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ON HOLOMORPHICALLY PROJECTIVE MAPPINGS OF GENERALIZED KÄHLERIAN SPACES

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Abstract. In this work we define generalized Kählerian spaces and for them consider holomorphically projective mappings with an invariant complex structure. Also we consider equitorsion holomorphically projective mappings and for them we find some invariant geometric objects.

1. Introduction

A generalized Riemannian space GR_N in the sense of Eisenhart's definition [1] is a differentiable N-dimensional manifold, equipped with nonsymmetric basic tensor g_{ij}. Connection coefficients of this space are generalized Cristoffel's symbols of the second kind. Generally it is $\Gamma^i_{jk} \neq \Gamma^i_{kj}$.

In a generalized Riemannian space one can define four kinds of covariant derivatives [3], [4]. For example, for a tensor a^i_j in GR_N we have

$$a^i_{j\underset{1}{|}m} = a^i_{j,m} + \Gamma^i_{pm} a^p_j - \Gamma^p_{jm} a^i_p, \quad a^i_{j\underset{2}{|}m} = a^i_{j,m} + \Gamma^i_{mp} a^p_j - \Gamma^p_{mj} a^i_p,$$
$$a^i_{j\underset{3}{|}m} = a^i_{j,m} + \Gamma^i_{pm} a^p_j - \Gamma^p_{mj} a^i_p, \quad a^i_{j\underset{4}{|}m} = a^i_{j,m} + \Gamma^i_{mp} a^p_j - \Gamma^p_{jm} a^i_p.$$

In the case of the space GR_N we have five independent curvature tensors [5] (in [5] $\underset{5}{R}$ is denoted by $\underset{2}{\tilde{R}}$). In this paper we consider only curvature tensors: $\underset{1}{R}{}^i{}_{jmn} = \Gamma^i_{jm,n} - \Gamma^i_{jn,m} + \Gamma^p_{jm}\Gamma^i_{pn} - \Gamma^p_{jn}\Gamma^i_{pm}$, $\underset{2}{R}{}^i{}_{jmn} = \Gamma^i_{mj,n} - \Gamma^i_{nj,m} + \Gamma^p_{mj}\Gamma^i_{np} - \Gamma^p_{nj}\Gamma^i_{mp}$. The Kählerian spaces and their mappings were investigated by many authors, for example K. Yano [10], [11], M. Prvanović [7], N. S. Sinyukov [8], J. Mikeš [2] and many others.

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An N-dimensional Riemannian space with basic metric tensor $g_{ij}(x)$ is a Kählerian space if there exists an almost complex structure $F^i_j(x)$, such that

$$F^h_p(x)F^p_i(x) = -\delta^h_i\,, \; g_{pq}F^p_iF^q_j = g_{ij}, \quad g^{ij} = g^{pq}F^i_pF^j_q, \; F^h_{i;j} = 0,$$

where ; denotes the covariant derivative with respect to the basic metric tensor g_{ij}. This paper is devoted to the generalized Kählerian spaces and their mappings.

2. Generalized Kählerian spaces

A generalized N-dimensional Riemannian space with (non-symmetric) metric tensor g_{ij}, is a generalized Kählerian space GK_N if there exists an almost complex structure $F^i_j(x)$ [9], such that

$$F^h_p(x)F^p_i(x) = -\delta^h_i\,, \tag{2.1}$$

$$g_{\underline{pq}}F^p_iF^q_j = g_{\underline{ij}}, \quad g^{\underline{ij}} = g^{\underline{pq}}F^i_pF^j_q, \tag{2.2}$$

$$F^h_{i\underset{\theta}{|}j} = 0, \quad (\theta = 1, 2), \tag{2.3}$$

where $\underset{\theta}{|}$ denotes the covariant derivative of the kind θ with respect to the metric tensor g_{ij}. From (2.2), using (2.1), we get

$$g_{\underline{ip}}F^p_j + g_{\underline{pj}}F^p_i = 0, \quad g^{\underline{ip}}F^j_p + g^{\underline{jp}}F^i_p = 0. \tag{2.4,5}$$

Let us denote

$$F_{ji} = F^p_j g_{\underline{pi}}, \quad F^{ji} = F^j_p g^{\underline{pi}}. \tag{2.6}$$

Then from (2.4) and (2.5) one obtains $F_{ij} + F_{ji} = 0$, $F^{ij} + F^{ji} = 0$. From here we prove the following theorems [9].

THEOREM 2.1. *For the torsion tensor of a generalized Kählerian space the relation* $\Gamma^i_{\underset{\vee}{jm}} = -\Gamma^p_{\underset{\vee}{qm}}F^i_pF^q_j$ *is valid.*

THEOREM 2.2. *The curvature tensors* $\underset{\theta}{R}{}^h{}_{ijk}$ $(\theta = 1, 2)$ *in the space* GK_N *satisfy the next relations*

$$F^p_i\underset{\alpha}{R}{}^h{}_{pjk} = F^h_p\underset{\alpha}{R}{}^p{}_{ijk}, \; \alpha = 1, 2. \tag{2.7,8}$$

Proof. a) From (2.3) we have $F^h_{i\underset{1}{|}jk} - F^h_{i\underset{1}{|}kj} = 0$, and then, using the first Ricci identity [3], [4] we have $-F^h_p\underset{1}{R}{}^p{}_{ijk} + F^p_i\underset{1}{R}{}^h{}_{pjk} - 2\Gamma^p_{\underset{\vee}{jk}}F^h_{i\underset{1}{|}p} = 0$, i.e. $F^p_i\underset{1}{R}{}^h{}_{pjk} - F^h_p\underset{1}{R}{}^p{}_{ijk} = 0$. The relation (2.7) is proved.

b) Analogously, using the Ricci identity for $F^h_{i\underset{2}{|}jk} - F^h_{i\underset{2}{|}kj}$ and (2.3) we get $F^p_i\underset{2}{R}{}^h{}_{pjk} - F^h_p\underset{2}{R}{}^p{}_{ijk} = 0$, wherefrom (2.8) follows. ■

THEOREM 2.3. *For the curvature tensors* $\underset{\theta}{R}_{hijk}$ $(\theta = 1, 2)$ *of the space* GK_N *the next relations are valid*

$$F^p_h \underset{\alpha}{R}_{pijk} = F^p_i \underset{\alpha}{R}_{phjk}, \ \alpha = 1, 2. \tag{2.9,10}$$

Proof. By composition in (2.7) with F^q_h we get $F^p_i F^h_q \underset{1}{R}^q{}_{pjk} + \underset{1}{R}^h{}_{ijk} = 0$. From here we have $F^p_h F^q_i \underset{1}{R}_{pqjk} - \underset{1}{R}_{hijk} = 0$, and by composition with F^i_r we get

$$F^p_h \underset{1}{R}_{pijk} + F^p_i \underset{1}{R}_{hpjk} = 0. \tag{2.11}$$

The first kind curvature tensor satisfy the relation $\underset{1}{R}_{hijk} = -\underset{1}{R}_{ihjk}$. Now from (2.11) we get the relation (2.9). The relation (2.10) we get in the same manner from (2.8) by using of anti-symmetry for the tensors $\underset{2}{R}_{hijk}$ with respect to the two first indices. ■

THEOREM 2.4. *The curvature tensors* $\underset{\theta}{R}^i{}_{jmn}$ $(\theta = 1, 2)$ *of the space* GK_N *satisfy the next relations*

$$\underset{\alpha}{R}_{(pq)} F^p_j F^q_m = \underset{\alpha}{R}_{(jm)} - 2\underset{\vee}{\Gamma}^p_{rq} \underset{\vee}{\Gamma}^q_{ps} F^r_j F^s_m + 2\underset{\vee}{\Gamma}^p_{jq} \underset{\vee}{\Gamma}^q_{pm}, \ \alpha = 1, 2, \tag{2.12 a,b}$$

where (jm) *denotes the symmetrization without division with respect to the indices* j, m.

Proof. (a) From $F^h_{i\underset{1}{|}j} = 0$, $\quad F^h_{i\underset{2}{|}j} = 0$ by substitution and division with 2 we get $F^h_{i;j} = 0$, where ; denotes covariant derivative with respect to $g_{\underline{ij}}$. The integrability conditions of this equation give the relation $F^h_p R^p{}_{ijk} - F^p_i R^h{}_{pjk} = 0$, where $R^h{}_{ijk}$ is a curvature tensor with respect to symmetric basic tensor $g_{\underline{ij}}$. Using the condition (2.1) we get $F^h_p F^q_i R^p{}_{ijk} + R^h{}_{ijk} = 0$, and from here $F^p_h F^q_i R_{pqjk} - R_{hijk} = 0$. With respect to the condition (2.1), we get $F^p_h R_{pijk} - F^p_i R_{phjk} = 0$. By composition with $g^{\underline{ij}}$ and contraction by virtue of indices i, j, we get $F^p_h R_{pk} = F^p_q R_{ph.k}^{\ \ q}$. By symmetrization with respect to h, k we get

$$R_{hk} = F^p_h F^q_k R_{pq}. \tag{2.13}$$

We can express the tensor $\underset{1}{R}^i{}_{jmn}$ in the form [5]:

$$\underset{1}{R}^i{}_{jmn} = R^i{}_{jmn} + \underset{\vee}{\Gamma}^i_{jm;n} - \underset{\vee}{\Gamma}^i_{jn;m} + \underset{\vee}{\Gamma}^p_{jm} \underset{\vee}{\Gamma}^i_{pn} - \underset{\vee}{\Gamma}^p_{jn} \underset{\vee}{\Gamma}^i_{pm}.$$

By contraction with respect to indices i, n, and by symmetrization with respect to j, m, we get

$$\underset{1}{R}_{(jm)} = R_{(jm)} - 2\underset{\vee}{\Gamma}^p_{jq} \underset{\vee}{\Gamma}^q_{pm}. \tag{2.14}$$

From (2.13) and (2.14) we have (2.12a).

(b) The tensor $\underset{2}{R}{}^i{}_{jmn}$ we can express in the form [5]:

$$\underset{2}{R}{}^i{}_{jmn} = R^i{}_{jmn} - \Gamma^i_{\underset{\vee}{jm};n} + \Gamma^i_{\underset{\vee}{jn};m} - \Gamma^p_{\underset{\vee}{jm}}\Gamma^i_{\underset{\vee}{pn}} + \Gamma^p_{\underset{\vee}{jn}}\Gamma^i_{\underset{\vee}{pm}}.$$

By contraction with respect to i, n, and then by symmetrization with respect to j, m, we get $\underset{2}{R}{}_{(jm)} = R_{(jm)} - 2\Gamma^p_{\underset{\vee}{jq}}\Gamma^q_{\underset{\vee}{pm}}$, wherefrom, using (2.13), we get the relation (2.12b). ∎

3. Holomorphically projective mapings

Generalizing the concept of analytic planar curve in a Kählerian space [6], [8] we get an analogous notion for a generalized Kählerian space [9].

Definition 3.1. A curve $l : x^h = x^h(t)$, $(h = 1, 2, \dots, N)$ is an *an analytic planar curve* if the following relation is satisfied

$$\lambda^h{}_{\underset{\theta}{|}p}\lambda^p = a(t)\lambda^h + b(t)F^h_p\lambda^p, \quad (\theta = 1, 2) \tag{3.1}$$

where $\lambda^h = dx^h/dt$, and $a(t)$ and $b(t)$ are same function of a parameter t.

In GK_N it is $\lambda^h{}_{\underset{1}{|}p}\lambda^p = \frac{d\lambda^h}{dt} + \Gamma^h_{pq}\lambda^p\lambda^q = \lambda^h{}_{\underset{2}{|}p}\lambda^p$. Then the expression on the left-hand side in (3.1) is invariant with respect to the both kind of covariant derivative, and so we can define analytic planar curve in the space GK_N by one relation $\frac{d\lambda^h}{dt} + \Gamma^h_{pq}\lambda^p\lambda^q = a(t)\lambda^h + b(t)F^h_p\lambda^p$.

Consider two N-dimensional generalized Kählerian spaces GK_N and $G\overline{K}_N$ with almost complex structures F^h_i and $\overline{F}^h_i$, respectively, where $F^h_i = \overline{F}^h_i$.

Definition 3.2. A diffeomorfism $f \colon GK_N \to G\overline{K}_N$ is *holomorphically projective* or *analytic planar* if by this mapping analytic planar curves of the space GK_N are mapped into analytic planar curves of the space $G\overline{K}_N$.

Let us denote by $P^h_{ij} = \overline{\Gamma}^h_{ij} - \Gamma^h_{ij}$ the deformation tensor of connection for analytic planar mapping, where Γ^h_{ij} and $\overline{\Gamma}^h_{ij}$ are the second kind Cristophell's symbols of the spaces GK_N and $G\overline{K}_N$, respectively.

Analytic planar curves of the spaces GK_N and $G\overline{K}_N$ are given by relations

$$\frac{d\lambda^h}{dt} + \Gamma^h_{pq}\lambda^p\lambda^q = a(t)\lambda^h + b(t)F^h_p\lambda^p, \quad \frac{d\lambda^h}{dt} + \overline{\Gamma}^h_{pq}\lambda^p\lambda^q = \overline{a}(t)\lambda^h + \overline{b}(t)F^h_p\lambda^p,$$

respectively. From these relations we get $(\overline{\Gamma}^h_{pq} - \Gamma^h_{pq})\lambda^p\lambda^q = \psi(t)\lambda^h + \sigma(t)F^h_p\lambda^p$, where we denote $\psi(t) = \overline{a}(t) - a(t)$, $\sigma(t) = \overline{b}(t) - b(t)$. Putting $\psi(t) = \psi_p\lambda^p$, $\sigma(t) = \sigma_q\lambda^q$, we have $(\overline{\Gamma}^h_{pq} - \Gamma^h_{pq} - \psi_p\delta^h_q - \sigma_p F^h_q)\lambda^p\lambda^q = 0$, wherefrom it is $\overline{\Gamma}^h_{ij} = \Gamma^h_{ij} + \psi_{(i}\delta^h_{j)} + \sigma_{(i}F^h_{j)} + \xi^h_{ij}$, where (ij) denotes a symmetrization without division by indices i, j and ξ^h_{ij} is an anti-symmetric tensor. The vector σ_i we can select so that $\sigma_i = -\psi_p F^p_i$. Then we have

$$\overline{\Gamma}^h_{ij} = \Gamma^h_{ij} + \psi_{(i}\delta^h_{j)} - \psi_p F^p_{(i} F^h_{j)} + \xi^h_{ij}. \tag{3.2}$$

Contracting by indices h, i in (3.2) and using $F^p_p = 0$, $\xi^p_{pj} = 0$ we have

$$\overline{\Gamma}^p_{pj} - \Gamma^p_{pj} = (N+2)\psi_j. \tag{3.3}$$

From (3.3) we can see that ψ_j is a gradient vector. Substituting from (3.3) into (3.2) we get $H\overline{T}^h_{ij} = HT^h_{ij}$, where we denote

$$HT^h_{ij} = \Gamma^h_{\underline{ij}} - \frac{1}{N+2}(\Gamma^p_{p(i}\delta^h_{j)} - \Gamma^q_{qp} F^p_{(i} F^h_{j)}). \tag{3.4}$$

Here $H\overline{T}^h_{ij}$ denotes an object of the form (3.4) in the space $G\overline{K}_N$. The magnitude HT^h_{ij} is not a tensor. We shall call it *holomorphically projective parameter of the type of Thomas's projective parameter.* From the facts given above, we have

THEOREM 3.1. *Geometric objects* (3.4) *of the space* GK_N *are invariants of holomorphically projective mappings.*

4. Equitorsion holomorphically projective mappings

Let $f\colon GK_N \to G\overline{K}_N$ be a holomorphically projective mapping, and let the torsion tensors $\underset{\vee}{\Gamma}{}^h_{ij}$ and $\underset{\vee}{\overline{\Gamma}}{}^h_{ij}$ of the spaces GK_N and $G\overline{K}_N$ satisfy the condition $\underset{\vee}{\overline{\Gamma}}{}^h_{ij} = \underset{\vee}{\Gamma}{}^h_{ij}$. In this case the mapping f is called *an equitorsion holomorphically projective mapping* of the spaces GK_N and $G\overline{K}_N$. Then (3.2) implies $\xi^h_{ij} = 0$.

4.1.Holomorphically projective parameters of the first kind

Curvature tensors of the first kind $\underset{1}{R}$ and $\underset{1}{\overline{R}}$ of the spaces GK_N and $G\overline{K}_N$, respectively, are connected by the relation [5]

$$\underset{1}{\overline{R}}{}^i_{jmn} = \underset{1}{R}{}^i{}_{jmn} + P^i_{\underset{1}{jm|n}} - P^i_{\underset{1}{jn|m}} + P^p_{jm}P^i_{pn} - P^p_{jn}P^i_{pm} + 2\underset{\vee}{\Gamma}{}^p_{mn}P^i_{jp}, \tag{4.1}$$

where $P^h_{ij} = \overline{\Gamma}^h_{ij} - \Gamma^h_{ij}$ is a deformation tensor. Substituting (3.3) and $\xi^h_{ij} = 0$ into (4.1) we get

$$\begin{aligned}\underset{1}{\overline{R}}{}^i_{jmn} = \underset{1}{R}{}^i{}_{jmn} &+ \delta^i_m \underset{1}{\psi}_{jn} + \delta^i_j \underset{1}{\psi}_{[mn]} - \delta^i_j \underset{1}{\psi}_{jm} \\ &+ F^p_j(F^i_n \underset{1}{\psi}_{pm} - F^i_m \underset{1}{\psi}_{pn}) + F^i_j(F^p_n \underset{1}{\psi}_{pm} - F^p_m \underset{1}{\psi}_{pn}) \\ &+ 2\underset{\vee}{\Gamma}{}^i_{mn}\psi_j + 2\underset{\vee}{\Gamma}{}^p_{mn}\psi_p\delta^i_j - 2\underset{\vee}{\Gamma}{}^p_{mn}\psi_q F^q_j F^i_p - 2\underset{\vee}{\Gamma}{}^p_{mn}\psi_q F^q_p F^i_j,\end{aligned} \tag{4.2}$$

where we denote $\underset{1}{\psi}_{ij} = \psi_{i\underset{1}{|}j} - \psi_i\psi_j + \psi_p F_i^p \psi_q F_j^q$. Contracting with respect to indices i, n in (4.2) we obtain

$$\underset{1}{\overline{R}}_{jm} = \underset{1}{R}_{jm} + \underset{1}{\psi}_{[mj]} - N\underset{1}{\psi}_{jm} - F_j^p F_m^q \underset{1}{\psi}_{(pq)} + 2\Gamma^p_{\underset{\vee}{mj}}\psi_p - 2\Gamma^p_{\underset{\vee}{mr}}\psi_q F_j^q F_p^r - 2\Gamma^p_{\underset{\vee}{mr}}\psi_q F_p^q F_j^r. \tag{4.3}$$

Anti-symmetrization without division in (4.3) with respect to indices j, m yields

$$(N+2)\underset{1}{\psi}_{[jm]} = \underset{1}{R}_{[jm]} - \underset{1}{\overline{R}}_{[jm]} + 4\Gamma^p_{\underset{\vee}{mj}}\psi_p - 2\Gamma^p_{\underset{\vee}{mr}}\psi_q F_j^q F_p^r + 2\Gamma^p_{\underset{\vee}{jr}}\psi_q F_m^q F_p^r - 2\Gamma^p_{\underset{\vee}{mr}}\psi_q F_p^q F_j^r + 2\Gamma^p_{\underset{\vee}{jr}}\psi_q F_p^q F_m^r. \tag{4.4}$$

Symmetrizing without division with respect to j, m in (4.3) we arrive at

$$\underset{1}{\overline{R}}_{(jm)} = \underset{1}{R}_{(jm)} - N\underset{1}{\psi}_{(jm)} - 2F_j^p F_m^q \underset{1}{\psi}_{(pq)} - 2\Gamma^p_{\underset{\vee}{mr}}\psi_q F_j^q F_p^r - 2\Gamma^p_{\underset{\vee}{jr}}\psi_q F_m^q F_p^r - 2\Gamma^p_{\underset{\vee}{mr}}\psi_q F_p^q F_j^r - 2\Gamma^p_{\underset{\vee}{jr}}\psi_q F_p^q F_m^r. \tag{4.5}$$

By composition with $F_p^j F_q^m$, contraction with respect to j, m, and using the conditions (2.12a), we get from (4.5)

$$\underset{1}{\overline{R}}_{(jm)} = \underset{1}{R}_{(jm)} - N\underset{1}{\psi}_{(pq)} F_j^p F_m^q - 2\underset{1}{\psi}_{(jm)} + 2\Gamma^p_{\underset{\vee}{qr}}\psi_j F_p^r F_m^q + 2\Gamma^p_{\underset{\vee}{qr}}\psi_m F_p^r F_j^q + 2\Gamma^p_{\underset{\vee}{rj}}\psi_q F_p^q F_m^r + 2\Gamma^p_{\underset{\vee}{rm}}\psi_m F_p^q F_j^r. \tag{4.6}$$

Using (4.4,5,6) and (2.12a) we get

$$(N+2)\underset{1}{\psi}_{jm} = \underset{1}{R}_{jm} - \underset{1}{\overline{R}}_{jm} + 2\Gamma^p_{\underset{\vee}{mj}}\psi_p - \frac{2N-2}{N-2}\Gamma^p_{\underset{\vee}{mr}}\psi_q F_j^q F_p^r - \frac{2}{N-2}\Gamma^p_{\underset{\vee}{jr}}\psi_q F_m^q F_p^r - \frac{2}{N-2}\Gamma^p_{\underset{\vee}{qr}}\psi_j F_p^r F_m^q - \frac{2}{N-2}\Gamma^p_{\underset{\vee}{qr}}\psi_m F_p^r F_j^q - 2\Gamma^p_{\underset{\vee}{mr}}\psi_q F_p^q F_j^r. \tag{4.7}$$

Eliminating ψ_i by using the condition (3.3) we reduce the equation (4.7) to the form $(N+2)\underset{1}{\psi}_{jm} = \underset{1}{R}_{jm} - \underset{1}{\overline{R}}_{jm} + \underset{1}{\overline{P}}_{jm} - \underset{1}{P}_{jm}$, where we denote

$$\underset{1}{P}_{jm} = \frac{2}{N+2}\Big(\Gamma^p_{\underset{\vee}{mj}}\Gamma^q_{qp} - \frac{N-1}{N-2}\Gamma^p_{\underset{\vee}{mr}}\Gamma^s_{sq}F_j^q F_p^r - \frac{1}{N-2}\Gamma^p_{\underset{\vee}{jr}}\Gamma^s_{sq}F_m^q F_p^r - \frac{1}{N-2}\Gamma^p_{\underset{\vee}{qr}}\Gamma^s_{sj}F_p^r F_m^q - \frac{1}{N-2}\Gamma^p_{\underset{\vee}{qr}}\Gamma^s_{sm}F_p^r F_j^q - \Gamma^p_{\underset{\vee}{mr}}\Gamma^s_{sq}F_p^q F_j^r\Big). \tag{4.8}$$

In the same manner the geometric objects $\underset{1}{\overline{P}}_{jm}$ of the space $G\overline{K}_N$ is defined. Eliminating $\underset{1}{\psi}_{jm}$ from (4.2) we obtain $HP\underset{1}{\overline{W}}{}^{i}{}_{jmn} = HP\underset{1}{W}{}^{i}{}_{jmn}$, where the magnitude

$$\begin{aligned} HP\underset{1}{W}{}^{i}{}_{jmn} &= \underset{1}{R}{}^{i}{}_{jmn} + \frac{1}{N+2}[\delta^{i}_{m}(\underset{1}{R}_{jn} - \underset{1}{P}_{jn}) + \delta^{i}_{j}(\underset{1}{R}_{[mn]} - \underset{1}{P}_{[mn]}) \\ &- \delta^{i}_{n}(\underset{1}{R}_{jm} - \underset{1}{P}_{jm}) + F^{p}_{j}F^{i}_{n}(\underset{1}{R}_{pm} - \underset{1}{P}_{pm}) - F^{p}_{j}F^{i}_{m}(\underset{1}{R}_{pn} - \underset{1}{P}_{pn}) \\ &+ F^{i}_{j}F^{p}_{n}(\underset{1}{R}_{pm} - \underset{1}{P}_{pm}) - F^{i}_{j}F^{p}_{m}(\underset{1}{R}_{pn} - \underset{1}{P}_{pn}) - 2\Gamma^{i}_{\underset{\vee}{mn}}\Gamma^{q}_{qj} - 2\delta^{i}_{j}\Gamma^{p}_{\underset{\vee}{mn}}\Gamma^{q}_{qp} \\ &+ 2\Gamma^{p}_{\underset{\vee}{mn}}\Gamma^{s}_{sq}F^{q}_{j}F^{i}_{p} + 2\Gamma^{p}_{\underset{\vee}{mn}}\Gamma^{s}_{sq}F^{q}_{p}F^{i}_{j}] \end{aligned} \tag{4.9}$$

is expressed by geometric objects of the space GK_N. In the same manner the magnitude $HP\underset{1}{\overline{W}}{}^{i}{}_{jmn}$ is expressed by geometric objects of the space $G\overline{K}_N$. The magnitude $HP\underset{1}{W}{}^{i}{}_{jmn}$ is not a tensor, and we call it *the equitorsion holomorphically projective parameter of the first kind* of the space GK_N. From the facts given above, we have

THEOREM 4.1. *The equitorsion holomorphically projective parameter* (4.9) *is an invariant of equitorsion holomorphically projective mapping* $f\colon GK_N \to G\overline{K}_N$.

4.2. Holomorphically projective parameters of the second kind

For the curvature tensors $\underset{2}{R}$ and $\underset{2}{\overline{R}}$ of the spaces GK_N and $G\overline{K}_N$ the relation

$$\underset{2}{\overline{R}}{}^{i}_{jmn} = \underset{2}{R}{}^{i}{}_{jmn} + P^{i}_{mj\underset{2}{|}n} - P^{i}_{nj\underset{2}{|}m} + P^{p}_{mj}P^{i}_{np} - P^{p}_{nj}P^{i}_{mp} + 2\Gamma^{p}_{\underset{\vee}{nm}}P^{i}_{pj} \tag{4.10}$$

is valid [5], where P^{i}_{jm} is a deformation tensor.

Analogously to the previous case we obtain $HP\underset{2}{\overline{W}}{}^{i}{}_{jmn} = HP\underset{2}{W}{}^{i}{}_{jmn}$, where we denote

$$\begin{aligned} HP\underset{2}{W}{}^{i}{}_{jmn} &= \underset{2}{R}{}^{i}{}_{jmn} + \frac{1}{N+2}[\delta^{i}_{m}(\underset{2}{R}_{jn} - \underset{2}{P}_{jn}) + \delta^{i}_{j}(\underset{2}{R}_{[mn]} - \underset{2}{P}_{[mn]}) \\ &- \delta^{i}_{n}(\underset{2}{R}_{jm} - \underset{2}{P}_{jm}) + F^{p}_{j}F^{i}_{n}(\underset{2}{R}_{pm} - \underset{2}{P}_{pm}) - F^{p}_{j}F^{i}_{m}(\underset{2}{R}_{pn} - \underset{2}{P}_{pn}) \\ &+ F^{i}_{j}F^{p}_{n}(\underset{2}{R}_{pm} - \underset{2}{P}_{pm}) - F^{i}_{j}F^{p}_{m}(\underset{2}{R}_{pn} - \underset{2}{P}_{pn}) - 2\delta^{i}_{j}\Gamma^{p}_{\underset{\vee}{nm}}\Gamma^{q}_{qp} - 2\Gamma^{i}_{\underset{\vee}{nm}}\Gamma^{q}_{qj} \\ &+ 2\Gamma^{p}_{\underset{\vee}{nm}}\Gamma^{s}_{sq}F^{q}_{p}F^{i}_{j} + 2\Gamma^{p}_{\underset{\vee}{nm}}\Gamma^{s}_{sq}F^{q}_{j}F^{i}_{p}], \end{aligned} \tag{4.11}$$

$$\begin{aligned} \underset{2}{P}_{jm} &= \frac{2}{N+2}(\Gamma^{p}_{\underset{\vee}{jm}}\Gamma^{q}_{qp} - \Gamma^{p}_{\underset{\vee}{rm}}\Gamma^{s}_{sq}F^{q}_{p}F^{r}_{j} - \frac{N-1}{N-2}\Gamma^{p}_{\underset{\vee}{rm}}\Gamma^{s}_{sq}F^{q}_{j}F^{r}_{p} \\ &- \frac{1}{N-2}\Gamma^{p}_{\underset{\vee}{rj}}\Gamma^{s}_{sq}F^{q}_{m}F^{r}_{p} - \frac{1}{N-2}\Gamma^{p}_{\underset{\vee}{rq}}\Gamma^{s}_{sj}F^{r}_{p}F^{q}_{m} - \frac{1}{N-2}\Gamma^{p}_{\underset{\vee}{rq}}\Gamma^{s}_{sm}qF^{r}_{p}F^{q}_{j}). \end{aligned} \tag{4.12}$$

The magnitude $HP\underset{2}{W}{}^{i}{}_{jmn}$ is not a tensor, and we call it *the equitorsion holomorphically projective parameter of the second kind* of the space GK_N. From the facts given above, we have

THEOREM 4.2. *The equitorsion holomorphically projective parameter of the second kind is an invariant of equitorsion holomorphically projective mapping of the spaces GK_N and $G\overline{K}_N$.*

4.3. The case of Kählerian spaces

In the case of holomorphically projective mappings of Kählerian spaces the magnitudes $\underset{\theta}{HPW}{}^i{}_{jmn}$, $(\theta = 1, 2)$, given by (4.9,11) reduce to the holomorphically projective curvature tensor [8]

$$HPW^i{}_{jmn} = R^i{}_{jmn} + \frac{1}{N+2}(R_{j[n}\delta^i_{m]} + F^p_j R_{p[m}F^i_{n]} + 2F^i_j F^p_n R_{pm}).$$

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