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Fixed points for occasionally weakly biased mappings of type (A)

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ABSTRACT. In this paper, in the first step, we will introduce the concept of occasionally weakly biased mappings of type (A) which is a convenient generalization of the concept of weakly biased mappings of type (A). In the second step, we will show that this new definition coincides with our concept of occasionally weakly biased mappings given in [8]. In the third and last step we will give an example which verifies the validity of our result.

1. INTRODUCTION AND PRELIMINARIES

In their paper [17], Jungck and Pathak gave a generalization of compatible mappings called biased mappings.

Definition 1 ([17])**.** Let f and g be self-mappings of a metric space $(\mathcal{X}, d)$. The pair (f, g) is g-biased if and only if whenever $\{x_n\}$ is a sequence in $\mathcal{X}$ and fx_n, $gx_n \to t \in \mathcal{X}$, then

$$\alpha d(gfx_n, gx_n) \leq \alpha d(fgx_n, fx_n),$$

if $\alpha = \liminf$ and $\alpha = \limsup$.

Also, the same authors [17], introduced the concept of weakly biased mappings which represents a convenient generalization of biased mappings.

Definition 2 ([17])**.** Let f and g be self-mappings of a metric space $(\mathcal{X}, d)$. The pair (f, g) is weakly g-biased if and only if $fp = gp$ implies

$$d(gfp, gp) \leq d(fgp, fp).$$

In our paper [8], we introduced the concept of occasionally weakly biased mappings which is a legitimate generalization of weakly biased mappings given by Jungck and Pathak in [17].

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Definition 3 ([8])**.** Let f and g be self-mappings of a set $\mathcal{X}$. The pair (f, g) is said to be **occasionally weakly f-biased and g-biased**, respectively, if and only if, there exists a point p in $\mathcal{X}$ such that $fp = gp$ implies

$$d(fgp, fp) \leq d(gfp, gp), \tag{1}$$

$$d(gfp, gp) \leq d(fgp, fp), \tag{2}$$

respectively.

In 1993, Jungck et al. [16] introduced the concept of compatible mappings of type (A) which is equivalent to compatible mappings under some conditions and gave some common fixed point theorems.

Definition 4 ([16])**.** Self-mappings f and g of a metric space $(\mathcal{X}, d)$ are said to be compatible of type (A) if

$$\lim_{n\to\infty} d(gfx_n, ffx_n) = 0, \lim_{n\to\infty} d(fgx_n, ggx_n) = 0,$$

whenever $\{x_n\}$ is a sequence in $\mathcal{X}$ such that fx_n and $gx_n \to t \in \mathcal{X}$.

To generalize the above definition, Pathak et al. [20] introduced the concept of biased mappings of type (A) proving some fixed point theorems for certain contractions of four mappings which improved some results.

Definition 5 ([20])**.** Let f and g be self-mappings of a metric space $(\mathcal{X}, d)$. The pair (f, g) is said to be g-biased and f-biased of type (A), respectively, if, whenever $\{x_n\}$ is a sequence in $\mathcal{X}$ and fx_n, $gx_n \to t \in \mathcal{X}$,

$$\alpha d(ggx_n, fx_n) \leq \alpha d(fgx_n, gx_n),$$
$$\alpha d(ffx_n, gx_n) \leq \alpha d(gfx_n, fx_n),$$

where $\alpha = \liminf_{n\to\infty}$ and if $\alpha = \limsup_{n\to\infty}$ respectively.

Again in the same paper [20], the authors gave the definition of weakly g-biased of type (A) as follows.

Definition 6 ([20])**.** Let f and g be self-mappings of a metric space $(\mathcal{X}, d)$. The pair (f, g) is said to be weakly g-biased of type (A) if $fp = gp$ implies

$$d(ggp, fp) \leq d(fgp, gp).$$

Now, we are ready to present our main results.

2. MAIN RESULTS

2.1. Occasionally weakly biased mappings of type (A).

Definition 7. Let f and g be self-mappings of a non-empty set $\mathcal{X}$. The pair (f, g) is said to be *occasionally weakly f-biased of type* (A) *and occasionally weakly g-biased of type* (A), respectively, if and only if there exists a point p in $\mathcal{X}$ such that $fp = gp$ implies

$$d(ffp, gp) \leq d(gfp, fp), \tag{3}$$

$$d(ggp, fp) \leq d(fgp, gp), \tag{4}$$

respectively.

Of course, weakly f-biased of type (A) and weakly g-biased of type (A) are occasionally weakly f-biased of type (A) and occasionally weakly g-biased of type (A), respectively. However, the converses are not true in general. Also, it is clear from the definitions that if f and g are weakly compatible or occasionally weakly compatible then f and g are both occasionally weakly f-biased of type (A) and occasionally weakly g-biased of type (A). Therefore, weakly compatible and occasionally weakly compatible mappings are subclasses of occasionally weakly biased of type (A) mappings. The following example testifies.

Example 1. Let $\mathcal{X} = [0, \infty)$ with the usual metric $d(x, y) = |x - y|$. Define $f, g : \mathcal{X} \to \mathcal{X}$ by

$$fx = \begin{cases} 2x^2, & \text{if } x \in [0,1], \\ \frac{4}{x}, & \text{if } x \in (1, \infty), \end{cases} \qquad gx = \begin{cases} 1, & \text{if } x \in [0,1], \\ 2x, & \text{if } x \in (1, \infty). \end{cases}$$

Consider a sequence $\{x_n\} = \left\{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}n}\right\}$ in $\mathcal{X}$. Then

$$\begin{aligned} &\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = 1, \\ &\lim_{n\to\infty} d(fgx_n, gfx_n) = |2 - 1| = 1 \neq 0, \\ &\lim_{n\to\infty} d(fgx_n, ggx_n) = |2 - 1| = 1 \neq 0 \end{aligned}$$

and

$$\lim_{n\to\infty} d(gfx_n, ffx_n) = |1 - 2| = 1 \neq 0.$$

Thus f and g are neither compatible nor compatible of type (A).

Also, we have $fx = gx$ if and only if $x = \frac{1}{\sqrt{2}}$ or $x = \sqrt{2}$, and

$$\begin{aligned} fg(\sqrt{2}) &= \sqrt{2} \neq 4\sqrt{2} = gf(\sqrt{2}), \\ fg\left(\frac{1}{\sqrt{2}}\right) &= 2 \neq 1 = gf\left(\frac{1}{\sqrt{2}}\right), \end{aligned}$$

i.e. f and g are neither weakly compatible nor occasionally weakly compatible.

Again we observe that

$$0 = d\left(gg\left(\frac{1}{\sqrt{2}}\right), f\left(\frac{1}{\sqrt{2}}\right)\right) \leq d\left(fg\left(\frac{1}{\sqrt{2}}\right), g\left(\frac{1}{\sqrt{2}}\right)\right) = 1,$$

that is, the pair (f, g) is occasionally weakly g-biased of type (A). However,

$$2\sqrt{2} = d\left(gg(\sqrt{2}), f(\sqrt{2})\right) \nleq d\left(fg(\sqrt{2}), g(\sqrt{2})\right) = \sqrt{2},$$

then f and g are not weakly g-biased of type (A).

On the other hand we have

$$\sqrt{2} = d\left(ff(\sqrt{2}), g(\sqrt{2})\right) \leq d\left(gf(\sqrt{2}), f(\sqrt{2})\right) = 2\sqrt{2},$$

i.e. the pair (f, g) is occasionally weakly f-biased of type (A). But, as

$$1 = d\left(ff\left(\frac{1}{\sqrt{2}}\right), g\left(\frac{1}{\sqrt{2}}\right)\right) \nleq d\left(gf\left(\frac{1}{\sqrt{2}}\right), f\left(\frac{1}{\sqrt{2}}\right)\right) = 0,$$

i.e., the pair (f, g) is not weakly f-biased of type (A).

Remark 1. It is known that the notions of weak compatibility and occasionally weak compatibility are the minimal conditions for the existence of a unique common fixed point. In the current settings, we assert that our notion of occasionally weakly biased mappings of type (A) has an edge over weak and occasionally weak compatibility, i.e. weakly (respectively occasionally weakly) compatible mappings are both occasionally weakly f-biased and g-biased of type (A), however the converses are false in general. Indeed, let $\mathcal{X}$ be a nonempty set endowed with a metric d and let f and g be self-mappings of $\mathcal{X}$. Suppose that f and g are weakly compatible or occasionally weakly compatible, then, $fu = gu$ implies that $fgu = gfu$, $u \in \mathcal{X}$. We have

$$d(ggu, fu) = d(gfu, gu) \leq d(gfu, fgu) + d(fgu, gu) = d(fgu, gu),$$

i.e. f and g are occasionally weakly g-biased of type (A). Similarly,

$$d(ffu, gu) = d(fgu, fu) \leq d(fgu, gfu) + d(gfu, fu) = d(gfu, fu),$$

i.e. f and g are occasionally weakly f-biased of type (A). However the converses are not true (see the above example).

Remark 2. In (3) inside Definition 7, if we replace fp with gp and gp with fp in the left hand side and fp with gp in the right hand side, we obtain

$$d(fgp, fp) \leq d(gfp, gp),$$

i.e. we get inequality (1) of Definition 3. Again, in (4) inside Definition 7, if we replace gp with fp and fp with gp in the left hand side and gp with fp in the right hand side, we get

$$d(gfp, gp) \leq d(fgp, fp),$$

i.e. we obtain inequality (2) of Definition 3. That is to say that occasionally weakly f-biased and occasionally weakly g-biased are equivalent to occasionally weakly f-biased of type (A) and occasionally weakly g-biased of type (A).

2.2. Unique common fixed points on metric spaces.

2.2.1. *Implicit relations.* According to [24], several classical fixed point theorems and common fixed point theorems have been unified considering a general condition by an implicit relation in ([22, 23]) and in other papers. Motivated by the three cited papers and the next ones [1, 2, 4, 5, 11, 18, 25, 26, 27] and so on, we introduce the new type of implicit relations.

Let Φ be a family of all functions $\varphi : \mathbb{R}_+^6 \to \mathbb{R}$ such that φ is non-increasing in variables t_2, t_3, t_4, t_5 and t_6, and satisfies the next conditions:

(1) $\varphi(t, t, 0, 0, t, t) > 0 \ \forall t > 0$,
(2) $\varphi(t, t, 2t, 0, t, t) > 0 \ \forall t > 0$,
(3) $\varphi(t, t, 0, 2t, t, t) > 0 \ \forall t > 0$.

Example 2. $\varphi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - k \max\{t_2, t_3, t_4, t_5, t_6\}$, where $k \in \left(0, \frac{1}{2}\right)$.

It is clear to see that φ is non-increasing in variables t_2, t_3, t_4, t_5 and t_6, and

(1) $\varphi(t, t, 0, 0, t, t) = t - k \max\{t, 0\} = t(1 - k) > 0 \ \forall t > 0$,
(2) $\varphi(t, t, 2t, 0, t, t) = t - k \max\{t, 0, 2t\} = t(1 - 2k) > 0 \ \forall t > 0$,
(3) $\varphi(t, t, 0, 2t, t, t) = t - k \max\{t, 0, 2t\} = t(1 - 2k) > 0 \ \forall t > 0$.

Example 3. $\varphi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - k(t_2 + t_3 + t_4 + t_5 + t_6)$, where $k \in \left(0, \frac{1}{5}\right)$.

Clearly φ is non-increasing in variables t_2, t_3, t_4, t_5 and t_6, and

(1) $\varphi(t, t, 0, 0, t, t) = t - 3kt = t(1 - 3k) > 0 \ \forall t > 0$,
(2) $\varphi(t, t, 2t, 0, t, t) = t - 5kt = t(1 - 5k) > 0 \ \forall t > 0$,
(3) $\varphi(t, t, 0, 2t, t, t) = t - 5kt = t(1 - 5k) > 0 \ \forall t > 0$.

Example 4. $\varphi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \alpha t_2 - \beta t_3 - \gamma t_4 - \delta t_5 - \lambda t_6$, where $\alpha > 0$, β, γ, δ, $\lambda \geq 0$, $\alpha + 2\beta + 2\gamma + \delta + \lambda < 1$.

It is obvious to see that φ is non-increasing in variables t_2, t_3, t_4, t_5 and t_6, and

(1) $\varphi(t, t, 0, 0, t, t) = t(1 - \alpha - \delta - \lambda) > 0 \ \forall t > 0$,
(2) $\varphi(t, t, 2t, 0, t, t) = t(1 - \alpha - 2\beta - \delta - \lambda) > 0 \ \forall t > 0$,
(3) $\varphi(t, t, 0, 2t, t, t) = t(1 - \alpha - 2\gamma - \delta - \lambda) > 0 \ \forall t > 0$.

2.2.2. *A unique common fixed point theorem for four mappings.*

Theorem 1. *Let f, g, h and k be self-mappings of a metric space $\mathcal{X}$ satisfying the following condition*

$$\begin{aligned}(5) \qquad & \varphi(d(fx, gy), d(hx, ky), d(fx, hx), d(gy, ky), \\ & d(hx, gy), d(fx, ky)) \leq 0,\end{aligned}$$

for all x, $y \in \mathcal{X}$, where $\varphi \in \Phi$. If the pair (f, h) as well as (g, k) is occasionally weakly h-biased of type (A) and occasionally weakly k-biased of type (A), respectively, then f, g, h and k have a unique common fixed point.

Proof. By hypothesis, there are two points u and v in $\mathcal{X}$ such that $fu = hu$ implies $d(hhu, fu) \leq d(fhu, hu)$ and $gv = kv$ implies $d(kkv, gv) \leq d(gkv, kv)$.

First, we are going to prove that $fu = gv$. Suppose that $fu \neq gv$. Then, from inequality (5) we have

$$\begin{aligned}&\varphi(d(fu,gv),d(hu,kv),d(fu,hu),d(gv,kv),d(hu,gv),d(fu,kv))\\&\quad=\varphi(d(fu,gv),d(fu,gv),0,0,d(fu,gv),d(fu,gv))\leq 0\end{aligned}$$

contradicts condition (1), hence, $fu = gv$.

Now, we assert that $ffu = fu$. If not, then the use of condition (5) gives

$$\begin{aligned}&\varphi(d(ffu,gv),d(hfu,kv),d(ffu,hfu),d(gv,kv),d(hfu,gv),d(ffu,kv))\\&\quad=\varphi(d(ffu,fu),d(hfu,fu),d(ffu,hfu),0,d(hfu,fu),d(ffu,fu))\leq 0.\end{aligned}$$

Since the pair (f, h) is occasionally weakly h-biased of type (A) and φ is non-increasing in t_2, t_3 and t_5, and using triangle inequality we get

$$\varphi(d(ffu,fu),d(ffu,fu),2d(ffu,fu),0,d(ffu,fu),d(ffu,fu))\leq 0,$$

which contradicts condition (2), therefore $ffu = fu$ and so $hfu = fu$.

Now, suppose that $ggv \neq gv$. Using inequality (5) we obtain

$$\begin{aligned}&\varphi(d(fu,ggv),d(hu,kgv),d(fu,hu),d(ggv,kgv),d(hu,ggv),d(fu,kgv))\\=\;&\varphi(d(gv,ggv),d(gv,kgv),0,d(ggv,kgv),d(gv,ggv),d(gv,kgv))\leq 0.\end{aligned}$$

As φ is non-increasing in t_2, t_4 and t_6, and the pair (g, k) is occasionally weakly k-biased of type (A), and using the triangle inequality, we get

$$\varphi(d(gv,ggv),d(gv,ggv),0,2d(gv,ggv),d(gv,ggv),d(gv,ggv))\leq 0,$$

a contradiction with condition (3). This implies that $ggv = gv$ and so $kgv = gv$, i.e. $gfu = fu$ and $kfu = fu$. Put $fu = hu = gv = kv = w$, therefore w is a common fixed point of mappings f, g, h and k.

Finally, let w and t be two distinct common fixed points of mappings f, g, h and k. Then, $w = fw = gw = hw = kw$ and $t = ft = gt = ht = kt$. From (5) we have

$$\begin{aligned}&\varphi(d(fw,gt),d(hw,kt),d(fw,hw),d(gt,kt),d(hw,gt),d(fw,kt))\\&\quad=\varphi(d(w,t),d(w,t),0,0,d(w,t),d(w,t))\leq 0,\end{aligned}$$

a contradiction, hence $t = w$. □

Corollary 1. *Let f, g, h and k be self-mappings of a metric space $\mathcal{X}$ satisfying the following condition*

$$d(fx,gy)\leq k\max\{d(hx,ky),d(fx,hx),d(gy,ky),d(hx,gy),d(fx,ky)\},$$

for all $x, y \in \mathcal{X}$, where $k \in \left(0, \frac{1}{2}\right)$. If the pair (f, h) as well as (g, k) is occasionally weakly h-biased of type (A) and occasionally weakly k-biased of type (A), respectively, then f, g, h and k have a unique common fixed point.

Proof. Use Theorem 1 and Example 2. □

Corollary 2. *Let f, g, h and k be self-mappings of a metric space $\mathcal{X}$ satisfying the following condition*

$$d(fx,gy) \le k(d(hx,ky) + d(fx,hx) + d(gy,ky) + d(hx,gy) + d(fx,ky)),$$

for all $x, y \in \mathcal{X}$, where $k \in \left(0, \frac{1}{5}\right)$. If the pair (f, h), as well as (g, k), is occasionally weakly h-biased of type (A) and occasionally weakly k-biased of type (A), respectively, then f, g, h and k have a unique common fixed point.

Proof. Use Theorem 1 and Example 3. □

Corollary 3. *Let f, g, h and k be self-mappings of a metric space $\mathcal{X}$ satisfying the following condition*

$$d(fx,gy) \le \alpha d(hx,ky) + \beta d(fx,hx) + \gamma d(gy,ky) + \delta d(hx,gy) + \lambda d(fx,ky)),$$

for all $x, y \in \mathcal{X}$, where $\alpha > 0$, β, γ, δ, $\lambda \ge 0$, $\alpha + 2\beta + 2\gamma + \delta + \lambda < 1$. If the pair (f, h) as well as (g, k) is occasionally weakly h-biased of type (A) and occasionally weakly k-biased of type (A), respectively, then f, g, h and k have a unique common fixed point.

Proof. Use Theorem 1 and Example 4. □

2.2.3. *A unique common fixed point theorem for a sequence of mappings.*

Theorem 2. *Let h, k and $\{f_n\}_{n=1,2,\ldots}$ be self-mappings of a metric space $\mathcal{X}$ satisfying the following condition*

$$\varphi(d(f_n x, f_{n+1} y), d(hx, ky), d(f_n x, hx), d(f_{n+1} y, ky), d(hx, f_{n+1} y), d(f_n x, ky)) \le 0,$$

for all $x, y \in \mathcal{X}$, where $\varphi \in \Phi$. If the pair (f_n, h) as well as (f_{n+1}, k) is occasionally weakly h-biased of type (A) and occasionally weakly k-biased of type (A), respectively, then h, k and $\{f_n\}_{n=1,2,\ldots}$ have a unique common fixed point.

2.2.4. *Illustrative example.*

Example 5. Let $\mathcal{X} = [0, 5)$ with the metric $d(x, y) = |x - y|$. Define

$$fx = \begin{cases} \frac{3}{4}, & \text{if } x \in [0,1), \\ 1, & \text{if } x \in [1,5), \end{cases} \qquad gx = \begin{cases} \frac{2}{3}, & \text{if } x \in [0,1), \\ 1, & \text{if } x \in [1,5), \end{cases}$$

and

$$hx = \begin{cases} 3, & \text{if } x \in [0,1), \\ \frac{1}{x^2}, & \text{if } x \in [1,5), \end{cases} \qquad kx = \begin{cases} 4, & \text{if } x \in [0,1), \\ \frac{1}{x}, & \text{if } x \in [1,5). \end{cases}$$

First it is clear to see that f and h are occasionally weakly h-biased of type (A) and g and k are occasionally weakly k-biased of type (A). Define $\varphi(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \frac{1}{4} \max\{t_2, t_3, t_4, t_5, t_6\}$, we get

(1) for x, $y \in [0,1)$, we have $fx = \frac{3}{4}$, $gy = \frac{2}{3}$, $hx = 3$, $ky = 4$ and

$$\begin{aligned}
&\varphi(d(fx,gy),d(hx,ky),d(fx,hx),d(gy,ky),d(hx,gy),d(fx,ky))\\
&= \varphi\left(\frac{1}{12},1,\frac{9}{4},\frac{10}{3},\frac{7}{3},\frac{13}{4}\right)\\
&= \frac{1}{12} - \frac{1}{4}\max\left\{1,\frac{9}{4},\frac{10}{3},\frac{7}{3},\frac{13}{4}\right\}\\
&= \frac{1}{12} - \frac{1}{4}\times\frac{10}{3}\\
&= -\frac{3}{4}\\
&\leq 0;
\end{aligned}$$

(2) for x, $y \in [1,5)$, we have $fx = gy = 1$, $hx = \dfrac{1}{x^2}$, $ky = \dfrac{1}{y}$ and

$$\begin{aligned}
&\varphi(d(fx,gy),d(hx,ky),d(fx,hx),d(gy,ky),d(hx,gy),d(fx,ky))\\
&= \varphi\left(0,\left|\frac{1}{x^2}-\frac{1}{y}\right|,\left|1-\frac{1}{x^2}\right|,\left|1-\frac{1}{y}\right|,\left|1-\frac{1}{x^2}\right|,\left|1-\frac{1}{y}\right|\right)\\
&= -\frac{1}{4}\max\left\{\left|\frac{1}{x^2}-\frac{1}{y}\right|,\left|1-\frac{1}{x^2}\right|,\left|1-\frac{1}{y}\right|\right\}\\
&\leq 0;
\end{aligned}$$

(3) for $x \in [0,1)$, $y \in [1,5)$, we have $fx = \frac{3}{4}$, $gy = 1$, $hx = 3$, $ky = \dfrac{1}{y}$ and

$$\begin{aligned}
&\varphi(d(fx,gy),d(hx,ky),d(fx,hx),d(gy,ky),d(hx,gy),d(fx,ky))\\
&= \varphi\left(\frac{1}{4},\left|3-\frac{1}{y}\right|,\frac{9}{4},\left|1-\frac{1}{y}\right|,2,\left|\frac{3}{4}-\frac{1}{y}\right|\right)\\
&= \frac{1}{4} - \frac{1}{4}\max\left\{\left|3-\frac{1}{y}\right|,\frac{9}{4},\left|1-\frac{1}{y}\right|,2,\left|\frac{3}{4}-\frac{1}{y}\right|\right\}\\
&\leq 0;
\end{aligned}$$

(4) finally, for $x \in [1,5)$, $y \in [0,1)$, we have $fx = 1$, $gy = \frac{2}{3}$, $hx = \dfrac{1}{x^2}$, $ky = 4$ and

$$\begin{aligned}
&\varphi(d(fx,gy),d(hx,ky),d(fx,hx),d(gy,ky),d(hx,gy),d(fx,ky))\\
&= \varphi\left(\frac{1}{3},\left|4-\frac{1}{x^2}\right|,\left|1-\frac{1}{x^2}\right|,\frac{10}{3},\left|\frac{2}{3}-\frac{1}{x^2}\right|,3\right)\\
&= \frac{1}{3} - \frac{1}{4}\max\left\{\left|4-\frac{1}{x^2}\right|,\left|1-\frac{1}{x^2}\right|,\frac{10}{3},\left|\frac{2}{3}-\frac{1}{x^2}\right|,3\right\}\\
&\leq 0.
\end{aligned}$$

So, all the conditions of Theorem 1 are satisfied and 1 is the unique common fixed point of mappings f, g, h and k.

3. Conclusion

Our results unify, extend and improve many related common fixed point theorems from the literature especially Theorems 2.2 and 2.6 of [29], Theorem 1 of [6], Theorem 1 of [28], Corollary 1 of [10], Theorem 2 of [23], Theorem 1 of [14], Theorem 3.1 of [12], Theorem 3.1 of [3], Theorem 3.2 of [13], Theorems 3.1 and 3.2 of [21] and others.

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