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A generalization of the array type polynomials

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ABSTRACT. We introduce a generalization of the array type polynomials by using two specific generating functions and investigate some of its basic properties in the sequel. A recurrence relation and two summation formulas involving these polynomials are also given.

1. INTRODUCTION

The Appell polynomials $A_n(x)$ defined by

$$f(t)\mathrm{e}^{xt} = \sum_{n=0}^{\infty} A_n(x) \frac{t^n}{n!},$$

where $f(t)$ is a formal power series in t, have found many considerable applications in mathematics, theoretical physics and chemistry [1, 23]. One the well-known examples of these polynomials is the Bernoulli polynomials for the so-called Bose-Einstein function

$$f(t) = \varepsilon(t) = \frac{t}{\mathrm{e}^t - 1}.$$

This function was also used in [5] as a weight function on $(0, +\infty)$ in construction of Gaussian quadrature formulas for numerical calculation of integrals, which frequently occur in connection with the evaluation, in the independent particle approximation, of thermodynamic variables for solid state physics problems for boson systems. This kind of weighted integrals, with the Bose-Einstein weight functions $\varepsilon(t)$ are also found to provide very effective tools for the summation of slowly convergent series whose general

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term is expressible in terms of a Laplace transform or its derivative. Such method is known as the *Laplace transform method* (cf. [17, pp. 398–401]).

Another example known in the literature as the generalized Apostol–Bernoulli polynomials can be defined as [9–11]

$$\left(\frac{t}{\lambda \mathrm{e}^t - 1}\right)^m \mathrm{e}^{xt} = \sum_{n=0}^{\infty} B_n^{(m)}(x,\lambda)\frac{t^n}{n!}, \tag{1}$$

where $\lambda \in \mathbb{C}$, and $|t| < 2\pi$ for $\lambda = 1$ and $|t| < |\log \lambda|$ for $\lambda \neq 1$. For $x = 0$ in (1), the generalized Apostol–Bernoulli numbers are derived (see also [13, 14, 21, 22] in this sense).

On the other hand, the generalized $\lambda-$Stirling numbers of the second kind $S_m^n(\lambda)$ were introduced in [12] as

$$\frac{\left(\lambda \mathrm{e}^t - 1\right)^m}{m!} = \sum_{n=0}^{\infty} S_m^n(\lambda)\frac{t^n}{n!}, \tag{2}$$

for $\lambda \in \mathbb{C}$ and $m \in \mathbb{N}_0 = \{0, 1, 2, \ldots, \}$, where $\lambda = 1$ gives the well known Stirling numbers of the second kind.

By referring to (2), the $\lambda-$array type polynomials $S_m^n(x, \lambda)$ are defined as [3]

$$\frac{\left(\lambda \mathrm{e}^t - 1\right)^m}{m!}\mathrm{e}^{xt} = \sum_{n=0}^{\infty} S_m^n(x,\lambda)\frac{t^n}{n!}, \tag{3}$$

see also [4, 18–20]. In [2], the authors deduced some notable identities associated with $\lambda-$array type polynomials, $\lambda-$Stirling numbers of the second kind and the Apostol–Bernoulli numbers. Furthermore, they studied $\lambda-$array polynomials via $\lambda-$delta operator. For more details see [6–8] and references therein.

In this paper, we define two parametric kinds of $\lambda-$array type polynomials as follows:

$$\frac{\left(\lambda \mathrm{e}^t - 1\right)^m}{m!}\mathrm{e}^{xt}\cos yt = \sum_{n=0}^{\infty} S_m^{n,c}(x,y,\lambda)\frac{t^n}{n!}, \tag{4}$$

and

$$\frac{\left(\lambda \mathrm{e}^t - 1\right)^m}{m!}\mathrm{e}^{xt}\sin yt = \sum_{n=0}^{\infty} S_m^{n,s}(x,y,\lambda)\frac{t^n}{n!},$$

which are, in fact, the real and the imaginary part of

$$\frac{\left(\lambda \mathrm{e}^t - 1\right)^m}{m!}\mathrm{e}^{(x+\mathrm{i}y)t},$$

respectively.

Motivated by some of above-mentioned papers, we then introduce a bivariate kind of $\lambda-$array type polynomials and investigate its general properties. We also derive a recurrence relation and some summation formulas including these polynomials and other special polynomials and numbers such as parametric-kind Apostol–Bernoulli, generalized Apostol–Bernoulli numbers, and generalized $\lambda-$Stirling numbers of the second kind.

2. A family of λ-array type polynomials

For two sequences $\{a_n\}_{n\in\mathbb{N}_0}$ and $\{b_n\}_{n\in\mathbb{N}_0}$ such that

$$A(t)=\sum_{n=0}^{\infty}a_n\frac{t^n}{n!}\quad\text{and}\quad B(t)=\sum_{n=0}^{\infty}b_n\frac{t^n}{n!},$$

their binomial convolution is given by

$$c_n=a_n*b_n=\sum_{k=0}^{n}\binom{n}{k}a_kb_{n-k}.$$

Also, we have the following generating function for the sequence $\{c_n\}_{n\in\mathbb{N}_0}$ as

$$C(t)=A(t)B(t)=\sum_{n=0}^{\infty}c_n\frac{t^n}{n!}.$$

By noting these comments, the Taylor–Maclaurin expansions of the two functions $\mathrm{e}^{xt}\cos yt$ and $\mathrm{e}^{xt}\sin yt$ are explicitly computed as (see e.g. [15] and [16])

$$\mathrm{e}^{xt}\cos(yt)=\sum_{n=0}^{\infty}C_n(x,y)\frac{t^n}{n!}, \tag{5}$$

and

$$\mathrm{e}^{xt}\sin(yt)=\sum_{n=0}^{\infty}S_n(x,y)\frac{t^n}{n!},$$

where $C_n(x,y)$ and $S_n(x,y)$ are defined respectively as follows:

$$C_n(x,y)=\sum_{k=0}^{[\frac{n}{2}]}(-1)^k\binom{n}{2k}x^{n-2k}y^{2k},$$

and

$$S_n(x,y)=\sum_{k=0}^{[\frac{n-1}{2}]}(-1)^k\binom{n}{2k+1}x^{n-2k-1}y^{2k+1},$$

in which $[x]$ stands for the greatest integer of x.

By observing the definitions of $\lambda-$Stirling numbers of the second kind $S_m^n(\lambda)$, $C_n(x,y)$ and $S_n(x,y)$, we can now introduce two parametric kinds of $\lambda-$array type polynomials as follows

$$S_m^{n,c}(x,y,\lambda) = S_m^n(\lambda) * C_n(x,y),$$

and

$$S_m^{n,s}(x,y,\lambda) = S_m^n(\lambda) * S_n(x,y),$$

whose generating functions are respectively defined by

$$\sum_{n=0}^{\infty} S_m^{n,c}(x,y,\lambda)\frac{t^n}{n!} = \frac{\left(\lambda \mathrm{e}^t - 1\right)^m}{m!}\mathrm{e}^{xt}\cos yt,$$

and

$$\sum_{n=0}^{\infty} S_m^{n,s}(x,y,\lambda)\frac{t^n}{n!} = \frac{\left(\lambda \mathrm{e}^t - 1\right)^m}{m!}\mathrm{e}^{xt}\sin yt,$$

from which it is readily deduced that

$$S_m^{n,c}(x,y,\lambda) = \sum_{k=0}^{n}\binom{n}{k} S_m^k(\lambda)\, C_{n-k}(x,y),$$

and

$$S_m^{n,s}(x,y,\lambda) = \sum_{k=0}^{n}\binom{n}{k} S_m^k(\lambda)\, S_{n-k}(x,y).$$

Theorem 1. *For every $n \in \mathbb{N}$, we have*

(6) $$S_m^{n,c}(2x,y,\lambda) = \sum_{k=0}^{n}\binom{n}{k} S_m^{k,c}(x,y,\lambda)\, x^{n-k},$$

and

(7) $$S_m^{n,s}(2x,y,\lambda) = \sum_{k=0}^{n}\binom{n}{k} S_m^{k,s}(x,y,\lambda)\, x^{n-k}.$$

Proof. From (4), one can write

$$\begin{aligned}\sum_{n=0}^{\infty} S_m^{n,c}(2x,y,\lambda)\frac{t^n}{n!} &= \frac{\left(\lambda \mathrm{e}^t - 1\right)^m}{m!}\mathrm{e}^{2xt}\cos yt \\ &= \left(\sum_{n=0}^{\infty} S_m^{n,c}(x,y,\lambda)\frac{t^n}{n!}\right)\left(\sum_{n=0}^{\infty}\frac{(xt)^n}{n!}\right) \\ &= \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n}\binom{n}{k} S_m^{k,c}(x,y,\lambda)\, x^{n-k}\right)\frac{t^n}{n!}.\end{aligned}$$

Comparing the coefficients of $t^n/n!$ on the both sides yields the desired identity (6). Equality (7) can be obtained similarly. □

Theorem 2. *For $n \in \mathbb{N}$ and $z \in \mathbb{R}$ we have*

$$S_m^{n,c}(x+z,y,\lambda) = \sum_{k=0}^{n} \binom{n}{k} S_m^{k,c}(x,y,\lambda)\, z^{n-k}$$

and

$$S_m^{n,s}(x+z,y,\lambda) = \sum_{k=0}^{n} \binom{n}{k} S_m^{k,s}(x,y,\lambda)\, z^{n-k}.$$

Proof. Using (4), one has

$$\sum_{n=0}^{\infty} S_m^{n,c}(x+z,y,\lambda)\frac{t^n}{n!} = \frac{\left(\lambda \mathrm{e}^t - 1\right)^m}{m!}\mathrm{e}^{(x+z)t}\cos yt,$$

and then, the right hand side in this equality can be written as

$$\left(\sum_{n=0}^{\infty} S_m^{n,c}(x,y,\lambda)\frac{t^n}{n!}\right)\left(\sum_{n=0}^{\infty}\frac{(zt)^n}{n!}\right) = \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n}\binom{n}{k} S_m^{k,c}(x,y,\lambda)\, z^{n-k}\right)\frac{t^n}{n!},$$

which gives the first identity after equating the coefficients $t^n/n!$. The second one can be derived in a similar way. □

Theorem 3. *For $r, n \in \mathbb{N}$, with $n \geq r$, the following derivative formulas hold:*

$$\frac{\partial^r}{\partial x^r}\left(S_m^{n,c}(x,y,\lambda)\right) = \langle n\rangle_r\, S_m^{n-r,c}(x,y,\lambda), \tag{8}$$

$$\frac{\partial^r}{\partial x^r}\left(S_m^{n,s}(x,y,\lambda)\right) = \langle n\rangle_r\, S_m^{n-r,s}(x,y,\lambda), \tag{9}$$

where $\langle x\rangle_r$ denotes the falling factorial, defined for $x \in \mathbb{R}$ by

$$\langle x\rangle_r = \prod_{k=0}^{r-1}(x-k) = \begin{cases} x(x-1)\cdots(x-r+1), & \text{if } r \geq 1,\\ 1, & \text{if } r = 0.\end{cases}$$

Proof. Using (4), we have

$$\begin{aligned}\sum_{n=0}^{\infty}\frac{\partial^r}{\partial x^r}\left(S_m^{n,c}(x,y,\lambda)\right)\frac{t^n}{n!} &= \frac{\partial^r}{\partial x^r}\frac{\left(\lambda \mathrm{e}^t - 1\right)^m}{m!}\mathrm{e}^{xt}\cos yt\\ &= \sum_{n=0}^{\infty}\langle n\rangle_r\, S_m^{n-r,c}(x,y,\lambda)\frac{t^n}{n!},\end{aligned}$$

which concludes the proof of (8).

The assertion (9) can be proved similarly. □

Corollary 1. *For $n \in \mathbb{N}$, the following relations are valid:*

$$\frac{\partial}{\partial y} S_m^{n,c}(x,y,\lambda) = -nS_m^{n-1,s}(x,y,\lambda),$$

and

$$\frac{\partial}{\partial y} S_m^{n,s}(x,y,\lambda) = nS_m^{n-1,c}(x,y,\lambda).$$

Theorem 4. *For $n \in \mathbb{N}$, two parametric kinds of $\lambda-$array type polynomials $S_m^{n,c}(x,y,\lambda)$ and $S_m^{n,s}(x,y,\lambda)$ satisfy the following recurrent relations:*

$$\lambda S_m^{n,c}(x+1,y,\lambda) = (m+1)S_{m+1}^{n,c}(x,y,\lambda) + S_m^{n,c}(x,y,\lambda), \tag{10}$$

and

$$\lambda S_m^{n,s}(x+1,y,\lambda) = (m+1)S_{m+1}^{n,s}(x,y,\lambda) + S_m^{n,s}(x,y,\lambda). \tag{11}$$

Proof. From (4), we have

$$\begin{aligned}
\lambda \sum_{n=0}^{\infty} S_m^{n,c}(x+1,y,\lambda)\frac{t^n}{n!} &= \lambda \frac{\left(\lambda e^t - 1\right)^m}{m!} e^{(x+1)t}\cos yt \\
&= \frac{\left(\lambda e^t - 1\right)^m}{m!} e^{xt}\cos yt \left\{\lambda e^t - 1 + 1\right\} \\
&= (m+1)\frac{\left(\lambda e^t - 1\right)^{m+1}}{(m+1)!} e^{xt}\cos yt + \frac{\left(\lambda e^t - 1\right)^m}{m!} e^{xt}\cos yt \\
&= (m+1)\sum_{n=0}^{\infty} S_{m+1}^{n,c}(x,y,\lambda)\frac{t^n}{n!} + \sum_{n=0}^{\infty} S_m^{n,c}(x,y,\lambda)\frac{t^n}{n!}.
\end{aligned}$$

Comparing the coefficients of $\frac{t^n}{n!}$ on the both sides of the last equality leads to desired identity (10). The relation (11) follows easily in a similar way. □

Theorem 5. *The following summation formulas are true*

$$\begin{aligned}
&\sum_{k=0}^{n+m} \binom{n+m}{k} S_m^{n+m-k,c}(x,y,\lambda) B_k^{(m)}(\lambda) \\
&\qquad = \begin{cases} \binom{n+m}{m} C_n(x,y), & \text{if } n \geq 0; \\ 0, & \text{if } -m \leq n \leq -1, \end{cases}
\end{aligned} \tag{12}$$

and

$$\begin{aligned}
&\sum_{k=0}^{n+m} \binom{n+m}{k} S_m^{n+m-k,s}(x,y,\lambda) B_k^{(m)}(\lambda) \\
&\qquad = \begin{cases} \binom{n+m}{m} S_n(x,y), & \text{if } n \geq 0; \\ 0, & \text{if } -m \leq n \leq -1, \end{cases}
\end{aligned} \tag{13}$$

where $B_k^{(m)}(\lambda)$ is the generalized Apostol–Bernoulli numbers.

Proof. Consider the equality

$$\left(\frac{t}{\lambda e^t - 1}\right)^m \frac{\left(\lambda e^t - 1\right)^m}{m!} e^{xt}\cos yt = \frac{t^m}{m!} e^{xt}\cos yt.$$

Making use of (1) for $x = 0$, (4) and (5), we find that

$$\sum_{n=0}^{\infty} B_n^{(m)}(\lambda)\frac{t^n}{n!}\sum_{n=0}^{\infty} S_m^{n,c}(x,y,\lambda)\frac{t^n}{n!} = \frac{t^m}{m!}\sum_{n=0}^{\infty} C_n(x,y)\frac{t^n}{n!}.$$

So, we have

$$\sum_{n=-m}^{\infty}\sum_{k=0}^{n+m}\binom{n+m}{k} B_k^{(m)}(\lambda) S_m^{n+m-k,c}(x,y,\lambda)\frac{t^n}{(n+m)!} = \frac{1}{m!}\sum_{n=0}^{\infty} C_n(x,y)\frac{t^n}{n!}. \tag{14}$$

Now, if we compare the coefficients of t^n on the both sides of (14), we reach the formula (12). The relation (13) can be derived similarly. □

Theorem 6. *The following relations are valid*

$$\sum_{k=0}^{n+m}\binom{n+m}{k} B_{k,c}(x,y,\lambda)\, S_m^{n+m-k}(\lambda) = \binom{n+m}{m} S_{m-1}^{n+m-1,c}(x,y,\lambda),$$

and

$$\sum_{k=0}^{n+m}\binom{n+m}{k} B_{k,s}(x,y,\lambda)\, S_m^{n+m-k}(\lambda) = \binom{n+m}{m} S_{m-1}^{n+m-1,s}(x,y,\lambda),$$

where $B_{k,c}(x,y,\lambda)$ and $B_{k,s}(x,y,\lambda)$ are two parametric kinds of Apostol–Bernoulli polynomials, presented in [22, Eq.8], and $S_m^n(\lambda)$ is the generalized $\lambda-$Stirling numbers of the second kind, given by (2).

Proof. Consider the equality

$$\frac{1}{t^m}\left(\frac{t}{\lambda e^t - 1}\right)\left(\lambda e^t - 1\right)^m e^{xt}\cos yt = \left(\frac{\lambda e^t - 1}{t}\right)^{m-1} e^{xt}\cos yt.$$

Now, utilizing [22, Eq.8], (2) and (4) yields that

$$\frac{1}{t^m}\left(\sum_{n=0}^{\infty} B_{n,c}(x,y,\lambda)\frac{t^n}{n!}\right)\left(\sum_{n=0}^{\infty} m! S_m^n(\lambda)\frac{t^n}{n!}\right) = \sum_{n=0}^{\infty}(m-1)! S_{m-1}^{n,c}(x,y,\lambda)\frac{t^{n-m+1}}{n!}. \tag{15}$$

Thus, while the left hand side of (15) can be written as

$$\sum_{n=0}^{\infty}\left(\sum_{k=0}^{n}\frac{B_{k,c}(x,y,\lambda)}{k!}\frac{m!S_m^{n-k}(\lambda)}{(n-k)!}\right)t^{n-m} \tag{16}$$

$$=\sum_{n=-m}^{\infty}\left(\sum_{k=0}^{n+m}\frac{B_{k,c}(x,y,\lambda)}{k!}\frac{m!S_m^{n+m-k}(\lambda)}{(n+m-k)!}\right)t^{n},$$

the right hand side can be expressed as

$$\sum_{n=-(m-1)}^{\infty}(m-1)!\frac{S_{m-1}^{n+m-1,c}(x,y,\lambda)}{(n+m-1)!}t^{n}. \tag{17}$$

Note that for $n=-m$, since $S_m^0(\lambda)\,B_{0,c}(x,y,\lambda)=0$, comparing the coefficients of t^n of (16) and (17) gives the first identity of the theorem. The second one can be obtained in a similar way. □

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