma_n})$ does not tend to zero because $L_n^{\gamma_n}(f)\to f$ for all $f\in C_{\varrho_1}(\mathbb{R}^+)$ and (u_n) is divergent. Thus, $||P_n^{\gamma_n}(f)-f||_{\varrho_2}$ does not tend to zero either, but it is an A-statistically null sequence for all $f\in C_{\varrho_1}(\mathbb{R}^+)$.

Now, by giving some illustrated examples and numerical results, we shall try to provide some comparisons concerning the convergence of the operators we have used.

Example 4.3. In this example, we shall see how the change of n values affects the convergence. Let us consider the specific function $10000\times 2^{-x+10}$ in the interval $x\in[25,27]$ and (γ_n) be the sequence defined by

$$\gamma_n=\begin{cases}1-\frac{1}{n+1}, & n=m^2 \;\; (m\in\mathbb{N})\\ \frac{1}{n+1}, & n\neq m^2.\end{cases}$$

One can see in Figure 1 the approximation process for the different values of n.

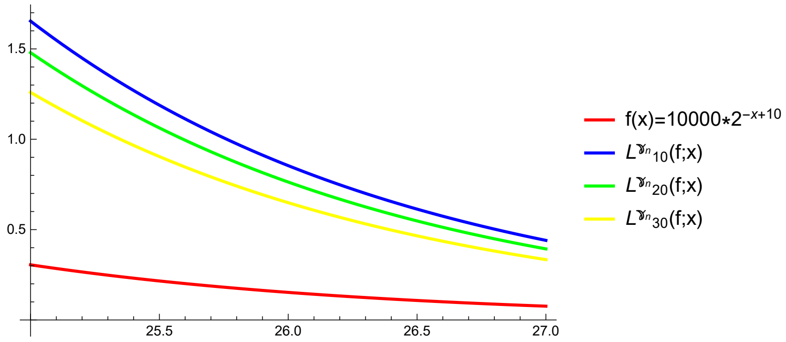

Figure 1: Approximation process of $L_n^\gamma(f;x)$ for $n = 10, 20, 30$.

Now, we present Table 1 to illustrate how various values of x affect the function $100\cos(2x)2^{-x}$, $\gamma = \frac{1}{1000}$.

x	$\lvert L_n^\gamma(f;x) - f(x)\rvert$
18.00	0.00026037
18.20	0.00024388
18.40	0.00021882
18.60	0.00018858
18.80	0.00015595
19.00	0.00012319
19.20	0.00009203
19.40	0.00006368
19.60	0.00003891
19.80	0.00001814
20.00	0.00001470

Table 1: Error of approximation by $L_{10}^\gamma(f;x)$ for different x values.

One can directly see from Table 2 that the higher values of n approximation. It may be useful to take a look at Table 2 to see the effect of n numerically for the function $100\cos(2x)2^{-x}$, $\gamma = \frac{1}{1000}$ and $x = 5$, respectively.

n	$\lvert L_n^\gamma(f;5) - f(5)\rvert$
10	0.00232127
20	0.00220399
30	0.00200169
40	0.00174536
50	0.00146127

Table 2: Error of approximation by $L_n^\gamma(f;5)$ for n values.

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