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SOME FUZZY SP-TOPOLOGICAL PROPERTIES

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Abstract. The concepts of fuzzy SP-irresolute continuous, fuzzy SP-irresolute open (closed) mappings, and a fuzzy SP-homeomorphism are being introduced and studied. Some of their characteristic properties are being considered. Finally, the claim that fuzzy semiregular properties are fuzzy SP-topological is being proved.

1. Introduction

The classes of fuzzy strongly preopen sets and fuzzy strongly precontinuous mappings are introduced in [8]. As a continuation of this work, in Section 3 and Section 4 we introduce the concepts of a fuzzy SP-irresolute continuous mapping and a fuzzy SP-irresolute open (closed) mapping.

As a pendant of the notion of a semihomeomorphism introduced in [5], Yalvac [13] introduced the concept of a fuzzy semihomeomorphism. In the section 5 we introduce the notion of a fuzzy SP-homeomorphism and we prove that fuzzy semiregular properties and fuzzy separability are fuzzy SP-topological.

2. Preliminatries

Now we introduce some basic notions and results that are used in the sequel.

In this work by (X,τ) or simply by X we will denote a fuzzy topological space (*fts*) due to Chang [6]. The interior, the closure and the complement of a fuzzy set A will be denoted by $\operatorname{int} A$, $\operatorname{cl} A$ and A^c, respectively.

DEFINITION 2.1. A fuzzy set A of an *fts* X is called:

1) fuzzy semiopen (semiclosed) if there exists a fuzzy open (closed) set U of X such that $U \leqslant A \leqslant \operatorname{cl} U$ ($\operatorname{int} U \leqslant A \leqslant U$) [1];

2) fuzzy preopen (preclosed) if $A \leqslant \operatorname{int}(\operatorname{cl} A)$ ($A \geqslant \operatorname{cl}(\operatorname{int} A)$) [4];

3) fuzzy strongly semiopen (strongly semiclosed) if $A \leqslant \operatorname{int}(\operatorname{cl}(\operatorname{int} A))$ ($A \geqslant \operatorname{cl}(\operatorname{int}(\operatorname{cl} A))$) [4];

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4) fuzzy strongly preopen (strongly preclosed) if $A \leqslant \operatorname{int}(\operatorname{pcl} A)$ $(A \geqslant \operatorname{cl}(\operatorname{pint} A))$ [8];

5) fuzzy regular open (regular closed) if $A = \operatorname{int}(\operatorname{cl} A)$ $(A = \operatorname{cl}(\operatorname{int} A))$ [1].

The family of all fuzzy semiopen (fuzzy preopen, fuzzy strongly semiopen, fuzzy strongly preopen, fuzzy regular open, fuzzy semiclosed, fuzzy preclosed, fuzzy strongly semiclosed, fuzzy strongly preclosed, fuzzy regular closed) sets of an *fts* (X, τ) will be denoted by $FSO(\tau)$ ($FPO(\tau)$, $FSSO(\tau)$, $FSPO(\tau)$, $FRO(\tau)$, $FSC(\tau)$, $FPC(\tau)$, $FSSC(\tau)$, $FSPC(\tau)$, $FRC(\tau)$).

Definition 2.2. Let $f\colon (X, \tau_1) \to (Y, \tau_2)$ be a mapping from an *fts* (X, τ_1) into an *fts* (Y, τ_2). The mapping f is called:

1) fuzzy continuous if $f^{-1}(B) \in \tau_1$, for each $B \in \tau_2$ [6];

2) fuzzy weakly continuous if $f^{-1}(B) \leqslant \operatorname{int} f^{-1}(\operatorname{cl} B)$, for each $B \in \tau_2$ [1];

3) fuzzy strongly precontinuous if $f^{-1}(B) \in FSPO(\tau_1)$, for each $B \in \tau_2$ [8];

4) fuzzy irresolute continuous if $f^{-1}(B) \in FSO(\tau_1)$, for each $B \in FSO(\tau_2)$ [13];

5) fuzzy preirresolute continuous if $f^{-1}(B) \in FPO(\tau_1)$, for each $B \in FPO(\tau_2)$ [7];

6) fuzzy strongly irresolute continuous if $f^{-1}(B) \in FSSO(\tau_1)$, for each $B \in FSSO(\tau_2)$ [2];

7) fuzzy R-continuous if $f^{-1}(B) \in FRO(\tau_1)$, for each $B \in FRO(\tau_2)$ [3];

8) fuzzy open (closed) if $f(A) \in \tau_2$ $(f(A)^c \in \tau_2)$, for each $A \in \tau_1$ $(A^c \in \tau_1)$ [12];

9) fuzzy strongly preopen (preclosed) if $f(A) \in FSPO(\tau_2)$ $(f(A) \in FSPC(\tau_2))$, for each $A \in \tau_1$ $(A^c \tau_1)$ [8];

10) fuzzy irresolute open (closed) if $f(A) \in FSO(\tau_2)$ $(f(A) \in FSC(\tau_2))$, for each $A \in FSO(\tau_1)$ $(A \in FSC(\tau_1))$ [13];

11) fuzzy preirresolute open (closed) if $f(A) \in FPO(\tau_2)$ $(f(A) \in FPC(\tau_2))$, for each $A \in FPO(\tau_1)$ $(A \in FPC(\tau_1))$ [7];

12) fuzzy strongly irresolute open (closed) if $f(A) \in FSSO(\tau_2)$ $(f(A) \in FSSC(\tau_2))$, for each $A \in FSSO(\tau_1)$ $(A \in FSSC(\tau_1))$ [2];

13) fuzzy R-open (R-closed) if $f(A) \in FRO(\tau_2)$ $(f(A) \in FRC(\tau_2))$, for each $A \in FRO(\tau_1)$ $(A \in FRC(\tau_1))$ [3];

14) fuzzy homeomorphism if it is bijective and f and f^{-1} are fuzzy continuous [6].

Definition 2.3. [10] A fuzzy point x_α of an *fts* X is a fuzzy set of X defined as follows:

$$x_\alpha(y) = \begin{cases} \alpha, & \text{if } y = x; \\ 0, & \text{otherwise.} \end{cases} \qquad 0 < \alpha \leqslant t.$$

Definition 2.4. [10] A fuzzy point x_α is said to be contained in a fuzzy set A of an *fts* X $(x_\alpha \in A)$ if $\alpha \leqslant A(x)$, for each $x \in X$.

Definition 2.5. [10] A fuzzy set A of an *fts* X is said to be a neighbourhood of a fuzzy point x_α of X, if there exists a fuzzy open set U of X, such that $x_\alpha \leqslant U \leqslant A$.

Lemma 2.1. [10] *Let A be a fuzzy set of an fts X. Then* $A = \bigvee_{x_\alpha \in A} x_\alpha$. ■

Definition 2.6. [8] Let A be a fuzzy set of an *fts* X.

(1) The union of all fuzzy strongly preopen sets contained in A is called a fuzzy strong preinterior of A, denoted by $\operatorname{spint} A$.

(2) The intersection of all fuzzy strongly preclosed sets containing A is called a fuzzy strong preclosure of A, denoted by $\operatorname{spcl} A$.

Lemma 2.2. [8] *Let A be a fuzzy set of an fts X. Then,*

1) $\operatorname{spcl} A^c = (\operatorname{spint} A)^c$; *2)* $\operatorname{spint} A^c = (\operatorname{spcl} A)^c$; *3)* $\operatorname{pcl} A^c = (\operatorname{pint} A)^c$; *4)* $\operatorname{pint} A^c = (\operatorname{pcl} A)^c$.

Lemma 2.3. [8] *Let A be a fuzzy set of an fts X. Then,*

$\operatorname{int} A \leqslant \operatorname{spint} A \leqslant \operatorname{pint} A \leqslant A \leqslant \operatorname{pcl} A \leqslant \operatorname{spcl} A \leqslant \operatorname{cl} A$. ■

Lemma 2.4. [8] *Let A be a fuzzy set of an fts X.*

1) If A is fuzzy strongly preopen, then $\operatorname{pcl} A = \operatorname{spcl} A = \operatorname{cl} A$.

2) If A is fuzzy strongly preclosed, then $\operatorname{pint} A = \operatorname{spint} A = \operatorname{int} A$. ■

Lemma 2.5. *Let X be an fts. If $A \neq 0_X$ is a fuzzy strongly preopen set, then* $\operatorname{int} A \neq 0_X$.

Proof. If $\operatorname{int} A = 0_X$, then from $\operatorname{cl}(\operatorname{int} A) \leqslant A$ it follows that A is a fuzzy preclosed set. Thus $\operatorname{int}(\operatorname{pcl} A) = \operatorname{int} A = 0_X < A$. Hence A is not fuzzy strongly preopen. ■

Lemma 2.6. [8] *Let A be a fuzzy set of an fts X. Then,*

1) $\operatorname{spint}(\operatorname{cl} A) = \operatorname{int}(\operatorname{cl} A)$; *2)* $\operatorname{spcl}(\operatorname{int} A) = \operatorname{cl}(\operatorname{int} A)$. ■

Lemma 2.7. [8] *Let (X, τ) be an fts. Then,*

1) $FSPO(\tau) \cap FSC(\tau) = FRO(\tau)$; *2)* $FSPC(\tau) \cap FSO(\tau) = FRC(\tau)$;

3) $FSPO(\tau) \cap FSO(\tau) = FSSO(\tau)$; *4)* $FSPC(\tau) \cap FSC(\tau) = FSSC(\tau)$.

Definition 2.7. [11] An *fts* (X, τ) is called fuzzy separable if there exists a countable sequence of fuzzy points $\{p_i\}_{i \in \mathbf{N}}$ such that for each fuzzy open set $A \neq 0_X$, there exists $p_i \in A$.

Let (X, τ) be an *fts*. Since the intersection of two fuzzy regular open sets is a regular open set, the family $FRO(\tau)$ forms a base for a smaller topology $S(X)$ on X, called the semiregularization of X.

Definition 2.8. [1] An *fts* (X, τ) is called fuzzy semiregular if the family of all fuzy regular open sets of X forms a base for the fuzzy topology τ.

Definition 2.9. [9] A topological property P is called semiregular if an *fts* (X,τ) has P if and only if $(X,S(X))$ has P.

3. Fuzzy SP-irresolute continuous mappings

Definition 3.1. A mapping $f\colon (X,\tau_1)\to(Y,\tau_2)$ from an *fts* (X,τ_1) into an *fts* (X,τ_2) is called fuzzy SP-irresolute continuous if $f^{-1}(B)\in FSPO(\tau_1)$, for each $B\in FSPO(\tau_2)$.

Remark 3.1. If f is fuzzy SP-irresolute continuous, then f is fuzzy strong precontinuous. The converse may not be true. The concepts of fuzzy SP-irresolute continuity and fuzzy continuity are independent.

Example 3.1. Let $X=\{a,b,c\}$ and A, B, C, D be fuzzy sets of X defined as follows:

$$\begin{array}{lll} A(a)=0{,}5 & A(b)=0{,}3 & A(c)=0{,}6;\\ B(a)=0{,}3 & B(b)=0{,}4 & B(c)=0{,}3;\\ C(a)=0{,}5 & C(b)=0{,}4 & C(c)=0{,}6;\\ D(a)=0{,}5 & D(b)=0{,}5 & D(c)=0{,}6. \end{array}$$

We put $\tau_1=\{0,A,B,A\wedge B,A\vee B,1\}$, $\tau_2=\{0,C,1\}$ and $f=id\colon(X,\tau_1)\to(X,\tau_2)$. Then f is fuzzy strong precontinuous, but f is not fuzzy SP-irresolute continuous. Furthermore, f is a fuzzy continuous mapping, which is not SP-irresolute continuous.

Example 3.2. Let $X=\{a,b,c\}$ and A, B, C, be fuzzy sets of X defined as follows:

$$\begin{array}{lll} A(a)=0{,}3 & A(b)=0{,}2 & A(c)=0{,}7;\\ B(a)=0{,}8 & B(b)=0{,}8 & B(c)=0{,}4;\\ C(a)=0{,}8 & C(b)=0{,}7 & C(c)=0{,}6. \end{array}$$

We put $\tau_1=\{0,A,B,A\wedge B,A\vee B,1\}$, $\tau_2=\{0,C,1\}$ and $f=id\colon(X,\tau_1)\to(X,\tau_2)$. Then f is fuzzy SP-irresolute continuous, but f is not fuzzy continuous.

Theorem 3.1. *Let $f\colon(X,\tau_1)\to(Y,\tau_2)$ be a mapping from an fts (X,τ_1) into an fts (Y,τ_2). Then the following statements are equivalent:*

(i) f iz fuzzy SP-irresolute continuous;

(ii) $f^{-1}(B)\in FSPC(\tau_1)$, for each $B\in FSPC(\tau_2)$;

(iii) $f(\operatorname{spcl} A)\leqslant \operatorname{spcl} f(A)$, for each fuzzy set A of X;

(iv) $\operatorname{spcl} f^{-1}(B)\leqslant f^{-1}(\operatorname{spcl} B)$, for each fuzzy set B of Y;

(v) $f^{-1}(\operatorname{spint} B)\leqslant \operatorname{spint} f^{-1}(B)$, for each fuzzy set B of Y.

Proof. (i) $\Longrightarrow$ (ii) can be proved by using Theorem 4.1 [14].

(ii) $\implies$ (iii). Let A be a fuzzy set of X. Then $\operatorname{spcl} f(A) \in FSPC(\tau_2)$. According to the assumption, $f^{-1}(\operatorname{spcl} f(A)) \in FSPC(\tau_1)$. Hence, $\operatorname{spcl} A \leqslant \operatorname{spcl} f^{-1}f(A) \leqslant \operatorname{spcl} f^{-1}(\operatorname{spcl} f(A)) = f^{-1}(\operatorname{spcl} f(A))$. Thus, $f(\operatorname{spcl} A) \leqslant \operatorname{spcl} f(A)$.

(iii) $\implies$ (iv). Let B be a fuzzy set of Y. According to the assumption, $f(\operatorname{spcl} f^{-1}(B)) \leqslant \operatorname{spcl} ff^{-1}(B) \leqslant \operatorname{spcl} B$. Thus, $\operatorname{spcl} f^{-1}(B) \leqslant f^{-1}f(\operatorname{spcl} f^{-1}(B)) \leqslant f^{-1}(\operatorname{spcl} B)$.

(iv) $\implies$ (v) can be proved by using Lemma 2.2 and Theorem 4.1 [14].

(v) $\implies$ (i). Let $B \in FSPO(\tau_2)$. Then $B = \operatorname{spint} B$. From (v) we obtain $f^{-1}(B) = f^{-1}(\operatorname{spint} B) \leqslant \operatorname{spint} f^{-1}(B) \leqslant f^{-1}(B)$. Thus, $f^{-1}(B) = \operatorname{spint} f^{-1}(B)$. Hence, f is a fuzzy SP-irresolute continuous mapping. ∎

THEOREM 3.2. *Let* $f\colon (X,\tau_1) \to (Y,\tau_2)$ *be a bijective mapping from an fts* (X,τ_1) *into an fts* (Y,τ_2). *The mapping* f *is fuzzy SP-irresolute continuous if and only if* $\operatorname{spint} f(A) \leqslant f(\operatorname{spint} A)$, *for each fuzzy set* A *of* X.

Proof. Let f be fuzzy SP-irresolute continuous. Then $f^{-1}(\operatorname{spint} f(A)) \in FSPO(\tau_1)$, for any fuzzy set A of X. Since f is injective, from Theorem 3.1 we obtain that $f^{-1}(\operatorname{spint} f(A)) \leqslant \operatorname{spint} f^{-1}f(A) = \operatorname{spint} A$. Again, since f is surjective, we have $\operatorname{spint} f(A) = ff^{-1}(\operatorname{spint} f(A)) \leqslant f(\operatorname{spint} A)$.

Conversely, let $B \in FSPO(\tau_2)$. Then $\operatorname{spint} B = B$. Since f is surjective, from the assumption we obtain that $f(\operatorname{spint} f^{-1}(B)) \geqslant \operatorname{spint} ff^{-1}(B) = \operatorname{spint} B = B$. This implies that $f^{-1}f(\operatorname{spint} f^{-1}(B)) \geqslant f^{-1}(B)$. Since f is injective, we have $\operatorname{spint} f^{-1}(B) = f^{-1}f(\operatorname{spint} f^{-1}(B)) \geqslant f^{-1}(B)$. Thus, $\operatorname{spint} f^{-1}(B) = f^{-1}(B)$. Hence, f is fuzzy SP-irresolute continuous. ∎

THEOREM 3.3. *Let* $f\colon (X,\tau_1) \to (Y,\tau_2)$ *be a mapping from an fts* (X,τ_1) *into an fts* (Y,τ_2). *Then the following statements are equivalent:*

(i) f *is fuzzy SP-irresolute continuous;*

(ii) $\operatorname{cl}(\operatorname{pint} f^{-1}(B)) \leqslant f^{-1}(\operatorname{spcl} B)$, *for each fuzzy set* B *of* Y;

(iii) $f^{-1}(\operatorname{spint} B) \leqslant \operatorname{int}(\operatorname{pcl} f^{-1}(B))$, *for each fuzzy set* B *of* Y;

(iv) $f(\operatorname{cl}(\operatorname{pint} A)) \leqslant \operatorname{spcl} f(A)$, *for each fuzzy set* A *of* X.

Proof. (i) $\implies$ (ii). Let B be a fuzzy set of Y. According to the assumption $f^{-1}(\operatorname{spcl} B) \in FSPC(\tau_1)$. Hence, $f^{-1}(\operatorname{spcl} B) \geqslant \operatorname{cl}(\operatorname{pint} f^{-1}(\operatorname{spcl} B)) \geqslant \operatorname{cl}(\operatorname{pint} f^{-1}(B))$.

(ii) $\implies$ (iii) can be proved by using Lemma 2.2 and Theorem 4.1 [14].

(iii) $\implies$ (iv). Let A be a fuzzy set of X. Let us put $B = f(A)$, then $A \leqslant f^{-1}(B)$. According to the assumption, $(\operatorname{int}(\operatorname{pcl} A^c))^c \leqslant (\operatorname{int}(\operatorname{pcl} f^{-1}(B^c)))^c \leqslant (f^{-1}(\operatorname{spint} B^c))^c$. Thus, $\operatorname{cl}(\operatorname{pint} A) \leqslant \operatorname{cl}(\operatorname{pint} f^{-1}(B)) \leqslant f^{-1}(\operatorname{spcl} B)$. Hence, $f(\operatorname{cl}(\operatorname{pint} A)) \leqslant ff^{-1}(\operatorname{spcl} B) \leqslant \operatorname{spcl} B = \operatorname{spcl} f(A)$.

(iv) $\implies$ (i). Let $B \in FSPC(\tau_2)$. According to the assumption, $f(\operatorname{cl}(\operatorname{pint} f^{-1}(B))) \leqslant \operatorname{spcl} ff^{-1}(B) \leqslant \operatorname{spcl} B = B$. Then $\operatorname{cl}(\operatorname{pint} f^{-1}(B)) \leqslant f^{-1}f(\operatorname{cl}(\operatorname{pint} f^{-1}(B))) \leqslant f^{-1}(B)$. Thus $f^{-1}(B) \in FSPC(\tau_1)$, hence f is fuzzy SP-irresolute continuous. ∎

THEOREM 3.4. *Let $f\colon (X,\tau_1)\to(Y,\tau_2)$ be a mapping from an fts (X,τ_1) into an fts (Y,τ_2). If f is SP-irresolute continuous, then $f^{-1}(B)\leqslant \operatorname{spint} f^{-1}(\operatorname{int}(\operatorname{pcl} B))$, for each $B\in FSPO(\tau_2)$.*

Proof. Let $B\in FSPO(\tau_2)$. Then, $f^{-1}(B)\leqslant f^{-1}(\operatorname{int}(\operatorname{pcl} B))$. Since $f^{-1}(B)\in FSPO(\tau_1)$, we have $f^{-1}(B)\leqslant \operatorname{spint} f^{-1}(\operatorname{int}(\operatorname{pcl} B))$. ∎

THEOREM 3.5. *Let $f\colon (X,\tau_1)\to(Y,\tau_2)$ be a mapping from an fts (X,τ_1) into an fts (Y,τ_2). Then the following statements are equivalent:*

(i) f is fuzzy SP-irresolute continuous;

(ii) for any fuzzy point x_α of X and any $B\in FSPO(\tau_2)$ containing $f(x_\alpha)$, there exists $A\in FSPO(\tau_1)$ containing x_α such that $A\leqslant f^{-1}(B)$;

(iii) for any fuzzy point x_α of X and any $B\in FSPO(\tau_2)$ containing $f(x_\alpha)$, there exists $A\in FSPO(\tau_1)$ containing x_α such that $f(A)\leqslant B$.

Proof. (i) $\Longrightarrow$ (ii). Let f be fuzzy SP-irresolute continuous. Let x_α be a fuzzy point of X and let $B\in FSPO(\tau_2)$ containing $f(x_\alpha)$. Then $x_\alpha\in f^{-1}(B)=\operatorname{spint} f^{-1}(B)$. The result follows for $A=\operatorname{spint} f^{-1}(B)$.

(ii) $\Longrightarrow$ (iii). It follows from the relation $f(A)\leqslant ff^{-1}(B)\leqslant B$.

(iii) $\Longrightarrow$ (i). Let $B\in FSPO(\tau_2)$ and let x_α be a fuzzy point of X such that $x_\alpha\in f^{-1}(B)$. Then $f(x_\alpha)\in B$. According to the assumption, there exists $A\in FSPO(\tau_1)$ containing x_α such that $f(A)\leqslant B$. Then $x_\alpha\in A\leqslant f^{-1}f(A)\leqslant f^{-1}(B)$ and $x_\alpha\in A=\operatorname{spint} A\leqslant \operatorname{spint} f^{-1}(B)$. Since x_α is an arbitrary fuzzy point and $f^{-1}(B)$ is the union of all fuzzy points which belong in $f^{-1}(B)$, it sollows that $f^{-1}(B)\leqslant \operatorname{spint} f^{-1}(B)$. Hence, f is fuzzy SP-irresolute continuous. ∎

THEOREM 3.6. *Let $f\colon (X,\tau_1)\to(Y,\tau_2)$ be a bijective mapping from an fts (X,τ_1) into an fts (Y,τ_2). The mapping f is fuzzy SP-irresolute continuous if and only if for any fuzzy point x_α of X and any $B\in FSPO(\tau_2)$ containing $f(x_\alpha)$,* $\operatorname{pcl} f^{-1}(B)$ *is a fuzzy neighbourhood of x_α.*

Proof. Let f be fuzzy SP-irresolute continuous. Let x_α be a fuzzy point of X and $B\in FSPO(\tau_2)$ containing $f(x_\alpha)$. Then $x_\alpha\in f^{-1}(B)\leqslant \operatorname{int}(\operatorname{pcl} f^{-1}(B))\leqslant \operatorname{pcl} f^{-1}(B)$. Hence $\operatorname{pcl} f^{-1}(B)$ is a fuzzy neighbourhood of x_α.

Conversely, let $B\in FSPO(\tau_2)$ and let $x_\alpha\in f^{-1}(B)$. According to the assumption, $\operatorname{pcl} f^{-1}(B)$ is a fuzzy neighbourhood of x_α. Thus, $x_\alpha\in \operatorname{int}(\operatorname{pcl} f^{-1}(B))$. Hence, $f^{-1}(B)\leqslant \operatorname{int}(\operatorname{pcl} f^{-1}(B))$, i.e. f is fuzzy SP-irresolute continuous. ∎

THEOREM 3.7. *Every fuzzy SP-irresolute continuous mapping is a fuzzy weakly continuous mapping.*

Proof. Let $f\colon (X,\tau_1)\to(Y,\tau_2)$ be fuzzy SP-irresolute continuous and $B\in FO(\tau_2)$. Then, from Lemma 2.4, Lemma 2.6 and Theorem 3.1 it follows that $f^{-1}(B)=\operatorname{spint} f^{-1}(B)\leqslant \operatorname{spint}(\operatorname{cl} f^{-1}(B))=\operatorname{int}(\operatorname{cl} f^{-1}(B))=\operatorname{int}(\operatorname{spcl} f^{-1}(B))\leqslant \operatorname{int} f^{-1}(\operatorname{spcl} B)\leqslant \operatorname{int} f^{-1}(\operatorname{cl} B)$. Hence, f is fuzzy weakly continuous. ∎

Theorem 3.8. *A mapping $f\colon (X,\tau_1) \to (Y,\tau_2)$ from an fts (X,τ_1) into an fts (Y,τ_2) is fuzzy strongly irresolute continuous if it is fuzzy irresolute continuous and fuzzy SP-irresolute continuous.*

Proof. Let $B \in FSSO(\tau_2)$. From Lemma 2.7 it follows that $B \in FSPO(\tau_2)$ and $B \in FSO(\tau_2)$. Then $f^{-1}(B) \in FSPO(\tau_1)$ and $f^{-1}(B) \in FSO(\tau_1)$. Thus, $f^{-1}(B) \in FSPO(\tau_1) \cap FSO(\tau_1)$. Again, from Lemma 2.7 it follows that $f^{-1}(B) \in FSSO(\tau_1)$. ■

Theorem 3.9. *Let $f\colon (X,\tau_1) \to (Y,\tau_2)$ be a mapping from an fts (X,τ_1) into an fts (Y,τ_2). If f is fuzzy SP-irresolute continuous and fuzzy irresolute continuous, then f is fuzzy R-continuous.*

Prrof. Let $B \in FRO(\tau_2)$. From Lemma 2.7 it follows that $B \in FSPO(\tau_2)$ and $B \in FSC(\tau_2)$. According to the assumption, $f^{-1}(B) \in FSPO(\tau_1)$ and $f^{-1}(B) \in FSC(\tau_1)$. Again, from Lemma 2.7 it follows that $f^{-1}(B) \in FRO(\tau_1)$. ■

Theorem 3.10. *Let $f\colon X \to Y$ and $g \in Y \to Z$ be mappings, where X, Y and Z are fts's.*

(1) If f and g are fuzzy SP-irresolute continuous, then gf is fuzzy SP-irresolute continuous as well.

(2) If f is fuzzy SP-irresolute continuous and g is fuzzy strong precontinuous, then gf is fuzzy strong precontinuous.

(3) If f is fuzzy SP-irresolute continuous and g is fuzzy continuous, then gf is fuzzy strong precontinuous.

(4) If gf is fuzzy SP-irresolute continuous and g is fuzzy strongly preopen (preclosed) and injective, then f is fuzzy strong precontinuous.

(5) If gf is fuzzy strongly preopen (preclosed) and g is fuzzy SP-irresolute continuous and injective, then f is fuzzy strongly preopen (preclosed).

Proof. The results follow from the relations $(gf)^{-1}(C) = f^{-1}(g^{-1}(C))$, for any fuzzy set C of Z, $f^{-1}(B) = (gf)^{-1}g(B)$, for any fuzzy set B of Y, when g is injective and $f(A) = g^{-1}(gf)(A)$, for any fuzzy set A of X, when g is injective. ■

Corollary 3.11. *Let X, X_1 and X_2 be fts's and $p_i\colon X_1 \times X_2 \to X_i$ $(i = 1,2)$ be the projection of $X_1 \times X_2$ onto X_i. If $f\colon X \to X_1 \times X_2$ is fuzzy SP-irresolute continuous, then $p_i f$ is fuzzy strong precontinuous.*

Proof. The projections p_i, $i = 1,2$, are fuzzy continuous mappings. ■

4. Fuzzy SP-irresolute open and SP-irresolute closed mappings

Definition 4.1. Let $f\colon (X,\tau_1) \to (Y,\tau_2)$ be a mapping from an *fts* (X,τ_1) into an *fts* (Y,τ_2). The mapping f is called SP-irresolute open (closed) if $f(A) \in FSPO(\tau_2)$ $(f(A) \in FSPC(\tau_2))$, for each $A \in FSPO(\tau_1)$ $(A \in FSPC(\tau_1))$.

Remark 4.1. The concept of a fuzzy SP-irresolute open (closed) mapping is strictly stronger then the concept of a fuzzy strongly preopen (preclosed) mapping.

The concepts of a fuzzy SP-irresolute open (closed) mapping and a fuzzy open (closed) mapping are independent.

Example 4.1. We consider the *fts*'s (X, τ_1) and (X, τ_2) from Example 3.1. The mapping $f = id\colon (X, \tau_2) \to (X, \tau_1)$ is fuzzy strongly preopen (preclosed) but it is not fuzzy SP-irresolute open (closed). Furthermore, f is a fuzzy open (closed) mapping that is not fuzzy SP-irresolute open (closed). Let (X, τ_1) and (X, τ_2) be the *fts*'s from Example 3.2. Then the mapping $f = id\colon (X, \tau_2) \to (X, \tau_1)$ is fuzzy SP-irresolute open (closed) but it is not fuzzy open (closed).

Theorem 4.1. *Let $f\colon (X, \tau_1) \to (Y, \tau_2)$ be a mapping from an fts (X, τ_1) into an fts (Y, τ_2). Then the following statements are equivalent:*

(i) f is fuzzy SP-irresolute open;

(ii) $f(\operatorname{spint} A) \leqslant \operatorname{spint} f(A)$, for each fuzzy set A of X;

(iii) $\operatorname{spint} f^{-1}(B) \leqslant f^{-1}(\operatorname{spint} B)$, for each fuzzy set B of Y;

(iv) $f^{-1}(\operatorname{spcl} B) \leqslant \operatorname{spcl} f^{-1}(B)$, for each fuzzy set B of Y.

Proof. (i) $\Longrightarrow$ (ii). Let A be a fuzzy set of X. Then $f(\operatorname{spint} A) = \operatorname{spint} f(\operatorname{spint} A) \leqslant \operatorname{spint} f(A)$.

(ii) $\Longrightarrow$ (iii). Let B be a fuzzy set of Y. According to the assumption, $f(\operatorname{spint} f^{-1}(B)) \leqslant \operatorname{spint} f f^{-1}(B) \leqslant \operatorname{spint} B$. Hence $\operatorname{spint} f^{-1}(B) \leqslant f^{-1} f(\operatorname{spint}(f^{-1}(B))) \leqslant f^{-1}(\operatorname{spint} B)$.

(iii) $\Longrightarrow$ (iv) can be proved by using Lemma 2.2 and Theorem 4.1 [14].

(iv) $\Longrightarrow$ (i). Let $A \in FSPO(\tau_1)$. Then $A = \operatorname{spint} A$. According to the assumption, $A = \operatorname{spint} A \leqslant \operatorname{pint} f^{-1} f(A) = (\operatorname{spcl} f^{-1}(f(A)^c))^c \leqslant f^{-1}(\operatorname{spcl} f(A)^c)^c = f^{-1}(\operatorname{spint} f(A))$, hence $f(A) \leqslant f f^{-1}(\operatorname{spint} f(A)) \leqslant \operatorname{spint} f(A)$. Thus $f(A) = \operatorname{spint} f(A)$, i.e. f is fuzzy SP-irresolute open. ∎

Theorem 4.2. *Let $f\colon (X, \tau_1) \to (Y, \tau_2)$ be a mapping from an fts (X, τ_1) into an fts (Y, τ_2). The mapping f is fuzzy SP-irresolute closed if and only if $\operatorname{spcl} f(A) \leqslant f(\operatorname{spcl} A)$, for each fuzzy set A of X.*

Proof. Let f be fuzzy SP-irresolute closed and let A be any fuzzy set of X. Then $f(\operatorname{spcl} A) \in FSPC(\tau_2)$. From $f(A) \leqslant f(\operatorname{spcl} A)$ it follows that $\operatorname{spcl} f(A) \leqslant f(\operatorname{spcl} A)$.

Conversely, let $A \in FSPC(\tau_1)$. From $f(A) = f(\operatorname{spcl} A) \geqslant \operatorname{spcl} f(A)$, we obtain $\operatorname{spcl} f(A) = f(A)$, hence f is fuzzy SP-irresolute closed. ∎

Theorem 4.3. *Let $f\colon (X, \tau_1) \to (Y, \tau_2)$ be a mapping from an fts (X, τ_1) into an fts (Y, τ_2).*

1) The mapping f is fuzzy SP-irresolute open if and only if $f(\operatorname{spint} A) \leqslant \operatorname{int}(\operatorname{pcl} f(A))$, for each fuzzy set A of X.

2) The mapping f is fuzzy SP-irresolute closed if and only if $\operatorname{cl}(\operatorname{pint} f(A)) \leqslant f(\operatorname{spcl} A)$, for each fuzzy set A of X.

Proof. We prove only the statement 1). Let A be a fuzzy set of X. Then $f(\operatorname{spint} A) \in FSPO(\tau_2)$, hence $f(\operatorname{spint} A) \leqslant \operatorname{int}(\operatorname{pcl} f(\operatorname{spint} A)) \leqslant \operatorname{int}(\operatorname{pcl} f(A))$.

Conversely, let $A \in FSPO(\tau_1)$, From $f(A) = f(\operatorname{spint} A) \leqslant \operatorname{int}(\operatorname{pcl} f(A))$ it follows that $f(A) \in FSPO(\tau_2)$, hence f is fuzzy SP-irresolute open. ■

THEOREM 4.4. *Let $f\colon (X,\tau_1) \to (Y,\tau_2)$ be a mapping from an fts (X,τ_1) into an fts (Y,τ_2). Then the following statements hold:*

1) if $f(\operatorname{int}(\operatorname{pcl} A)) \leqslant \operatorname{int}(\operatorname{pcl} f(A))$, for each $A \in FSPO(\tau_1)$, then f is fuzzy SP-irresolute open;

2) if $f(\operatorname{cl}(\operatorname{pint} A)) \geqslant \operatorname{cl}(\operatorname{pint} f(A))$, for each $A \in FSPC(\tau_1)$, then f is fuzzy SP-irresolute closed.

Proof. We prove only the statement 1). Let $A \in FSPO(\tau_1)$. Then $A \leqslant \operatorname{int}(\operatorname{pcl} A)$. According to the assumption, $f(A) \leqslant f(\operatorname{int}(\operatorname{pcl} A)) \leqslant \operatorname{int}(\operatorname{pcl} f(A))$, hence $f(A) \in FSPO(\tau_2)$, i.e. f is SP-irresolute open. ■

THEOREM 4.5. *Let $f\colon X \to Y$ be a bijective mapping from an fts X into an fts Y. Then the following statements hold:*

1) f is fuzzy SP-irresolute open if and only if it is fuzzy SP-irresolute closed;

2) f is fuzzy SP-irresolute open (closed) if and only if f^{-1} is fuzzy SP-irresolute continuous.

Proof. 1) can be proved by using Lemma 4.1 [14].

2) It follows from the relation $(f^{-1})^{-1}(A) = f(A)$, for each fuzzy set A of X. ■

COROLLARY 4.6. *Let $f\colon X \to Y$ be a bijective mapping from an fts X into an fts Y. Then the following statements are equivalent:*

(i) f is fuzzy SP-irresolute closed;

(ii) $f(\operatorname{spint} A) \leqslant \operatorname{spint} f(A)$, for each fuzzy set A of X;

(iii) $\operatorname{spint} f^{-1}(B) \leqslant f^{-1}(\operatorname{spint} B)$, for each fuzzy set B of Y;

(iv) $f^{-1}(\operatorname{spcl} B) \leqslant \operatorname{spcl} f^{-1}(B)$, for each fuzzy set B of Y. ■

COROLLARY 4.7. *Let $f\colon X \to Y$ be a bijective mapping from an fts X into an fts Y. The mapping f is fuzzy SP-irresolute open if and only if $\operatorname{spcl} f(A) \leqslant f(\operatorname{spcl} A)$, for each fuzzy set A of X.* ■

THEOREM 4.8. *Let $f\colon (X,\tau_1) \to (Y,\tau_2)$ be a mapping from an fts (X,τ_1) into an fts (Y,τ_2). The mapping f is fuzzy SP-irresolute open if and only if for each fuzzy set B of Y and each $A \in FSPC(\tau_1)$, $f^{-1}(B) \leqslant A$, there exists $C \in FSPC(\tau_2)$ such that $B \leqslant C$ and $f^{-1}(C) \leqslant A$.*

Proof. Let B be a fuzzy set of Y and let $A \in FSPC(\tau_1)$ such that $f^{-1}(B) \leqslant A$. Then $A^c \leqslant f^{-1}(B^c)$, hence $f(A^c) \leqslant ff^{-1}(B^c) \leqslant B^c$. Since $A^c \in FSPO(\tau_1)$, we obtain that $f(A^c) \in FSPO(\tau_2)$, hence $f(A^c) \leqslant \operatorname{spint} B^c$. Thus $A^c \leqslant f^{-1}f(A^c) \leqslant f^{-1}(\operatorname{spint} B^c)$. It follows that $A \geqslant f^{-1}(\operatorname{spint} B^c)^c = f^{-1}(\operatorname{spcl} B)$. The result follows for $C = \operatorname{spcl} B$.

Conversely, let $U \in FSPO(\tau_1)$. We claim that $f(U) \in FSPO(\tau_2)$. From $U \leqslant f^{-1}f(U)$ it follows that $U^c \geqslant f^{-1}f(U)^c$, where $U^c \in FSPC(\tau_1)$. Hence there is $B \in FSPC(\tau_2)$ such that $B \geqslant f(U)^c$ and $f^{-1}(B) \leqslant U^c$. Since $B \geqslant f(U)^c$, it follows that $B \geqslant \operatorname{spcl} f(U)^c$ or $B^c \leqslant (\operatorname{spcl} f(U)^c)^c = \operatorname{spint} f(U)$. From $f^{-1}(B) \leqslant U^c$ we obtain $f^{-1}(B^c) \geqslant U$ or $B^c \geqslant ff^{-1}(B^c) \geqslant f(U)$. Since $f(U) \leqslant B^c \leqslant \operatorname{spint} f(U)$, we have $f(U) = \operatorname{spint} f(U)$. Thus, $f(U) \in FSPO(\tau_2)$, hence f is fuzzy SP-irresolute open. ∎

COROLLARY 4.9. *Let $f\colon (X,\tau_1) \to (Y,\tau_2)$ be a mapping from an fts (X,τ_1) into an fts (Y,τ_2). If f is fuzzy SP-irresolute open then:*

1) $f^{-1}(\operatorname{cl}(\operatorname{pint} B)) \leqslant \operatorname{spcl} f^{-1}(B)$, for each fuzzy set B of Y;

2) $f^{-1}(\operatorname{cl} B) \leqslant \operatorname{spcl} f^{-1}(B)$, for each $B \in FPO(\tau_2)$.

Proof. 1) Let B be a fuzzy set of Y. Then $\operatorname{spcl} f^{-1}(B) \in FSPC(\tau_1)$. From Theorem 4.8 it follows that there exists $C \in FSPC(\tau_2)$ such that $B \leqslant C$ and $f^{-1}(C) \leqslant \operatorname{spcl} f^{-1}(B)$. Thus $f^{-1}(\operatorname{cl}(\operatorname{pint} B)) \leqslant f^{-1}(\operatorname{cl}(\operatorname{pint} C)) \leqslant f^{-1}(C) \leqslant \operatorname{spcl} f^{-1}(B)$.

2) It follows immediately from 1). ∎

THEOREM 4.10. *Let $f\colon (X,\tau_1) \to (Y,\tau_2)$ be a mapping from an fts (X,τ_1) into an fts (Y,τ_2). The mapping f is fuzzy SP-irresolute closed if and only if for each fuzzy set B of Y and each $A \in FSPO(\tau_1)$, $f^{-1}(B) \leqslant A$, there exists $C \in FSPO(\tau_2)$ such that $B \leqslant C$ and $f^{-1}(C) \leqslant A$.*

Proof. Let B be a fuzzy set of Y and let $A \in FSPO(\tau_1)$ such that $f^{-1}(B) \leqslant A$. Then $f(A^c) \in FSPC(\tau_2)$. We put $C = f(A^c)^c$. Then $C \in FSPO(\tau_2)$, $B \leqslant C$ and $f^{-1}(C) = f^{-1}(f(A^c)^c) \leqslant f^{-1}f(A) \leqslant A$.

Conversely, let $A \in FSPC(\tau_1)$. Then $A^c \in FSPO(\tau_1)$ and $A^c \geqslant f^{-1}(f(A)^c)$. According to the assumption there exists $C \in FSPO(\tau_2)$ such that $f(A)^c \leqslant C$ and $f^{-1}(C) \leqslant A^c$. Hence, $f(A) = C^c \in FSPC(\tau_2)$. ∎

THEOREM 4.11. *Let $f\colon X \to Y$ be a mapping from an fts X into an fts Y. The mapping f is fuzzy SP-irresolute closed and fuzzy SP-irresolute continuous if and only if $f(\operatorname{spcl} A) = \operatorname{spcl} f(A)$, for each fuzzy set A of X.*

Proof. It follows from Theorem 3.1 and Theorem 4.2. ∎

THEOREM 4.12. *Let $f\colon X \to Y$ be a mapping from an fts X into an fts Y. The mapping f is fuzzy SP-irresolute open and fuzzy SP-irresolute continuous if and only if $f^{-1}(\operatorname{spcl} B) = \operatorname{spcl} f^{-1}(B)$, for each fuzzy set B of Y.*

Proof. It follows from Theorem 3.1 and Theorem 4.1. ∎

THEOREM 4.13. *Let $f\colon X \to Y$ be a mapping from an fts X into an fts Y. The mapping f is fuzzy SP-irresolute open and fuzzy SP-irresolute continuous if and only if $f^{-1}(\operatorname{spint} B) = \operatorname{spint} f^{-1}(B)$, for each fuzzy set B of Y.*

Proof. It follows from Theorem 3.1 and Theorem 4.1. ∎

THEOREM 4.14. *Let $f\colon X \to Y$ be a mapping from an fts X into an fts Y. If f is fuzzy SP-irresolute open (closed) and fuzzy irresolute open (closed), then it is fuzzy strongly irresolute open (closed).*

Proof. It is similar to the proof of Theorem 3.8. ■

THEOREM 4.15. *Let $f\colon X \to Y$ be a mapping from an fts X into an fts Y. If f is fuzzy SP-irresolute open (closed) and fuzzy irresolute closed (open), then it is fuzzy R-open (R-closed).*

Proof. It is similar to the proof of Theorem 3.9. ■

COROLLARY 4.16. *Let $f\colon X \to Y$ be a mapping from a semiregular fts X into an fts Y. If f is fuzzy SP-irresolute open (closed) and fuzzy irresolute closed (open), then it is fuzzy open (closed).*

THEOREM 4.17. *Let $f\colon X \to Y$ and $g\colon Y \to Z$ be mappings, where X, Y and Z are fts's. Then the following statements are true:*

1) If f and g are fuzzy SP-irresolute open (closed), then gf is fuzzy SP-irresolute open (closed) as well.

2) If f is fuzzy strongly preopen (preclosed) and g is fuzzy SP-irresolute open (closed), then gf is fuzzy strongly preopen (preclosed).

3) If gf is fuzzy SP-irresolute continuous and g is fuzzy SP-irresolute open (closed) and injective, then f is fuzzy SP-irresolute continuous.

4) If gf is fuzzy SP-irresolute continuous and g is fuzzy strongly preopen (preclosed) and injective, then f is fuzzy strong precontinuous.

5) If gf is fuzzy SP-irresolute open (closed) and g is fuzzy SP-irresolute continuous and injective, then f is fuzzy SP-irresolute open (closed).

6) If gf is fuzzy strongly preopen (preclosed) and g is fuzzy SP-irresolute continuous and injective, then f is fuzzy strongly preopen (preclosed).

7) If gf is fuzzy SP-irresolute continuous and f is SP-irresolute open (closed) and surjective, then g is fuzzy SP-irresolute continuous.

8) If gf is fuzzy strong precontinuous and f is fuzzy SP-irresolute open (closed) and surjective, then g is fuzzy strong precontinuous.

9) If gf is fuzzy SP-irresolute open (closed) and f is fuzzy SP-irresolute continuous and surjective, then g is fuzzy SP-irresolute open (closed).

10) If gf is fuzzy SP-irresolute open (closed) and f is strong precontinuous and surjective, then g is fuzzy strongly preopen (preclosed).

Proof. It follows from the relations $(gf)(A) = g(f(A))$, for each fuzzy set A of X, $f(A) = g^{-1}(gf)(A)$, for each fuzzy set A of X, when g is injective, $f^{-1}(B) = (gf)^{-1}g(B)$, for each fuzzy set B of Y, when g is injective, $g(B) = (gf)f^{-1}(B)$, for each fuzzy set B of Y, when f is surjective and $g^{-1}(C) = f(gf)^{-1}(C)$, for each fuzzy set C of Z, when f is surjective. ■

5. Fuzzy SP-homeomorphism

Definition 5.1. A bijective mapping $f\colon X \to Y$ from an *fts* X into an *fts* Y is called a fuzzy SP-homeomorphism if both f and f^{-1} are fuzzy SP-irresolute continuous.

Theorem 5.1. *Let $f\colon X \to Y$ be a bijective mapping from an fts X into an fts Y. The following statements are equivalent:*

(i) f is a fuzzy SP-homeomorphism;

(ii) f^{-1} is a fuzzy SP-homeomorphism;

(iii) f and f^{-1} are fuzzy SP-irresolute open (closed);

(iv) f is fuzzy SP-irresolute continuous and fuzzy SP-irresolute open (closed);

(v) $f(\operatorname{spcl} A) = \operatorname{spcl} f(A)$, *for each fuzzy set A of X;*

(vi) $f(\operatorname{spint} A) = \operatorname{spint} f(A)$, *for each fuzzy set A of X;*

(vii) $f^{-1}(\operatorname{spint} B) = \operatorname{spint} f^{-1}(B)$, *for each fuzzy set B of Y;*

(viii) $\operatorname{spcl} f^{-1}(B) = f^{-1}(\operatorname{spcl} B)$, *for each fuzzy set B of Y.*

Proof. (i) $\Longrightarrow$ (ii). It follows immediately from the definition of a SP-homeomorphism and the relation $(f^{-1})^{-1} = f$.

(ii) $\Longrightarrow$ (iii). It follows from Theorem 4.5.

(iii) $\Longrightarrow$ (iv). It follows from Theorem 4.5.

(iv) $\Longrightarrow$ (v). It follows from Theorem 4.5 and Theorem 4.11.

(v) $\Longrightarrow$ (vi) can be proved by using Lemma 2.2 and Theorem 4.1 [14].

(vi) $\Longrightarrow$ (vii). Let B be a fuzzy set of X. According to the assumption $f(\operatorname{spint} f^{-1}(B)) = \operatorname{spint} f f^{-1}(B) = \operatorname{spint} B$. Thus $f^{-1} f(\operatorname{spint} f^{-1}(B)) = f^{-1}(\operatorname{spint} B)$. Hence $\operatorname{spint} f^{-1}(B) = f^{-1}(\operatorname{spint} B)$.

(vii) $\Longrightarrow$ (viii) can be proved by using Lemma 2.2 and Theorem 4.1 [14].

(viii) $\Longrightarrow$ (i). It follows from Theorem 4.5 and Theorem 4.12. ∎

Definition 5.2. A property which is preserved under fuzzy SP-homeomorphisms is said to be a SP-topological property.

Theorem 5.2. *If $f\colon (X,\tau_1) \to (Y,\tau_2)$ is a fuzzy SP-homeomorphism from an fts (X,τ_1) into an fts (Y,τ_2), then it is R-continuous and R-open (closed).*

Proof. 1) Let $A \in FRO(\tau_1)$. Then $f(A) = f(\operatorname{int}(\operatorname{cl} A)) = f(\operatorname{spint}(\operatorname{cl} A)) = \operatorname{spint} f(\operatorname{cl} A) = \operatorname{spint} f(\operatorname{spcl} A) = \operatorname{spint}(\operatorname{spcl} f(A)) = \operatorname{spint}(\operatorname{cl} f(A)) = \operatorname{int}(\operatorname{cl} f(A))$.

2) can be proved in a similar manner as 1). ∎

Corollary 5.3. *Let $f\colon X \to Y$ be a mapping from an fts X into an fts Y. If f is a fuzzy SP-homeomorphism then $f\colon (X, S(X)) \to (Y, S(Y))$ is a fuzzy homeomorphism.*

Corollary 5.4. *Fuzzy semiregular properties are SP-topological.*

REMARK 5.1. If we refer to [9], as a consequence of Corollary 5.4, the following properties are fuzzy SP-topological: fuzzy Hausdorfness, fuzzy Urysohness, fuzzy connectedness, fuzzy disconectedness and fuzzy extremely disconnectedness. Furthermore, according to [9] the following properties are fuzzy SP-topological: fuzzy almost regularity, fuzzy nearly compactness, fuzzy nearly paracompactness, fuzzy locally nearly compactness, fuzzy nearly Lindelöfness, fuzzy locally nearly Lindelöfness, fuzzy almost locally connectedness.

THEOREM 5.5. *Fuzzy separability is a SP-topological property.*

Proof. Let $f\colon (X,\tau_1) \to (Y,\tau_2)$ be a fuzzy SP-homeomorphism from a fuzzy separable space X into an *fts* Y. According to the assumption there exists a countable sequence $\{p_i\}_{i\in\mathbf{N}}$ of fuzzy poinys of X such that for any fuzzy open set $A \neq 0_X$ there exists $p_i \in A$. Then $\{f(p_i)\}_{i\in\mathbf{N}}$ is a countable sequence of fuzzy points of Y. Let $B \neq 0_Y$ be a fuzzy open set. Since $f^{-1}(B) \neq 0_X$ and $f^{-1}(B) \in FSPO(\tau_1)$ from Lemma 2.5 we have $\operatorname{int} f^{-1}(B) \neq 0_X$. Then there exists a fuzzy point $p_i \in \operatorname{int} f^{-1}(B)$. Thus $f(p_i) \in f(\operatorname{int} f^{-1}(B)) \leqslant f(\operatorname{spint} f^{-1}(B)) = ff^{-1}(\operatorname{spint} B) = B$. Hence Y is fuzzy separable. ■

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