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On almost topological groups

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Abstract. We introduce and study the almost topological groups which are fundamentally a generalization of topological groups. Almost topological groups are defined by using almost continuous mappings in the sense of Singal and Singal. We investigate some permanence properties of almost topological groups. It is proved that translation of a regularly open (resp. regularly closed) set in an almost topological group is regularly open (resp. regularly closed). And this fact gives us a lot of important and useful results of almost topological groups.

1. Introduction

A topological group is a group endowed with a topology which turns out the group operation and the inversion mapping (that is, $x \to x^{-1}$) continuous. Since the advent of this concept, it has been captured a great attention from different researchers and mathematicians. Many mathematicians have contributed significantly in the field of topological groups. Some first contributors to the theory of topological groups are A.D. Alexandroff, N. Bourbaki, M.I. Graev, S. Kakutani, E. van Kampen, A.N. Kolmogorov, A.A. Markov, Pontryagin, etc. Among those who contributed greatly to this field are A.V. Arhangel'skii, M.M. Choban, W.W. Comfort, D. Dikranjan, E. van Douwen, V.I. Malykhin, J. van Mill, B.A. Pasynkov, D. Shakhmatov, M. Tkachenko.

Recent literature of mathematics contains several similar notions and generalizations of topological groups. Semi-topological groups [10, 11], s-topological groups [1, 4, 8], S-topological groups [1], quasi S-topological groups [6], irresolute topological groups [5, 9], quasi-irresolute topological groups [12] and paratopological groups [16] are well-known.

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In this paper, we will study a new class of spaces, which we shall call almost topological groups. Basically, almost topological groups are a generalization of topological groups. We will present some examples of almost topological groups which are not topological groups. We also set forth some basic properties of almost topological groups. It is proved that translation of a regularly open (resp. regularly closed) set in an almost topological group is regularly open (resp. regularly closed), that any group homomorphism of almost topological groups which is R-continuous at the identity is almost continuous everywhere.

2. Preliminaries

Throughout the present paper, (X, τ) (or simply X) means a topological space. For $A \subseteq X$, $Cl(A)$ denotes the closure of A and $Int(A)$ denotes the interior of A. By $f : X \to Y$, we denote a map f from a topological space X to a topological space Y.

In 1963, N. Levine introduced the notion of semi-open sets in the literature of mathematics. He defined a set A in a topological space X to be semi-open [7] if there exists an open set U in X such that $U \subseteq A \subseteq Cl(U)$; or equivalently, a subset $A \subseteq X$ is semi-open if and only if $A \subseteq Cl(Int(A))$. The complement of a semi-open set is semi-closed set; or equivalently, a subset A of X is semi-closed if $Int(Cl(A)) \subseteq A$. Every open set is semi-open but the converse is not true, in general. A subset A of a topological space X is called regularly open if $A = Int(Cl(A))$. The complement of a regularly open set is said to be regularly closed, or equivalently, a subset A of X is regularly closed if $A = Cl(Int(A))$. It is clear that every regularly open (closed) set is open (closed). However, the converse need not be true, in general. The finite intersection of regularly open sets is regularly open. Also, it is known that the closure of a semi-open set in X is regularly closed and the interior of a semi-closed set in X is regularly open.

A subset A of a topological space X is said to be $\delta-$open [17] if for each $x \in A$, there exists a regular open set U in X such that $x \in U \subseteq A$. The complement of a $\delta-$open set is called $\delta-$closed set [17]. The intersection of all $\delta-$closed sets in X containing a subset $A \subseteq X$ is called the $\delta-$closure of A and is denoted by $Cl_\delta(A)$. It is known that a subset A of X is $\delta-$closed if and only if $A = Cl_\delta(A)$. A point $x \in Cl_\delta(A)$ if and only if $A \cap Int(Cl(U)) \neq \emptyset$ for each open set U in X containing x. The union of all $\delta-$open sets in X that are contained in $A \subseteq X$ is called the $\delta-$interior of A and is denoted by $Int_\delta(A)$. The family of all regularly open (resp. regularly closed) sets of X is denoted by $RO(X)$ (resp. $RC(X)$). If $A \in RO(X)$ and $B \in RO(Y)$, then $A \times B \in RO(X \times Y)$, where X, Y are topological spaces.

In 1968, Singal and Singal [15] defined and investigated the notion of almost continuous mappings. They defined a function $f : X \to Y$ is said to be almost continuous if, for each $x \in X$ and for each open neighborhood V

of $f(x)$ in Y, there exists an open neighborhood U of x such that $f(U) \subseteq Int(Cl(V))$. In the same work, they proved a classical result saying that a function $f : X \to Y$ is almost continuous if and only if $f^{-1}(V)$ is open (resp. closed) in X, for every regular open (resp. regular closed) set V in Y. Since then, many authors have accomplished an excellent work in the field of almost continuous mappings.

3. Almost topological groups

Before we start our main work, we introduce some notations. By G, we mean the group $(G, *)$ where '$*$' is a binary operation on G under which G is a group. For $x, y \in G$, we denote $x * y$ by xy unless stated explicitly. We denote the inverse of an element x in G by x^{-1} (i.e., under addition operation on G, x^{-1} means $-x$ and under multiplication operation, x^{-1} retains its notation and meaning). Translation of a set A in a group $(G, *)$ (or simply, G) by an element g is denoted by gA, defined by $gA = \{g * a : a \in A\}$. In this section, we introduce the notion of almost topological groups, present hosts of examples of almost topological groups and investigate several of their basic properties. Along with other results, we prove that translation of a regularly open (resp. regularly closed) set in an almost topological group is regularly open (resp. regularly closed).

Definition 3.1. Let G be a group endowed with a topology τ such that

1. the mapping $\phi : G \times G \to G$ defined by $\phi(x, y) = xy$, and
2. the mapping $\psi : G \to G$ defined by $\psi(x) = x^{-1}$

are almost continuous, where $G \times G$ carries the product topology. Then the pair (G, τ) is called almost topological group. Equivalently, (G, τ) is an almost topological group if the following conditions are satisfied:

1. For each $x, y \in G$ and each regularly open set W in G containing xy, there exist open neighborhoods U and V of x and y, respectively, in G such that $U * V \subseteq W$, and
2. For each $x \in G$ and each regularly open set V in G containing x^{-1}, there exists an open neighborhood U of x in G such that $U^{-1} \subseteq V$.

Remark 3.1. For any $A, B \subseteq G$, we denote $A * B$ by AB and define as $AB = \{ab : a \in A, b \in B\}$ and $A^{-1} = \{a^{-1} : a \in A\}$. If $A = \{g\}$ for some $g \in G$, we denote $A * B$ by gB and $B * A$ by Ag.

Before we start discussing some general properties of almost topological groups. Let us present some examples of them.

Example 3.1. Consider the group $(G, +)$, where $G = \mathbb{R}$ be the set of real numbers, endowed with the standard topology $\mathcal{U}$. Then $(G,\mathcal{U})$ is an almost topological group.

Example 3.2. Let $(G, *)$ be any group with the discrete topology $\mathcal{D}$. Then $(G, \mathcal{D})$ is an almost topological group.

In general, it is obvious from the definition that every topological group is an almost topological group, but the converse is not true in general. The following are the examples of almost topological groups which are not topological groups.

Example 3.3. Consider the group $G = \mathbb{Z}_2$ with the topology $\tau = \{\emptyset, \{0\}, \mathbb{Z}_2\}$. Then (G, τ) is an almost topological group which is not a topological group.

In fact, if G is any group, A proper subset of G and $\tau = \{\emptyset, A, G\}$. Then (G, τ) is an almost topological group which is not a topological group.

Example 3.4. Let $G = \mathbb{R}$ be the additive group of real numbers and let τ be the topology on G generated by the family $\{(a, b) : a, b \in \mathbb{R}\} \cup \{(c, d) \cap D : c, d \in \mathbb{R}\}$ where D denotes the set of irrational numbers. Then (G, τ) is an almost topological group which is not a topological group.

Henceforth, we denote an almost topological group by (G, τ) (or simply G when there is no chance of confusion). We turn now to some general properties of almost topological groups.

Theorem 3.1. *Let (G, τ) be an almost topological group and let $g \in G$ be any element of G. Then:*

1. *the mapping $h_g : G \to G$ defined by $h_g(x) = gx$, $\forall x \in G$, is almost continuous;*
2. *the mapping $l_g : G \to G$ defined by $l_g(x) = xg$, $\forall x \in G$, is almost continuous.*

Proof. (1) Let x be any element of G and let W be a regular open set in G containing gx. By definition 3.1, there exist open neighborhoods U and V of g and x respectively, in G such that $UV \subseteq W$. In particular, $gV \subseteq W$ which means that $h_g(V) \subseteq W$. This indicates that h_g is almost continuous at x and hence h_g is almost continuous.

(2) Pick up $x \in G$ and let $W \in RO(G)$ containing xg. Then there exist open sets U in G containing x and V in G containing g such that $UV \subseteq W$. This gives $Ug \subseteq W$, i.e., $l_g(U) \subseteq W$. This shows that l_g is almost continuous at x. Since x was an arbitrary element of G, l_g is almost continuous. □

Theorem 3.2. *Let A be any regularly open set in an almost topological group (G, τ). The following are valid:*

1. *$gA \in RO(G)$, for all $g \in G$.*
2. *$Ag \in RO(G)$, for all $g \in G$.*
3. *$A^{-1} \in RO(G)$.*

Proof. (1) Firstly, we show that $gA \in \tau$. Let $x \in gA$ be any element. Then by definition of almost topological groups, there exist open neighborhoods U of g^{-1} and V of x in G such that $UV \subseteq A$. In particular, $g^{-1}V \subseteq A$. This is equivalent to the relation $V \subseteq gA$. This indicates that $x \in Int(gA)$ and thus,

$Int(gA) = gA$. That is, $gA \in \tau$. Consequently, $gA \subseteq Int(Cl(gA))$. With an eye to the necessity of the problem, we have to show that $Int(Cl(gA)) \subseteq gA$. Since A is open, $Cl(A) \in RC(G)$. By Theorem 3.1, $h_{g^{-1}} : G \to G$ is almost continuous and as a result of this, it follows that $gCl(A)$ is closed. Therefore, $Int(Cl(gA)) \subseteq Cl(gA) \subseteq gCl(A)$. This means that $g^{-1}Int(Cl(gA)) \subseteq Cl(A)$. Since $Int(Cl(gA))$ is regularly open, it follows that $g^{-1}Int(Cl(gA)) \subseteq Int(Cl(A)) = A$, i.e., $Int(Cl(gA)) \subseteq gA$. Thus $gA = Int(Cl(gA))$. Hence, job is done.

(2) The proof follows by similar arguments as in part (1) above.

(3) Let y be any element of A^{-1}. Then there exists an open set U in G containing y such that $U^{-1} \subseteq A \Rightarrow U \subseteq A^{-1}$. This proves that y is an interior point of A^{-1}. Hence A^{-1} is open. This results in $A^{-1} \subseteq Int(Cl(A^{-1}))$. It remains to prove $Int(Cl(A^{-1})) \subseteq A^{-1}$. Since A is open, $Cl(A)$ is regularly closed and hence $Cl(A)^{-1}$ is closed in G. Therefore, $Int(Cl(A^{-1})) \subseteq Cl(A^{-1}) \subseteq Cl(A)^{-1} \Rightarrow Int(Cl(A^{-1})) \subseteq Int(Cl(A))^{-1} = A^{-1}$. Thus, $A^{-1} = Int(Cl(A^{-1}))$, showing that $A^{-1} \in RO(G)$. □

Corollary 3.1. *Let F be any regularly closed set in an almost topological group G. Then:*

(1) *$gF \in RC(G)$, for each $g \in G$.*
(2) *$F^{-1} \in RC(G)$.*

Definition 3.2. A mapping $f : X \to Y$ is called almost open [15] if the image of any regularly open set is open, i.e., if for every $U \in RO(X)$, $f(U)$ is open in Y.

Remark 3.2. Due to Theorem 3.2, it immediately follows that the mappings h_g and l_g in Theorem 3.1 are almost open.

Theorem 3.3. *Let A be any regularly open set in an almost topological group G. Then:*

(1) *$Cl(gA) = gCl(A)$, for each $g \in G$.*
(2) *$Cl(Ag) = Cl(A)g$, for each $g \in G$.*
(3) *$Cl(A^{-1}) = Cl(A)^{-1}$.*

Proof. (1) Pick $x \in Cl(gA)$ and consider $y = g^{-1}x$. Let W be any open set in G containing y. Then there exist open sets U and V in G containing g^{-1} and x respectively, such that $UV \subseteq Int(Cl(W))$. By assumption, there is $a \in gA \cap V \Rightarrow g^{-1}a \in A \cap UV \subseteq A \cap Int(Cl(W)) \Rightarrow A \cap Int(Cl(W)) \neq \emptyset \Rightarrow A \cap Cl(W) \neq \emptyset$.

Since A is open, $A \cap W \neq \emptyset$. That is, $x \in gCl(A)$. For the converse, let y be any element of $gCl(A)$. Then $y = gx$ for some $x \in Cl(A)$. In order to show that $gCl(A) \subseteq Cl(gA)$, let W be an open set in G containing gx. Then there exist open sets U in G containing g and V in G containing x such that $UV \subseteq Int(Cl(W))$. Since $x \in Cl(A)$, $A \cap V \neq \emptyset$. There is $a \in A \cap V$. This gives $ga \in (gA) \cap Int(Cl(W)) \Rightarrow (gA) \cap Cl(W) \neq \emptyset$. By Theorem 3.2, gA

is open and thus $(gA) \cap W \neq \emptyset$. Therefore, $y \in Cl(gA)$. Combining the facts from above, we get $Cl(gA) = gCl(A)$.

(2) Following up the same steps as in part (1) above, we can readily show $Cl(Ag) = Cl(A)g$.

(3) Since $Cl(A)$ is regularly closed, $Cl(A)^{-1}$ is closed set in G. Therefore, the containment $A^{-1} \subseteq Cl(A)^{-1}$ implies that $Cl(A^{-1}) \subseteq Cl(A)^{-1}$. Next, let $y \in Cl(A)^{-1}$ be an arbitrary element. Then $y = x^{-1}$ for some $x \in Cl(A)$. Let V be any open set in G containing y. Then there exists an open set U in G such that $x \in U$ with $U^{-1} \subseteq Int(Cl(V))$. Also, there is $a \in A \cap U$ which yields $a^{-1} \in A^{-1} \cap Int(Cl(V))$. That is, $A^{-1} \cap Int(Cl(V)) \neq \emptyset \Rightarrow A^{-1} \cap Cl(V) \neq \emptyset \Rightarrow A^{-1} \cap V \neq \emptyset$, since A^{-1} is open. Therefore, $y \in Cl(A^{-1})$. Hence $Cl(A^{-1}) = Cl(A)^{-1}$. □

Theorem 3.4. *Let A be any semi-open set in an almost topological group G. Then:*

(1) $Cl(gA) \subseteq gCl(A)$, *for all* $g \in G$.
(2) $Cl(Ag) \subseteq Cl(A)g$, *for all* $g \in G$.
(3) $Cl(A^{-1}) \subseteq Cl(A)^{-1}$.

Proof. Here we give the proof of part (1) only. The proof for part (2) and (3) follow along similar lines.

(1) Since A is semi-open, $Cl(A)$ is regularly closed. By Theorem 3.1, $h_{g^{-1}} : G \to G$ is almost continuous. Therefore, $gCl(A)$ is closed. Hence $Cl(gA) \subseteq gCl(A)$. □

Theorem 3.5. *Let A be any regularly closed subset of an almost topological group G. The following assertions are valid:*

(1) $Int(gA) = gInt(A)$, *for all* $g \in G$.
(2) $Int(Ag) = Int(A)g$, *for all* $g \in G$.
(3) $Int(A^{-1}) = Int(A)^{-1}$.

Proof. (1) Since A is regularly closed, $Int(A)$ is regularly open in G. Consequently, $gInt(A) \subseteq Int(gA)$. Conversely, let $y \in Int(gA)$ be arbitrary. Suppose that $y = gx$ for some $x \in A$. By hypothesis, it follows that gA is closed and thereby $Int(gA)$ is regularly open in G. Let U and V be open sets in G containing g and x respectively, such that $UV \subseteq Int(gA)$. Then $gV \subseteq gA$, whence it follows that $gV \subseteq gInt(A)$. Thus, $Int(gA) \subseteq gInt(A)$. Hence the assertion follows.

Similarly, part (2) and (3) can be proved. □

Theorem 3.6. *Let A be both semi-open and semi-closed subset of an almost topological group G. Then the following are valid:*

(1) $Cl(gA) = gCl(A)$, *for each* $g \in G$.
(2) $Cl(Ag) = Cl(A)g$, *for each* $g \in G$.
(3) $Cl(A^{-1}) = Cl(A)^{-1}$.

Proof. We only prove (1) and (3). The proof of part (2) follows along similar lines.

(1) Since A is semi-open, $Cl(A)$ is regularly closed, wherefrom it follows that $Cl(gA) \subseteq gCl(A)$. Further, semi-openness of A yields $Cl(A) = Cl(Int(A)) \Rightarrow gCl(A) = gCl(Int(A))$. Since A is semi-closed, $Int(A)$ is regularly open in G. By Theorem 3.4, $gCl(A) = gCl(Int(A)) = Cl(gInt(A)) \subseteq Cl(gA)$. From above calculations, we conclude that $Cl(gA) = gCl(A)$.

(3) By hypothesis, it follows that $Cl(A)$ is regularly closed and hence $Cl(A)^{-1}$ is closed. Consequently, $Cl(A^{-1}) \subseteq Cl(A)^{-1}$. On other way round, since A is semi-open, $Cl(A) = Cl(Int(A)) \Rightarrow Cl(A)^{-1} = Cl(Int(A))^{-1}$. Also, since A is semi-closed, $Int(A)$ is regularly open. By Theorem 3.3, $Cl(A)^{-1} = Cl(Int(A)^{-1}) \subseteq Cl(A^{-1})$. This proves that $Cl(A^{-1}) = Cl(A)^{-1}$. □

Using similar arguments, we obtain the following result:

Theorem 3.7. *Under the same hypothesis of Theorem* 3.6*, the following are valid:*

(1) $Int(gA) = gInt(A)$, *for each* $g \in G$.
(2) $Int(Ag) = Int(A)g$, *for each* $g \in G$.
(3) $Int(A^{-1}) = Int(A)^{-1}$.

Proof. (1) Since A is semi-closed, $Int(A)$ is regularly open. By Theorem 3.1, $h_{g^{-1}} : G \to G$ is almost continuous. Therefore, $h_{g^{-1}}^{-1}(Int(A)) = gInt(A)$ is open. Thus, $gInt(A) \subseteq Int(gA)$. Next, by hypothesis, it follows that $Int(A) = Int(Cl(A)) \Rightarrow gInt = gInt(Cl(A))$. Since A is semi-open, $Cl(A)$ is regularly closed. By Theorem 3.5, $gInt(Cl(A)) = Int(gCl(A)) \supseteq Int(gA)$. That is, $gInt(A) \supseteq Int(gA)$. Therefore, $Int(gA) = gInt(A)$.

Analogously, (2) and (3) can be proved. □

Remark 3.3. By virtue of Examples 3.3 and 3.4, we can observe that both conditions 'semi-openness' and 'semi-closedness' on the set A in Theorem 3.6 and Theorem 3.7 are essential.

Theorem 3.8. *Let A be any open set in an almost topological group G. Then $gA \subseteq Int(gInt(Cl(A)))$ for each $g \in G$.*

Proof. Since A is open, $A \subseteq Int(Cl(A)) \Rightarrow gA \subseteq gInt(Cl(A))$. By Theorem 3.2, $gInt(Cl(A))$ is open (in fact, regularly open). Hence $gA \subseteq Int(gInt(Cl(A)))$. □

In a similar style, we obtain the following result.

Theorem 3.9. *Let B be any closed subset of an almost topological group. Then $Cl(gCl(Int(B))) \subseteq gB$ for each $g \in G$.*

Proof. Petty. □

Theorem 3.10. *Let G be an almost topological group. For any $A \subseteq G$, the following hold:*

(1) $Cl(gA) \subseteq gCl_\delta(A)$*, for each* $g \in G$.

(2) $gInt_\delta(A) \subseteq Int(gA)$*, for each* $g \in G$.

Proof. (1) Let x be ny element of $Cl(gA)$. Consider $y = g^{-1}x$ and let W be any open neighborhood of y in G. Then there exist open neighborhoods U of g^{-1} and V of x in G such that $UV \subseteq Int(Cl(W))$.

Since $x \in Cl(gA)$, there is $p \in (gA) \cap V$ and thus we always have, $g^{-1}p \in A \cap (UV) \subseteq A \cap Int(Cl(W)) \Rightarrow A \cap Int(Cl(W)) \neq \emptyset$. Therefore, $y \in Cl_\delta(A)$; that is, $x \in gCl_\delta(A)$.

(2) Let $y \in gInt_\delta(A)$. Then $y = gx$ for some $x \in Int_\delta(A)$. This means that there exists a regular open set U in G such that $x \in U \subseteq A$. In a sense, $Int(Cl(U)) \subseteq A \Rightarrow gInt(Cl(U)) \subseteq gA$. This gives $y = gx \in gInt(Cl(U)) \subseteq gA$. By Theorem 3.2, $gInt(Cl(U))$ is open and thus, $y \in Int(gA)$. Hence $gInt_\delta(A) \subseteq Int(gA)$. □

Definition 3.3. A mapping $f : X \to Y$ is called R-continuous [2, 3] if for each $x \in X$ and each regularly open set V in Y containing $f(x)$, there exists a regularly open set U in X containing x such that $f(U) \subseteq V$.

Theorem 3.11. *Let G and H be almost topological groups and $f : G \to H$ be a group homomorphism. If f is R-continuous at e_G, then f is almost continuous everywhere.*

Proof. Let x be any element of G. Let V be a regularly open set in H containing $y = f(x)$. By Theorem 3.2, $y^{-1}V$ is regularly open set in H containing e_H. By hypothesis, there exists a regularly open set U in G containing e_G such that $f(U) \subseteq y^{-1}V$. This implies that $f(xU) \subseteq V$. By above observations, xU is open, and thereby it follows that f is almost continuous at x. Hence f is almost continuous everywhere. □

Theorem 3.12. *Let A be an open set in an almost topological group G. Then $Cl_\delta(gA) = gCl_\delta(A)$ for each $g \in G$.*

Proof. Since $Cl_\delta(A)$ is regularly closed, by Corollary 3.2.1, $gCl_\delta(A)$ is regularly closed. Consequently, $Cl_\delta(gA) \subseteq gCl_\delta(A)$. For the reverse inclusion, let $y \in gCl_\delta(A)$. Then $y = gx$ for some $x \in Cl_\delta(A)$. For any open neighborhood W of y, we obtain open neighborhoods U of g and V of x in G satisfying $UV \subseteq Int(Cl(W))$. Our aim is to show $(gA) \cap Int(Cl(W)) \neq \emptyset$. Now, since $x \in Cl_\delta(A)$, $A \cap Int(Cl(V)) \neq \emptyset \Rightarrow A \cap Cl(V) \neq \emptyset$. By hypothesis, $A \cap V \neq \emptyset$. Let $h \in A \cap V$. From this, we have $gh \in (gA) \cap (UV) \subseteq (gA) \cap Int(Cl(W)) \Rightarrow (gA) \cap Int(Cl(W)) \neq \emptyset$. Hence $y \in Cl_\delta(gA)$. This proves $gCl_\delta(A) \subseteq Cl_\delta(gA)$. In conclusion, $Cl_\delta(gA) = gCl_\delta(A)$. □

A regularly open subset U of an almost topological group G containing $x \in G$ is called regularly open neighborhood of x. The collection of all regularly open neighborhoods of $x \in G$ is denoted by $\mathcal{N}_x$.

Theorem 3.13. *Let G be an almost topological group. Then for every $V \in \mathcal{N}_{e_G}$, there exists an open neighborhood U of e_G such that $UU \subseteq V$.*

Proof. Since $e_G * e_G = e_G$, there exist open neighborhoods A and B of e_G such that $AB \subseteq V$. Let $U = A \cap B$. Then U is open neighborhood of e_G such that $UU \subseteq V$. It completes the proof. ☐

Theorem 3.14. *Let G be an almost topological group. Then for every $V \in \mathcal{N}_x$, there exists an open neighborhood U of e_G such that $xUU \subseteq V$.*

Proof. By Theorem 3.2, $x^{-1}V$ is regularly open neighborhood of e_G. By Theorem 3.13, there exists an open neighborhood U of e_G such that $UU \subseteq x^{-1}V \Rightarrow xUU \subseteq V$. ☐

Theorem 3.15. *Let G be an almost topological group. Then for every $U \in \mathcal{N}_{e_G}$, $Cl(U) \subseteq U^{-1}U$.*

Proof. Let $x \in Cl(U)$. Since xU is open neighborhood of x, $U \cap (xU) \neq \emptyset$. Therefore, we obtain an equation, $g = xh$ for some $g, h \in U$. This gives $x = h^{-1}g \in U^{-1}U$. Hence the assertion follows. ☐

Definition 3.4. A topological space X is called almost regular [13] if for each regularly closed set F in X and each $x \notin F$, there exist disjoint open sets U and V in X such that $x \in U$ and $F \subseteq V$.

Conjecture. Every almost topological group is almost regular.

Definition 3.5. A topological space X is called nearly compact [14] if every open cover of X has a finite subcover the interiors of the closures whose members cover X.

Theorem 3.16. *If A and B are compact subsets of an almost topological group G, then AB is nearly compact.*

Proof. Let A and B be compact subsets of G. Since the image of a compact set under almost continuous mappings is nearly compact, by definition of an almost topological group, it follows that AB is nearly compact. ☐

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