

## INFINITESIMAL DEFORMATIONS OF CURVATURE TENSORS AT NON-SYMMETRIC AFFINE CONNECTION SPACE

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**Abstract.** At the present work we consider infinitesimal deformations of geometric objects, especially of curvature tensors, at a space  $L_N$  of non-symmetric affine connection.

### 0. Introduction

We consider a space  $L_N$  of a non-symmetric affine connection  $L_{jk}^i$  with the torsion tensor  $T_{jk}^i = L_{jk}^i - L_{kj}^i$  at local coordinates  $x^i$  ( $i = 1, \dots, N$ ). Basic information on infinitesimal deformations one can find at [4–11].

Let  $x^i$ ,  $i = 1, \dots, N$  be local coordinates at  $L_N$  and let us define infinitesimal deformations of space  $L_N$  in the next manner.

**DEFINITION 0.1.** A transformation  $f: L_N \rightarrow L_N: x = (x^1, \dots, x^N) \equiv (x^i) \mapsto \bar{x} = (\bar{x}^1, \dots, \bar{x}^N) \equiv (\bar{x}^i)$ , where

$$\bar{x} = x + z(x)\varepsilon, \quad (0.1)$$

or in local coordinates

$$\bar{x}^i = x^i + z^i(x^j)\varepsilon, \quad i, j = 1, \dots, N, \quad (0.1')$$

where  $\varepsilon$  is an infinitesimal, is called *infinitesimal deformation of a space  $L_N$* , determined by the vector  $z = (z^i)$ , which is called *the field of infinitesimal deformation* (0.1').

Under deformation of a space according to (0.1') some geometrical objects are deformed. The fact that geometric object is given at point  $x$  at local coordinates  $x^i$ , is denoted by  $\mathcal{A}(i, x)$ . Under deformation (0.1') we get geometric object  $\bar{\mathcal{A}}$ ,

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which can be considered at the same point and at the same coordinate system (see [6,7,10,11]).

REMARK 0.1. In this study of infinitesimal deformations according to (0.1'), quantities of an order higher than the first with respect to  $\varepsilon$  are neglected.

DEFINITION 0.2. The magnitude  $\mathcal{DA}$ , the difference between deformed object  $\bar{\mathcal{A}}$  and initial object  $\mathcal{A}$  at the same coordinate system and at the same point with respect to (0.1'), i.e.

$$\mathcal{DA} = \bar{\mathcal{A}}(i, x) - \mathcal{A}(i, x), \quad (0.2)$$

is called *Lie difference (Lie differential)*, and the magnitude

$$\mathcal{L}_z \mathcal{A} = \lim_{\varepsilon \rightarrow 0} \frac{\mathcal{DA}}{\varepsilon} = \lim_{\varepsilon \rightarrow 0} \frac{\bar{\mathcal{A}}(i, x) - \mathcal{A}(i, x)}{\varepsilon} \quad (0.2')$$

is *Lie derivative* of geometric object  $\mathcal{A}(i, x)$  with respect to the vector field  $z = (z^i(x^j))$ .

Using the relation (0.2), for deformed object  $\bar{\mathcal{A}}(i, x)$  we have

$$\bar{\mathcal{A}}(i, x) = \mathcal{A}(i, x) + \mathcal{DA}, \quad (0.2'')$$

and thus we can express  $\bar{\mathcal{A}}$ , finding previously  $\mathcal{DA}$ .

From (0.2') we get

$$\frac{\mathcal{DA}}{\varepsilon} = \mathcal{L}_z \mathcal{A} + \varepsilon_1, \varepsilon_1 \rightarrow 0, \quad \text{if } \varepsilon \rightarrow 0,$$

and  $\mathcal{DA} = \varepsilon \mathcal{L}_z \mathcal{A} + \varepsilon \varepsilon_1$ , i.e.

$$\mathcal{DA} = \varepsilon \mathcal{L}_z \mathcal{A}, \quad (0.3)$$

where  $\varepsilon \varepsilon_1$ , as an infinitesimal value of the higher order with respect to  $\varepsilon$ , is omitted.

As it is known (see [6,7]), for an arbitrary tensor  $t_{j_1, \dots, j_v}^{i_1, \dots, i_u}$  we have

$$\begin{aligned} \mathcal{D}t_{j_1 \dots j_v}^{i_1 \dots i_u} &= [t_{j_1 \dots j_v, p}^{i_1 \dots i_u} z^p - \sum_{\alpha=1}^u z_{,p}^{i_\alpha} \binom{p}{i_\alpha} t_{j_1 \dots j_v}^{i_1 \dots i_u} + \sum_{\beta=1}^v z_{,j_\beta}^p \binom{j_\beta}{p} t_{j_1 \dots j_v}^{i_1 \dots i_u}] \varepsilon \\ &= \mathcal{L}_z t_{j_1 \dots j_v}^{i_1 \dots i_u} \varepsilon, \end{aligned} \quad (0.4)$$

where we denoted, for instance,  $z_{,p}^i = \partial z^i / \partial x^p$ , and

$$\binom{p}{i_\alpha} t_{j_1 \dots j_v}^{i_1 \dots i_u} = t_{j_1 \dots j_v}^{i_1 \dots i_{\alpha-1} p i_{\alpha+1} \dots i_u}, \quad \binom{j_\beta}{p} t_{j_1 \dots j_v}^{i_1 \dots i_u} = t_{j_1 \dots j_{\beta-1} p j_{\beta+1} \dots j_v}^{i_1 \dots i_u}.$$

For the *connection coefficients* we have

$$\mathcal{D}L_{jk}^i = (L_{jk,p}^i z^p + z_{,jk}^i - z_{,p}^i L_{jk}^p + z_{,j}^p L_{pk}^i + z_{,k}^p L_{jp}^i) \varepsilon = \mathcal{L}_z L_{jk}^i \varepsilon. \quad (0.5)$$

Because of non-symmetry of the connection, at  $L_N$  we can consider two types of covariant derivatives for a vector and four types for general tensor. So, denoting by  ${}_{\theta}^{i_1 \dots i_u} |_{j_1 \dots j_v, m}$  the derivative of the type  $\theta$ , we have ([1]–[3]):

$${}_{\theta}^{i_1 \dots i_u} |_{j_1 \dots j_v, m} = t_{j_1 \dots j_v, m}^{i_1 \dots i_u} + \sum_{\alpha=1}^u L_{pm}^{i_{\alpha}} \binom{p}{i_{\alpha}} t_{j_1 \dots j_v}^{i_1 \dots i_u} - \sum_{\beta=1}^v L_{j_{\beta} m}^p \binom{j_{\beta}}{p} t_{j_1 \dots j_v}^{i_1 \dots i_u}. \quad (0.6a-d)$$

By virtue of Ricci-type identities twelve curvature tensors in  $L_N$  are obtained [1,2,3]. Among these tensors only five of them are independent:

$$R_1^i{}_{jmn} = L_{jm,n}^i - L_{jn,m}^i + L_{jm}^p L_{pn}^i - L_{jn}^p L_{pm}^i, \quad (0.7)$$

$$R_2^i{}_{jmn} = L_{mj,n}^i - L_{nj,m}^i + L_{mj}^p L_{np}^i - L_{nj}^p L_{mp}^i, \quad (0.8)$$

$$R_3^i{}_{jmn} = L_{jm,n}^i - L_{nj,m}^i + L_{jm}^p L_{np}^i - L_{nj}^p L_{pm}^i + L_{nm}^p T_{pj}^i, \quad (0.9)$$

$$R_4^i{}_{jmn} = L_{jm,n}^i - L_{nj,m}^i + L_{jm}^p L_{np}^i - L_{nj}^p L_{pm}^i + L_{mn}^p T_{pj}^i, \quad (0.10)$$

$$R_5^i{}_{jmn} = \frac{1}{2}(L_{jm,n}^i + L_{mj,n}^i - L_{jn,m}^i - L_{nj,m}^i + L_{jm}^p L_{pn}^i + L_{mj}^p L_{np}^i - L_{jn}^p L_{mp}^i - L_{nj}^p L_{pm}^i). \quad (0.11)$$

If we define symmetric connection

$$L_0^i{}_{jk} = \frac{1}{2}(L_{jk}^i + L_{kj}^i), \quad (0.12)$$

its curvature tensor

$$R_0^i{}_{jmn} = L_0^i{}_{jm,n} - L_0^i{}_{jn,m} + L_0^p{}_{jm} L_0^i{}_{pn} - L_0^p{}_{jn} L_0^i{}_{pm} \quad (0.13)$$

is an ordinary Riemann-Christoffel's curvature tensor of symmetric connection. Denoting by semicolon (;) covariant derivative with respect to symmetric connection then according to [3], we have

$$R_1^i{}_{jmn} = R_0^i{}_{jmn} + \frac{1}{2}T_{jm;n}^i - \frac{1}{2}T_{jn;m}^i + \frac{1}{4}T_{jm}^p T_{pn}^i - \frac{1}{4}T_{jn}^p T_{pm}^i, \quad (0.14)$$

$$R_2^i{}_{jmn} = R_0^i{}_{jmn} - \frac{1}{2}T_{jm;n}^i + \frac{1}{2}T_{jn;m}^i + \frac{1}{4}T_{jm}^p T_{pn}^i - \frac{1}{4}T_{jn}^p T_{pm}^i, \quad (0.15)$$

$$R_3^i{}_{jmn} = R_0^i{}_{jmn} + \frac{1}{2}T_{jm;n}^i + \frac{1}{2}T_{jn;m}^i - \frac{1}{4}T_{jm}^p T_{pn}^i + \frac{1}{4}T_{jn}^p T_{pm}^i - \frac{1}{2}T_{mn}^p T_{pj}^i, \quad (0.16)$$

$$R_4^i{}_{jmn} = R_0^i{}_{jmn} + \frac{1}{2}T_{jm;n}^i + \frac{1}{2}T_{jn;m}^i - \frac{1}{4}T_{jm}^p T_{pn}^i + \frac{1}{4}T_{jn}^p T_{pm}^i + \frac{1}{2}T_{mn}^p T_{pj}^i \quad (0.17)$$

$$R_5^i{}_{jmn} = R_0^i{}_{jmn} + \frac{1}{4}T_{jm}^p T_{pn}^i + \frac{1}{4}T_{jn}^p T_{pm}^i. \quad (0.18)$$

At (0.14–18) all the addends at the right side are tensors.

At the present work we will consider infinitesimal deformations of cited five curvature tensors.

### 1. Infinitesimal deformations of curvature tensor $\bar{R}_1$

1.1. According to the (0.7) for deformed first curvature tensor at  $L_N$ , i.e. for  $\bar{R}_1$ , we have

$$\bar{R}_1^i{}_{jmn}(x) = \bar{L}_{jm,n}^i - \bar{L}_{jn,m}^i + \bar{L}_{jm}^p \bar{L}_{pn}^i - \bar{L}_{jn}^p \bar{L}_{pm}^i, \quad (1.1)$$

and with respect to  $\bar{L}_{jm}^i(x) = L_{jm}^i(x) + \mathcal{D}L_{jm}^i$ , we have

$$\begin{aligned} \bar{R}_1^i{}_{jmn} &= (L_{jm}^i + \mathcal{D}L_{jm}^i)_{,n} - (L_{jn}^i + \mathcal{D}L_{jn}^i)_{,m} \\ &\quad + (L_{jm}^p + \mathcal{D}L_{jm}^p)(L_{pn}^i + \mathcal{D}L_{pn}^i) - (L_{jn}^p + \mathcal{D}L_{jn}^p)(L_{pm}^i + \mathcal{D}L_{pm}^i). \end{aligned}$$

Developing this and omitting the members of the form  $\mathcal{D}L_{\dots} \cdot \mathcal{D}L_{\dots}$ , as they include  $(\varepsilon)^2$ , we have

$$\begin{aligned} \bar{R}_1^i{}_{jmn} &= L_{jm,n}^i + (\mathcal{D}L_{jm}^i)_{,n} - L_{jn,m}^i - (\mathcal{D}L_{jn}^i)_{,m} + L_{jm}^p L_{pn}^i \\ &\quad + L_{jm}^p \mathcal{D}L_{pn}^i + (\mathcal{D}L_{jm}^p) L_{pn}^i - L_{jn}^p L_{pm}^i - L_{jn}^p \mathcal{D}L_{pm}^i - (\mathcal{D}L_{jn}^p) L_{pm}^i. \end{aligned} \quad (1.2)$$

As  $\mathcal{D}L_{jm}^i$  is a tensor, we can consider covariant derivative:

$$(\mathcal{D}L_{jm}^i)_{|n} = (\mathcal{D}L_{jm}^i)_{,n} + L_{pn}^i \mathcal{D}L_{jm}^p - L_{jn}^p \mathcal{D}L_{pm}^i - L_{mn}^p \mathcal{D}L_{jp}^i$$

where

$$(\mathcal{D}L_{jm}^i)_{,n} + L_{pn}^i \mathcal{D}L_{jm}^p = (\mathcal{D}L_{jm}^i)_{|n} + L_{jn}^p \mathcal{D}L_{pm}^i + L_{mn}^p \mathcal{D}L_{jp}^i \quad (1.3)$$

and in the same manner

$$(\mathcal{D}L_{jn}^i)_{,m} + L_{pm}^i \mathcal{D}L_{jn}^p = (\mathcal{D}L_{jn}^i)_{|m} + L_{jm}^p \mathcal{D}L_{pn}^i + L_{nm}^p \mathcal{D}L_{jp}^i \quad (1.3')$$

If we have in mind (0.7), (1.3, 3'), the equation (1.2) becomes

$$\begin{aligned} \bar{R}_1^i{}_{jmn} &= R_{jmn}^i + (\mathcal{D}L_{jm}^i)_{|n} + L_{jn}^p \mathcal{D}L_{pm}^i + L_{mn}^p \mathcal{D}L_{jp}^i \\ &\quad - (\mathcal{D}L_{jn}^i)_{|m} - L_{im}^p \mathcal{D}L_{pn}^i - L_{nm}^p \mathcal{D}L_{jp}^i + L_{jm}^p \mathcal{D}L_{pn}^i - L_{jn}^p \mathcal{D}L_{pm}^i, \end{aligned}$$

i.e.

$$\bar{R}_1^i{}_{jmn} = R_{jmn}^i + (\mathcal{D}L_{jm}^i)_{|n} - (\mathcal{D}L_{jn}^i)_{|m} + T_{mn}^p \mathcal{D}L_{jp}^i. \quad (1.4)$$

From here

$$\mathcal{D}R_{jmn}^i = \bar{R}_1^i{}_{jmn} - R_{jmn}^i = (\mathcal{D}L_{jm}^i)_{|n} - (\mathcal{D}L_{jn}^i)_{|m} + T_{mn}^p \mathcal{D}L_{jp}^i, \quad (1.4')$$

and (dividing with  $\varepsilon$ ):

$$\mathcal{L}_z R_{jmn}^i = (\mathcal{L}_z L_{jm}^i)_{|n} - (\mathcal{L}_z L_{jn}^i)_{|m} + T_{mn}^p \mathcal{L}_z L_{jp}^i. \quad (1.4'')$$

1.2. We can also start from (0.14), and then we have

$$\begin{aligned} \bar{R}_{1jmn}^i &= \bar{R}_{0jmn}^i + \frac{1}{2}\bar{T}_{jm;n}^i - \frac{1}{2}\bar{T}_{jn;m}^i + \frac{1}{4}\bar{T}_{jm}^p\bar{T}_{pn}^i - \frac{1}{4}\bar{T}_{jn}^p\bar{T}_{pm}^i \\ &= R_{0jmn}^i + \mathcal{D}R_{0jmn}^i + \frac{1}{2}(T_{jm}^i + \mathcal{D}T_{jm}^i)_{;n} - \frac{1}{2}(T_{jn}^i + \mathcal{D}T_{jn}^i)_{;m} \\ &\quad + \frac{1}{4}(T_{jm}^p + \mathcal{D}T_{jm}^p)(T_{pn}^i + \mathcal{D}T_{pn}^i) - \frac{1}{4}(T_{jn}^p + \mathcal{D}T_{jn}^p)(T_{pm}^i + \mathcal{D}T_{pm}^i). \end{aligned}$$

Omitting the members containing  $\mathcal{D}T \cdot \mathcal{D}T$ , we get

$$\begin{aligned} \bar{R}_{1jmn}^i &= R_{0jmn}^i + \mathcal{D}R_{0jmn}^i + \frac{1}{2}T_{jm;n}^i + \frac{1}{2}(\mathcal{D}T_{jm}^i)_{;n} - \frac{1}{2}T_{jn;m}^i - \frac{1}{2}(\mathcal{D}T_{jn}^i)_{;m} \\ &\quad + \frac{1}{4}(T_{jm}^p T_{pn}^i + T_{jm}^p \mathcal{D}T_{pn}^i + \mathcal{D}T_{jm}^p T_{pn}^i - T_{jn}^p T_{pm}^i - T_{jn}^p \mathcal{D}T_{pm}^i - \mathcal{D}T_{jn}^p T_{pm}^i). \end{aligned}$$

So, with respect to (0.14), one obtains

$$\bar{R}_{1jmn}^i = R_{1jmn}^i + \mathcal{D}R_{0jmn}^i + \frac{1}{2}(\mathcal{D}T_{jm}^i)_{;n} - \frac{1}{2}(\mathcal{D}T_{jn}^i)_{;m} + \frac{1}{4}\mathcal{D}(T_{jm}^p T_{pn}^i - T_{jn}^p T_{pm}^i), \quad (1.5)$$

wherefrom

$$\mathcal{D}R_{1jmn}^i = \mathcal{D}R_{0jmn}^i + \frac{1}{2}(\mathcal{D}T_{jm}^i)_{;n} - \frac{1}{2}(\mathcal{D}T_{jn}^i)_{;m} + \frac{1}{4}\mathcal{D}(T_{jm}^p T_{pn}^i - T_{jn}^p T_{pm}^i) \quad (1.5')$$

where ; denotes covariant derivative with respect to  $L_0$  (0.12), and (dividing with  $\varepsilon$ ):

$$\mathcal{L}_z R_{1jmn}^i = \mathcal{L}_z R_{0jmn}^i + \frac{1}{2}(\mathcal{L}_z T_{jm}^i)_{;n} - \frac{1}{2}(\mathcal{L}_z T_{jn}^i)_{;m} + \frac{1}{4}\mathcal{L}_z(T_{jm}^p T_{pn}^i - T_{jn}^p T_{pm}^i). \quad (1.5'')$$

### 2. Infinitesimal deformations of curvature tensor $R_2$

2.1. In the similar manner as for  $R_1$ , using (0.8), we get

$$\bar{R}_{2jmn}^i = R_{2jmn}^i + (\mathcal{D}L_{mj}^i)_{|n} - (\mathcal{D}L_{nj}^i)_{|m} + T_{nm}^p \mathcal{D}L_{pj}^i. \quad (2.1)$$

and corresponding expressions for  $\mathcal{D}R_{2jmn}^i$  and  $\mathcal{L}_z R_{2jmn}^i$ .

2.1. Starting from (0.15), we obtain

$$\bar{R}_{2jmn}^i = R_{2jmn}^i + \mathcal{D}R_{0jmn}^i - \frac{1}{2}(\mathcal{D}T_{jm}^i)_{;n} + \frac{1}{2}(\mathcal{D}T_{jn}^i)_{;m} + \frac{1}{4}\mathcal{D}(T_{jm}^p T_{pn}^i - T_{jn}^p T_{pm}^i). \quad (2.2)$$

### 3. Infinitesimal deformations of curvature tensor $R_3$

3.1. Starting from (0.9), we obtain

$$\begin{aligned} \bar{R}_{3jmn}^i &= L_{jm,n}^i + (\mathcal{D}L_{jm}^i)_{;n} - L_{nj,m}^i - (\mathcal{D}L_{nj}^i)_{;m} + L_{jm}^p L_{np}^i + L_{jm}^p \mathcal{D}L_{np}^i + \mathcal{D}L_{jm}^p L_{np}^i \\ &\quad - L_{nj}^p L_{pm}^i - L_{nj}^p \mathcal{D}L_{pm}^i - \mathcal{D}L_{nj}^p L_{pm}^i + L_{nm}^p T_{pj}^i + L_{nm}^p \mathcal{D}T_{pj}^i + \mathcal{D}L_{nm}^p T_{pj}^i. \end{aligned} \quad (3.1)$$

In relation to

$$\begin{aligned} (\mathcal{D}L_{jm}^i)|_n &= (\mathcal{D}L_{jm}^i)_{,n} + L_{np}^i \mathcal{D}L_{jm}^p - L_{jn}^p \mathcal{D}L_{pm}^i - L_{mn}^p \mathcal{D}L_{jp}^i, \\ (\mathcal{D}L_{nj}^i)|_m &= (\mathcal{D}L_{nj}^i)_{,m} + L_{pm}^i \mathcal{D}L_{nj}^p - L_{mn}^p \mathcal{D}L_{pj}^i - L_{mj}^p \mathcal{D}L_{np}^i, \end{aligned}$$

one obtains respectively

$$\begin{aligned} (\mathcal{D}L_{jm}^i)_{,n} + L_{np}^i \mathcal{D}L_{jm}^p &= (\mathcal{D}L_{jm}^i)|_n + L_{jn}^p \mathcal{D}L_{pm}^i + L_{mn}^p \mathcal{D}L_{jp}^i, \\ (\mathcal{D}L_{nj}^i)_{,m} + L_{pm}^i \mathcal{D}L_{nj}^p &= (\mathcal{D}L_{nj}^i)|_m + L_{mn}^p \mathcal{D}L_{pj}^i + L_{mj}^p \mathcal{D}L_{np}^i. \end{aligned}$$

Using these equations and (0.9), the equation (3.1) becomes

$$\begin{aligned} \bar{R}_{3jmn}^i &= R_{3jmn}^i + (\mathcal{D}L_{jm}^i)|_n + L_{jn}^p \mathcal{D}L_{pm}^i + L_{mn}^p \mathcal{D}L_{jp}^i - (\mathcal{D}L_{nj}^i)|_m \\ &\quad - L_{mn}^p \mathcal{D}L_{pj}^i - L_{mj}^p \mathcal{D}L_{np}^i + L_{jm}^p \mathcal{D}L_{np}^i - L_{nj}^p \mathcal{D}L_{pm}^i + L_{nm}^p \mathcal{D}T_{pj}^i + T_{pj}^i \mathcal{D}L_{nm}^p, \end{aligned}$$

that is

$$\begin{aligned} \bar{R}_{3jmn}^i &= R_{3jmn}^i + (\mathcal{D}L_{jm}^i)|_n - (\mathcal{D}L_{nj}^i)|_m \\ &\quad + T_{jn}^p \mathcal{D}L_{pm}^i + T_{jm}^p \mathcal{D}L_{np}^i + L_{mn}^p \mathcal{D}T_{jp}^i + \mathcal{D}(T_{pj}^i L_{nm}^p). \quad (3.2) \end{aligned}$$

**3.2.** According to (0.16), we get

$$\begin{aligned} \bar{R}_{3jmn}^i &= R_{0jmn}^i + \mathcal{D}R_{0jmn}^i + \frac{1}{2}T_{jm;n}^i + \frac{1}{2}(\mathcal{D}T_{jm}^i)_{;n} + \frac{1}{2}T_{jn;m}^i + \frac{1}{2}(\mathcal{D}T_{jn}^i)_{;m} \\ &\quad - \frac{1}{4}T_{jm}^p T_{pn}^i - \frac{1}{4}T_{jm}^p \mathcal{D}T_{pn}^i - \frac{1}{4}\mathcal{D}T_{jm}^p T_{pn}^i + \frac{1}{4}T_{jn}^p T_{pm}^i + \frac{1}{4}T_{jn}^p \mathcal{D}T_{pm}^i \\ &\quad + \frac{1}{4}\mathcal{D}T_{jn}^p T_{pm}^i - \frac{1}{2}T_{mn}^p T_{pj}^i - \frac{1}{2}T_{mn}^p \mathcal{D}T_{pj}^i - \frac{1}{2}\mathcal{D}T_{mn}^p T_{pj}^i \end{aligned}$$

i.e.

$$\begin{aligned} \bar{R}_{3jmn}^i &= R_{3jmn}^i + \mathcal{D}R_{0jmn}^i + \frac{1}{2}(\mathcal{D}T_{jm}^i)_{;n} + \frac{1}{2}(\mathcal{D}T_{jn}^i)_{;m} \\ &\quad + \frac{1}{4}\mathcal{D}(T_{pm}^i T_{jn}^p - T_{pn}^i T_{jm}^p - 2T_{pj}^i T_{mn}^p). \quad (3.3) \end{aligned}$$

#### 4. Infinitesimal deformations of curvature tensor $R_4$

**4.1.** Starting from (0.10) and using the procedure which is similar to that used for  $R_3$ , we obtain

$$\begin{aligned} \bar{R}_{4jmn}^i &= R_{4jmn}^i + (\mathcal{D}L_{jm}^i)|_n - (\mathcal{D}L_{nj}^i)|_m \\ &\quad + T_{jm}^p \mathcal{D}L_{np}^i + T_{jn}^p \mathcal{D}L_{pm}^i + L_{mn}^p \mathcal{D}T_{jp}^i + \mathcal{D}(T_{pj}^i L_{mn}^p) \quad (4.1) \end{aligned}$$

4.2. Using (0.17), we obtain for  $R_4$

$$\begin{aligned} \bar{R}_4^i{}_{jmn} = R_4^i{}_{jmn} + \mathcal{D}R_0^i{}_{jmn} + \frac{1}{2}(\mathcal{D}T_{jm}^i)_{;n} + \frac{1}{2}(\mathcal{D}T_{jn}^i)_{;m} \\ + \frac{1}{4}\mathcal{D}(T_{pm}^i T_{jn}^p - T_{pn}^i T_{jm}^p + 2T_{pj}^i T_{mn}^p). \end{aligned} \quad (4.2)$$

## 5. Infinitesimal deformations of the tensor $R_5$

5.1 According to (0.11) we have

$$\begin{aligned} 2\bar{R}_5^i{}_{jmn} = L_{jm,n}^i + (\mathcal{D}L_{jm}^i)_{;n} + L_{mj,n}^i + (\mathcal{D}L_{mj}^i)_{;n} - L_{jn,m}^i - (\mathcal{D}L_{jn}^i)_{;m} \\ - L_{nj,m}^i - (\mathcal{D}L_{nj}^i)_{;m} + L_{jm}^p L_{pn}^i + L_{jm}^p \mathcal{D}L_{pn}^i + (\mathcal{D}L_{jm}^p)L_{pn}^i + L_{mj}^p L_{np}^i \\ + L_{mj}^p \mathcal{D}L_{np}^i + (\mathcal{D}L_{mj}^p)L_{np}^i - L_{jn}^p L_{mp}^i - L_{jn}^p \mathcal{D}L_{mp}^i \\ - (\mathcal{D}L_{jn}^p)L_{mp}^i - L_{nj}^p L_{pm}^i - L_{nj}^p \mathcal{D}L_{pm}^i - (\mathcal{D}L_{nj}^p)L_{pm}^i. \end{aligned} \quad (5.1)$$

By differentiating the Lie differences one obtains

$$\begin{aligned} (\mathcal{D}L_{jm}^i)_{|n} = (\mathcal{D}L_{jm}^i)_{;n} + L_{pn}^i \mathcal{D}L_{jm}^p - L_{nj}^p \mathcal{D}L_{pm}^i - L_{nm}^p \mathcal{D}L_{jp}^i, \\ (\mathcal{D}L_{mj}^i)_{|n} = (\mathcal{D}L_{mj}^i)_{;n} + L_{np}^i \mathcal{D}L_{mj}^p - L_{mn}^p \mathcal{D}L_{pj}^i - L_{jn}^p \mathcal{D}L_{mp}^i, \\ (\mathcal{D}L_{nj}^i)_{|m} = (\mathcal{D}L_{nj}^i)_{;m} + L_{pm}^i \mathcal{D}L_{nj}^p - L_{mn}^p \mathcal{D}L_{pj}^i - L_{mj}^p \mathcal{D}L_{np}^i, \\ (\mathcal{D}L_{jn}^i)_{|m} = (\mathcal{D}L_{jn}^i)_{;m} + L_{mp}^i \mathcal{D}L_{jn}^p - L_{jm}^p \mathcal{D}L_{pn}^i - L_{nm}^p \mathcal{D}L_{jp}^i. \end{aligned}$$

If we substitute at (5.1) three members from the right sides of previous four equations, we get

$$\bar{R}_5^i{}_{jmn} = R_5^i{}_{jmn} + \frac{1}{2}[(\mathcal{D}L_{jm}^i)_{|n} - (\mathcal{D}L_{nj}^i)_{|m} + (\mathcal{D}L_{mj}^i)_{|n} - (\mathcal{D}L_{jn}^i)_{|m}]. \quad (5.2)$$

5.2. If we start from (0.18), we have

$$\begin{aligned} \bar{R}_5^i{}_{jmn} = R_0^i{}_{jmn} + \mathcal{D}R_0^i{}_{jmn} + \frac{1}{4}[T_{jm}^p T_{pn}^i + T_{jm}^p \mathcal{D}T_{pn}^i \\ + (\mathcal{D}T_{jm}^p)T_{pn}^i + T_{jn}^p T_{pm}^i + T_{jn}^p \mathcal{D}T_{pm}^i + (\mathcal{D}T_{jn}^p)T_{pm}^i]. \end{aligned}$$

According to (0.18) the previous equation becomes:

$$\bar{R}_5^i{}_{jmn} = R_5^i{}_{jmn} + \mathcal{D}R_0^i{}_{jmn} + \frac{1}{4}[T_{jm}^p \mathcal{D}T_{pn}^i + (\mathcal{D}T_{jm}^p)T_{pn}^i + T_{jn}^p \mathcal{D}T_{pm}^i + (\mathcal{D}T_{jn}^p)T_{pm}^i],$$

i.e.

$$\bar{R}_5^i{}_{jmn} = R_5^i{}_{jmn} + \mathcal{D}R_0^i{}_{jmn} + \frac{1}{4}\mathcal{D}(T_{jm}^p T_{pn}^i + T_{jn}^p T_{pm}^i). \quad (5.3)$$

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