Classification of Maps by Their Membership on Maximal Clones that Contain Minimum and Complement

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Abstract

In this paper, five-valued logic functions are classified according to their membership in the maximal clones which contain $min(x, y)$ and $\overline{x} = 4 - x$.

1 Notation and Preliminaries

Denote by $\mathbf{N}$ the set $\{1, 2, \ldots\}$ of positive integers. For $k, n \in \mathbf{N}$, $E_k = \{0, 1, \ldots, k-1\}$, denote by $P_k^{(n)}$ the set of all maps $E_k^n \to E_k$, and $P_k = \bigcup_{n \in \mathbf{N}} P_k^{(n)}$. We say that f is an *i-th projection of arity* n $(1 \leq i \leq n)$ if $f \in P_k^{(n)}$ and f satisfies the identity $f(x_1, \ldots, x_n) \approx x_i$. We say that $f \in P_k^{(n)}$ is *essential* if it depends on at least two variables and it takes all values from E_k. Let π_i^n denote the i-th projection of arity n, and let Π_k denote the set of all projections over E_k. For $n, m \geq 1, f \in P_k^{(n)}$ and $g_1, \ldots, g_n \in P_k^{(m)}$, *the superposition of* f and $g_1, \ldots, g_n$, denoted by $f(g_1, \ldots, g_n)$, is defined by $f(g_1, \ldots, g_n)(a_1, \ldots, a_m) = f(g_1(a_1, \ldots, a_m), \ldots, g_n(a_1, \ldots, a_m))$ for all $(a_1, \ldots, a_m) \in E_k^m$. A set $F \subseteq P_k$ is *a clone of operations on* E_k (or *clone* for short) if $\Pi_k \subseteq F$ and F is closed with respect to superposition. For $F \subseteq P_k$, $\langle F \rangle_{\mathrm{CL}}$ stands for the clone generated by F. We say that clone F is *maximal* if there is no clone G such that $F \subset G \subset P_k$. $F \subseteq P_k$ is *complete* if $\langle F \rangle_{\mathrm{CL}} = P_k$.

Let $\varrho \subseteq E_k^h$ be an h-ary relation and $f \in P_k^{(n)}$. We say that f *preserves* ϱ if for all h-tuples $(a_{11}, \ldots, a_{1h}), \ldots, (a_{n1}, \ldots, a_{nh})$ from ϱ we have $(f(a_{11}, \ldots, a_{n1}), \ldots, f(a_{1h}, \ldots, a_{nh})) \in \varrho$. $\mathrm{Pol}\,\varrho$ is the set of all $f \in P_k$ which preserve ϱ. For $F \subseteq P_k$, $\mathrm{Inv}\,F$ denotes the set of all the relations preserved by each $f \in F$.

Let C be a clone on E_k and $F \subseteq P_k$. F is a *complete relative* to C (or *C-complete*) if $\langle F \cup C \rangle_{\mathrm{CL}} = P_k$.

A relative complete set F in P_k is called a *relative base of* P_k *with respect to* C if no proper subset of F is relative complete in P_k with respect to C.

The functions from P_k may be classified into nonempty equivalence classes by their membership in the relative maximal clones. Then we can discuss the completeness properties in P_k in terms of these classes instead of individual functions: if a set is complete then replacing a function in the set by any function in the corresponding equivalence class yields another complete set.

Let $M_1, ..., M_m$ be relative maximal clones. Define the map $\phi : P_k \to \{0, 1\}^m$ by setting $\phi(f) := a_1...a_m$ where $a_i = 0$ if $f \in M_i$ and $a_i = 1$ if $f \notin M_i$. We call $\phi(f)$ *the characteristic vector of* f. We put $f \varrho g$ if $f, g \in P_k$ have the same characteristic vector, i.e. if $\phi(f) = \phi(g)$. It means that for all $j \in \{1, 2, ..., n\}$, either $\{f, g\} \subset M_j$ or $\{f, g\} \cap M_j = \emptyset$. Clearly, ϱ is an equivalence relation on P_k and so it partitions P_k into pairwise disjoint nonempty sets called *(equivalence) classes.*

To $a_1, ..., a_m \in \{0, 1\}^m$ we associate $A = \{i : a_i = 1\}$ and if $A_1, ..., A_l$ are the subsets of $\{1, ..., m\}$ corresponding to the characteristic vectors, the relative completeness problem is reduced to the listing of subsets of $\{A_1, ..., A_l\}$ covering $\{1, ..., m\}$.

All maximal clones in P_k containing the functions min and complement and all maximal clones in $P_k, 2 < k < 9$ containing two special unary operations are determined in [3].

2 Results and discussion

Using the statements of paper [3] it follows:

Theorem 2.1 *There are 11 relative maximal clones in* P_5 *:*

$$Q_1 = Pol\binom{012340134}{012341043} = Pol(E_5^2 - P_{02} - P_{03} - P_{04} - P_{12} - P_{13} - P_{14} - P_{23} - P_{24})$$
$$Q_2 = Pol\binom{01234121323}{01234213132} = Pol(E_5^2 - P_{01} - P_{02} - P_{03} - P_{04} - P_{14} - P_{24} - P_{34})$$
$$Q_3 = Pol(0134)$$
$$Q_4 = Pol(04)$$
$$Q_5 = Pol(123)$$
$$Q_6 = Pol(13)$$
$$Q_7 = Pol(024)$$
$$Q_8 = Pol(2)$$
$$Q_9 = Pol\binom{0123401020312131423243 4}{01234102030213141324243} = Pol(E_5^2 - P_{04})$$
$$Q_{10} = Pol\binom{0123401021213232434}{0123410202131324243} = Pol(E_5^2 - P_{04} - P_{03} - P_{14})$$
$$Q_{11} = Pol\binom{01234010212232434}{01234102021324243} = Pol(E_5^2 - P_{04} - P_{03} - P_{13} - P_{14})$$

From theorems 1-14 [4] it follows respectively:

Theorem 2.2

1. $Q_5Q_7 \subset Q_8 \;\wedge\; Q_3Q_5 \subset Q_6 \;\wedge\; Q_3Q_7 \subset Q_4$.

2. $\overline{Q}_9Q_{10} \subset Q_8 \;\wedge\; \overline{Q}_9Q_{11} \subset Q_8 \;\wedge\; \overline{Q}_{10}Q_{11} \subset Q_8$.

3. $Q_2Q_6 \subset Q_5 \;\wedge\; Q_2Q_8 \subset Q_5 \;\wedge\; Q_1Q_4 \subset Q_3 \;\wedge\; Q_1Q_6 \subset Q_3$.

4. $Q_1Q_7Q_9 \subset Q_{10} \;\wedge\; Q_1Q_7Q_{10} \subset Q_{11}$.

5. $Q_7\overline{Q}_8\overline{Q}_{10} \subset \overline{Q}_{11} \;\wedge\; Q_7\overline{Q}_8\overline{Q}_{11} \subset \overline{Q}_{10} \;\wedge\; Q_7\overline{Q}_8\overline{Q}_9 \subset \overline{Q}_{10} \;\wedge\; Q_7\overline{Q}_8\overline{Q}_9 \subset \overline{Q}_{11}$.

6. $Q_2\overline{Q}_5\overline{Q}_{11} \subset \overline{Q}_{10} \;\wedge\; Q_2\overline{Q}_5\overline{Q}_9 \subset \overline{Q}_{10} \;\wedge\; Q_2\overline{Q}_5\overline{Q}_9 \subset \overline{Q}_{11}$.

7. $Q_4Q_9\overline{Q}_{10} \subset \overline{Q}_{11}$

8. $Q_3Q_{10} \subset Q_9 \;\wedge\; Q_6Q_{10} \subset Q_9 \;\wedge\; Q_4Q_{10} \subset Q_{11}$.

9. $Q_1Q_2 \subset Q_9$.

10. $Q_2Q_{10} \subset Q_9 \;\wedge\; Q_2Q_{11} \subset Q_9$.

11. $Q_2Q_{11} \subset Q_{10}$.

12. $Q_1Q_2\overline{Q}_5 \subset Q_9 \;\wedge\; Q_1Q_2\overline{Q}_5 \subset Q_{10} \;\wedge\; Q_1Q_2\overline{Q}_5 \subset Q_{11}$.

13. $Q_1\overline{Q}_5\overline{Q}_9 \subset \overline{Q}_{10} \;\wedge\; Q_1\overline{Q}_5\overline{Q}_{11} \subset \overline{Q}_{10}$.

14. $Q_1Q_2\overline{Q}_5 \subset Q_3$.

Remarks It is important to notice that theorem 2.2. is useful for classification of functions in P_5 (according to their membership in all maximal clones), which is still an open problem.

Theorem 2.3 *There are 391 classes of functions in* P_5 *with respect to the clone generated by* $min(x,y)$ *and* $\overline{x} = 4 - x$.

Proof In P_5 we have 2048 possible classes of functions. According to the previous theorem we have found 1657 empty classes of functions. Representatives are efectively constructed for other 391 classes of functions. They are given in the following tables.

3	00000010011	#114	70	01011010111	#159	137	10100000000	#201	204	11001000110	#248
4	00000011000	#62	71	01011011000	#54	138	10100000011	#202	205	11001000111	#249
5	00000011011	#71	72	01011011011	#60	139	10100000111	#203	206	11001001000	#250
6	00001101000	#1	73	01011011111	#85	140	10100010000	#204	207	11001001011	#58
7	00001111000	#115	74	01011110000	#33	141	10100010011	#205	208	11001001111	#57
8	00010010000	#73	75	01011110010	#44	142	10100010111	#206	209	11001010000	#251
9	00010010001	#116	76	01011110011	#160	143	10100011000	#207	210	11001010011	#252
10	00010010011	#117	77	01011110110	#48	144	10100011011	#208	211	11001010110	#253
11	00010011000	#63	78	01011110111	#161	145	10100011111	#209	212	11001010111	#254
12	00010011001	#108	79	01011111000	#7	146	10100100000	#77	213	11001011000	#255
13	00010011011	#80	80	01011111011	#18	147	10100100011	#210	214	11001011011	#256
14	00011111000	#2	81	01011111111	#22	148	10100100111	#211	215	11001011111	#257
15	00110100000	#79	82	01110100000	#162	149	10100110000	#212	216	11001100000	#258
16	00110110000	#110	83	01110100100	#163	150	10100110011	#213	217	11001100011	#42
17	00110110001	#118	84	01110100110	#164	151	10100110111	#214	218	11001100110	#259
18	00110110011	#119	85	01110100111	#165	152	10100111000	#67	219	11001100111	#36
19	00110111000	#69	86	01110110000	#166	153	10100111011	#70	220	11001101000	#260
20	00110111001	#111	87	01110110001	#167	154	10100111111	#215	221	11001101011	#16
21	00110111011	#84	88	01110110010	#168	155	10101101000	#216	222	11001101111	#10
22	01000000000	#120	89	01110110011	#169	156	10101101011	#217	223	11001110000	#261
23	01000000110	#121	90	01110110100	#170	157	10101101111	#218	224	11001110011	#262
24	01000000111	#122	91	01110110101	#171	158	10101111000	#219	225	11001110110	#263
25	01000010000	#123	92	01110110110	#172	159	10101111011	#220	226	11001110111	#264
26	01000010011	#124	93	01110110111	#173	160	10101111111	#221	227	11001111000	#265
27	01000010110	#124	94	01110111000	#174	161	10110000000	#74	228	11001111011	#29
28	01000010111	#126	95	01110111001	#175	162	10110000001	#222	229	11001111111	#24
29	01000011000	#127	96	01110111011	#176	163	10110000011	#96	230	11010010000	#266
30	01000011011	#128	97	01110111111	#177	164	10110000111	#223	231	11010010001	#267
31	01000011111	#129	98	01111100000	#40	165	10110010000	#109	232	11010010010	#268
32	01001000000	#130	99	01111100110	#178	166	10110010001	#224	233	11010010011	#269
33	01001000110	#131	100	01111100111	#179	167	10110010011	#225	234	11010010110	#270
34	01001000111	#132	101	01111101000	#13	168	10110010111	#226	235	11010010111	#271
35	01001001000	#53	102	01111101111	#52	169	10110011000	#64	236	11010011000	#272
36	01001001111	#61	103	01111110000	#102	170	10110011001	#107	237	11010011001	#273
37	01001010000	#133	104	01111110010	#180	171	10110011011	#81	238	11010011011	#274
38	01001010011	#134	105	01111110011	#181	172	10110011111	#227	239	11010011111	#275
39	01001010110	#135	106	01111110110	#182	173	10110100000	#91	240	11011010000	#276
40	01001010111	#136	107	01111110111	#183	174	10110100001	#228	241	11011010001	#277
41	01001011000	#137	108	01111111000	#27	175	10110100011	#98	242	11011010010	#278
42	01001011011	#138	109	01111111011	#51	176	10110100111	#229	243	11011010011	#279
43	01001011111	#139	110	01111111111	#103	177	10110110000	#78	244	11011010110	#280
44	01001100000	#32	111	10000000000	#184	178	10110110001	#230	245	11011010111	#281
45	01001100110	#45	112	10000000011	#76	179	10110110011	#97	246	11011011000	#282
46	01001100111	#140	113	10000000111	#185	180	10110110111	#231	247	11011011001	#283
47	01001101000	#6	114	10000010000	#186	181	10110111000	#68	248	11011011011	#56
48	01001101111	#19	115	10000010011	#187	182	10110111001	#106	249	11011011111	#87
49	01001110000	#141	116	10000010111	#188	183	10110111011	#83	250	11011110000	#284
50	01001110011	#142	117	10000011000	#189	184	10110111111	#232	251	11011110001	#37
51	01001110110	#143	118	10000011011	#66	185	10111101000	#3	252	11011110010	#285
52	01001110111	#144	119	10000011111	#190	186	10111101011	#233	253	11011110011	#35
53	01001111000	#23	120	10001101000	#191	187	10111101111	#100	254	11011110110	#286
54	01001111011	#49	121	10001101011	#192	188	10111111000	#101	255	11011110111	#46
55	01001111111	#30	122	10001101111	#5	189	10111111011	#112	256	11011111000	#287
56	01010010000	#145	123	10001111000	#193	190	10111111111	#234	257	11011111001	#25
57	01010010001	#146	124	10001111011	#194	191	11000000000	#235	258	11011111011	#9
58	01010010010	#147	125	10001111111	#195	192	11000000011	#236	259	11011111111	#20
59	01010010011	#148	126	10010010000	#196	193	11000000110	#237	260	11100000000	#288
60	01010010110	#149	127	10010010001	#75	194	11000000111	#238	261	11100000011	#289
61	01010010111	#150	128	10010010011	#95	195	11000010000	#239	262	11100000110	#290
62	01010011000	#151	129	10010010111	#197	196	11000010011	#240	263	11100000111	#291
63	01010011001	#152	130	10010011000	#198	197	11000010110	#241	264	11100010000	#292
64	01010011011	#153	131	10010011001	#65	198	11000010111	#242	265	11100010011	#293
65	01010011111	#154	132	10010011011	#82	199	11000011000	#243	266	11100010110	#294
66	01011010000	#155	133	10010011111	#199	200	11000011011	#244	267	11100010111	#295
67	01011010010	#156	134	10011111000	#200	201	11000011111	#245	268	11100011000	#296

271	11100100000	#299	302	11101100111	#90	333	11110100011	#352	363	11111010110	#379
272	11100100011	#300	303	11101101000	#11	334	11110100100	#353	364	11111010111	#380
273	11100100100	#301	304	11101101011	#88	335	11110100101	#354	365	11111011000	#104
274	11100100110	#302	305	11101101111	#15	336	11110100110	#355	366	11111011001	#381
275	11100100111	#303	306	11101110000	#328	337	11110100111	#356	367	11111011011	#105
276	11100110000	#304	307	11101110011	#329	338	11110110000	#357	368	11111011111	#382
277	11100110011	#305	308	11101110100	#330	339	11110110001	#358	369	11111100000	#34
278	11100110100	#306	309	11101110110	#331	340	11110110010	#359	370	11111100001	#383
279	11100110110	#307	310	11101110111	#332	341	11110110011	#360	371	11111100010	#43
280	11100110111	#308	311	11101111000	#26	342	11110110100	#361	372	11111100011	#384
281	11100111000	#309	312	11101111011	#50	343	11110110101	#362	373	11111100100	#93
282	11100111011	#310	313	11101111111	#28	344	11110110110	#363	374	11111100101	#385
283	11100111111	#311	314	11110000000	#333	345	11110110111	#364	375	11111100110	#47
284	11101000000	#312	315	11110000001	#334	346	11110111000	#365	376	11111100111	#386
285	11101000011	#313	316	11110000010	#335	347	11110111001	#366	377	11111101000	#8
286	11101000110	#314	317	11110000011	#336	348	11110111011	#367	378	11111101011	#17
287	11101000111	#315	318	11110000110	#337	349	11110111111	#368	379	11111101111	#21
288	11101001000	#316	319	11110000111	#338	350	11111000000	#369	380	11111110000	#39
289	11101001011	#317	320	11110010000	#339	351	11111000001	#370	381	11111110001	#387
290	11101001111	#318	321	11110010001	#340	352	11111000010	#371	382	11111110010	#388
291	11101010000	#319	322	11110010010	#341	353	11111000011	#372	383	11111110011	#89
292	11101010011	#320	323	11110010011	#342	354	11111000110	#373	384	11111110100	#94
293	11101010110	#321	324	11110010110	#343	355	11111000111	#374	385	11111110101	#389
294	11101010111	#322	325	11110010111	#344	356	11111001000	#55	386	11111110110	#390
295	11101011000	#323	326	11110011000	#345	357	11111001011	#59	387	11111110111	#92
296	11101011011	#324	327	11110011001	#346	358	11111001111	#86	388	11111111000	#12
297	11101011111	#325	328	11110011011	#347	359	11111010000	#375	389	11111111001	#391
298	11101100000	#38	329	11110011111	#348	360	11111010001	#376	390	11111111011	#14
299	11101100011	#326	330	11110100000	#349	361	11111010010	#377	391	11111111111	#31

UNARY REPRESENTATIVES:

#	01234														
1	00000	15	00024	29	00130	43	00232	57	01014	71	01134	85	01411	99	04441
2	00001	16	00030	30	00134	44	00233	58	01030	72	01210	86	01412	100	04442
3	00002	17	00032	31	00142	45	00234	59	01032	73	01211	87	01413	101	10002
4	00003	18	00033	32	00200	46	00241	60	01033	74	01212	88	02030	102	10222
5	00004	19	00034	33	00201	47	00242	61	01034	75	01213	89	02203	103	10422
6	00010	20	00041	34	00202	48	00243	62	01110	76	01214	90	02204	104	11012
7	00011	21	00042	35	00203	49	00300	63	01111	77	01220	91	02222	105	11032
8	00012	22	00043	36	00204	50	00320	64	01112	78	01221	92	02241	106	11123
9	00013	23	00100	37	00213	51	00322	65	01113	79	01222	93	02242	107	11132
10	00014	24	00104	38	00220	52	00422	66	01114	80	01311	94	02243	108	11133
11	00020	25	00113	39	00221	53	01010	67	01120	81	01312	95	03211	109	11212
12	00021	26	00120	40	00222	54	01011	68	01121	82	01313	96	03212	110	11222
13	00022	27	00122	41	00224	55	01012	69	01122	83	01321	97	03221	111	11322
14	00023	28	00124	42	00230	56	01013	70	01124	84	01322	98	03222	112	14442

BINARY REPRESENTATIVES :

f(00)f(01)f(02)f(03)f(04) | f(10)f(11)f(12)f(13)f(14) | f(20)f(21)f(22)f(23)f(24) | f(30)f(31)f(32)f(33)f(34) | f(40)f(41)f(42)f(43)f(44)

113	43334	43334	43234	43334	44444	131	00000	11111	22222	33433	44444
114	01334	11333	22233	33333	43334	132	44444	43334	43244	43134	44044
115	01110	00000	00000	00000	00000	133	44444	43434	44243	43434	44444
116	11333	11333	33233	33333	33333	134	44444	43334	43244	43134	44144
117	10001	11111	33211	11111	11111	135	00000	11111	22222	33433	43334
118	22133	22133	22222	22222	22222	136	44344	43334	43244	43134	44044
119	10001	11111	33222	22222	22222	137	44444	43434	44344	43434	44444
120	44244	43234	22222	43234	44244	138	44444	43334	43344	43134	44144
121	00200	01210	22222	01234	00244	139	44444	43334	43344	43134	44044
122	44444	43333	43234	43133	44044	140	44444	43344	43244	43134	44044
123	01100	11111	11211	11111	01110	141	44444	44444	44243	44444	44444

152	11311	11311	11111	11111	11011
153	33333	33333	33311	33334	33333
154	33333	33333	33310	33334	33333
155	44443	43434	44244	43434	44444
156	11111	11011	22222	33333	33333
157	33333	33333	33211	33434	33333
158	11110	11111	22222	43434	44444
159	33333	33333	33200	33334	33333
160	33333	33343	33211	33333	33333
161	33333	33343	33201	33333	33333
162	22222	22222	22222	43222	44222
163	01222	01222	22222	22233	22244
164	22210	22211	22222	01234	00244
165	22222	22222	22210	22234	22244
166	22243	22234	22222	22222	22222
167	22022	22122	11222	33222	33222
168	22211	22211	22222	22234	22244
169	22222	22222	22211	22234	22244
170	11222	01222	22222	22233	22243
171	11133	01133	22222	22233	22243
172	01222	11222	22222	43222	33222
173	22222	22222	33210	22334	22444
174	22011	22111	11111	11111	11111
175	22133	22133	22322	22333	22344
176	22222	22222	22322	22133	22144
177	22222	22222	22333	22133	22044
178	22222	22222	00244	22222	22222
179	22222	22222	01244	22422	22422
180	22222	22222	11244	22222	22222
181	22222	22222	11244	22322	22322
182	01122	10122	22222	43333	33333
183	22222	22222	10243	22322	22322
184	01210	11210	22220	11210	01210
185	44444	43330	43230	43330	44444
186	00000	11211	11211	11111	01110
187	41114	11111	11211	11111	41114
188	40004	11111	11211	11111	41114
189	01110	11111	11112	11111	01110
190	40004	11111	11111	11111	41114
191	01210	00000	00000	00000	00000
192	41214	44444	44444	44444	44444
193	01210	10000	10000	10000	00000
194	41114	44444	44444	44444	44444
195	40004	44444	44444	44444	43334
196	11211	01211	02222	01211	11211
197	34443	33333	33233	33333	30004
198	33333	33233	32323	33233	33233
199	33333	33333	23333	33333	04443
200	11111	10001	20001	10001	11111
201	44444	23233	22222	23233	43234
202	41214	43233	42222	43233	42224
203	40004	43233	42222	43233	42224
204	02220	21212	22222	21212	02120
205	41114	23334	23234	23334	44444
206	40004	23333	23233	23333	43334
207	02220	21212	22122	21212	02220
208	41114	23333	23333	23333	43334
209	40004	23333	23333	23333	43334
210	44444	42224	42224	42124	44444
211	40004	42224	42224	42124	44444
212	02120	22222	22222	22222	02220
213	41114	22223	22223	22223	42224
214	40004	22223	22223	22223	42224
215	40004	22223	22323	22223	42224

219	01110	20000	20000	20000	00000
220	41114	24444	24444	24444	44444
221	02220	34444	34444	34444	04440
222	22222	11111	21232	21112	22222
223	22222	23334	23234	23334	20002
224	11121	11113	11211	11111	11111
225	12221	23234	22224	23234	23332
226	23333	33333	33233	33333	30004
227	23333	33333	33333	33333	30004
228	22222	22221	22232	22222	22222
229	22222	22224	22224	22324	20002
230	21111	12111	11213	11111	11111
231	02222	22220	12220	22220	24442
232	04442	22332	13332	23332	22222
233	44444	14444	24444	14444	24444
234	04443	44444	44444	44444	24440
235	43234	33233	22224	33233	43244
236	03030	33233	02220	33233	03030
237	00200	11110	22222	43333	43234
238	40000	01110	01212	01110	00000
239	00100	01110	21210	01110	00200
240	03330	33233	02223	33233	03330
241	01110	11211	22222	43233	44444
242	40200	01110	11214	01110	00000
243	00100	01110	21110	01110	00000
244	03330	33233	02323	33233	03330
245	40200	01110	01110	01114	00000
246	00000	01010	02200	01010	00000
247	03030	33033	03230	33333	03030
248	43434	43433	22222	11011	00200
249	00000	01210	00200	01014	00000
250	00000	01010	02000	01010	00000
251	00100	01010	00202	01010	00000
252	03330	33033	03233	33333	03330
253	43334	43433	22222	11011	00200
254	40444	41414	44242	41414	44344
255	00200	01010	00100	01010	00100
256	03330	33033	03233	33333	03330
257	00000	01010	00104	01010	00000
258	00000	00000	02200	00000	00000
259	00200	00000	22222	44444	44444
260	00200	00000	20002	00000	00200
261	00100	00000	00202	00000	00000
262	03330	33033	03233	30333	03330
263	01110	11211	22222	44444	44444
264	00200	00200	42223	00000	00200
265	01210	10111	01111	11111	01110
266	11111	11211	01211	11111	11111
267	11111	11111	01213	11111	11111
268	11211	11111	22222	43333	43333
269	33333	03333	33233	33333	33333
270	01211	11111	22222	43333	33333
271	11411	11111	21210	11111	11111
272	11111	11111	01111	11111	11211
273	11111	11111	01113	11111	11111
274	33333	03333	33333	33333	33333
275	11411	11111	21110	11111	11111
276	10000	01010	02200	01010	00000
277	11111	11111	10213	11111	11111
278	11111	11211	22222	43434	44444
279	03333	33033	03233	33333	33333
280	01111	11011	22222	43233	33333
281	31211	11210	22224	11011	11211
282	11111	11111	20111	11111	11111

283	11111	11111	20113	11111	11111
284	11111	10211	01211	11111	11111
285	33233	34333	22222	11111	11111
286	11211	10111	22222	44444	44444
287	10000	00200	22100	00000	00000
288	00220	21212	22222	21212	02220
289	02030	33333	03230	33333	03030
290	02220	11212	22222	43232	42224
291	02420	01112	21212	23112	02220
292	00100	21110	01210	01110	00000
293	02330	33333	03233	33333	03330
294	02120	11212	22222	43232	42224
295	02020	21114	11211	21111	02120
296	00000	21110	01110	01110	00000
297	02330	33333	03333	33333	03330
298	02010	11114	11111	11111	01110
299	02220	22222	22220	22222	02220
300	02230	32332	03230	33333	03230
301	42200	22222	22222	22222	02220
302	02220	12222	22222	42222	42224
303	02020	22224	02220	22222	02020
304	01110	22000	01211	11111	01110
305	02330	32333	03233	33333	03330
306	02120	22222	22222	42222	42224
307	02020	22112	31213	21112	02420
308	00000	22110	01110	01110	00000
309	02330	32333	03333	33333	03330
310	02020	22224	22323	22222	02420
311	00000	21010	00200	01010	00000
312	00000	21010	00200	01010	00000
313	02030	33333	03230	33033	03030
314	02220	21012	22222	23332	42424
315	00220	21014	20222	21212	02220
316	00000	21010	00000	01010	00000
317	02030	33333	03030	33333	03030
318	02420	21112	21412	23112	02220
319	00100	21010	00200	01010	00000
320	02330	33033	33233	33333	03330
321	02110	01011	22222	33333	44444
322	02020	21212	42223	21412	02220
323	01110	21011	01111	11111	01110
324	02330	33033	33333	33333	03330
325	02020	21212	42322	21412	02220
326	00030	23323	03200	33333	03030
327	02220	12222	22222	44222	42224
328	02120	20222	22222	22222	02220
329	03330	32023	33233	32323	03330
330	02220	00222	22222	22222	42324
331	02120	22022	22222	22322	42424
332	02020	22422	22223	22222	02220
333	20222	21212	22222	21212	22222
334	22222	01213	22222	21212	22222
335	02222	21210	22222	33232	22222
336	20230	33333	23230	33333	23232
337	22222	01212	22222	33233	42222
338	23030	33332	03230	33334	03030
339	20221	21212	22222	21212	22222
340	21111	01111	11211	11111	11311
341	22222	01212	22222	23232	22322
342	33333	23330	33233	33333	33333
343	22222	01212	22222	23232	42322
344	23330	33332	33230	33334	03330
345	23333	43333	23333	33333	33333
346	22211	43211	22313	23311	11111
347	33333	23330	33333	33333	33333
348	20222	43232	32122	23232	22222
349	20222	22222	22222	22222	22222
350	22222	42213	22222	22222	22222
351	22222	23222	22222	02222	22222
352	20202	02333	23232	33333	23232
353	42202	22222	22222	22222	22222
354	02242	22222	22224	12322	23222
355	42222	42222	22222	02222	22222
356	20202	22233	02224	22224	02224
357	20221	22222	22222	22222	22222
358	22222	42222	22213	22222	22222
359	22221	23222	22222	02222	22222
360	33333	02323	33233	32323	33333
361	42201	22222	22222	22222	22222
362	02242	22222	22224	22222	13222
363	42221	42222	22222	02222	22222
364	20202	22233	02223	22224	02223
365	22222	42222	22322	22222	22222
366	22222	21222	22322	22222	22422
367	33333	02323	33333	32323	33333
368	10211	01311	43131	11321	11121
369	22222	20222	22123	22222	22222
370	22222	21212	20212	21213	22222
371	22222	21112	22222	23432	22222
372	23202	33033	23202	33333	23232
373	22222	21012	22222	23232	22422
374	20222	41012	22222	21212	22222
375	11111	11011	11211	11111	21111
376	22222	21211	20223	21212	22222
377	22122	21012	22222	23332	22222
378	33332	33033	33233	33333	33333
379	22122	21012	22222	23232	22422
380	21111	01111	34211	11411	11111
381	11111	21111	10113	11111	11111
382	12011	11111	11104	11114	11111
383	22222	20222	22212	22223	22222
384	20000	02023	00200	02020	00000
385	22222	20223	22212	22222	24222
386	20202	22243	02224	22224	02224
387	22222	20221	22223	22222	22222
388	22122	20222	22222	23222	22222
389	22222	20221	22223	22222	24222
390	22122	20222	22222	24222	22222
391	22222	20222	22123	22222	22222

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