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Characterization of triple χ^3 sequence spaces via Orlicz functions

N. Subramanian and A. Esi

Abstract. In this paper we study of the characterization and general properties of triple gai sequence via Orlicz space of χ^3_M of χ^3 establishing some inclusion relations.

1. Introduction

Throughout w, χ and Λ denote the classes of all, gai and analytic scalar valued single sequences, respectively. We write w^3 for the set of all complex sequences (x_{mnk}), where $m, n, k \in \mathbb{N}$, the set of positive integers. Then, w^3 is a linear space under the coordinate wise addition and scalar multiplication.

We can represent triple sequences by matrix. In case of double sequences we write in the form of a square. In the case of a triple sequence it will be in the form of a box in three dimensional case.

Some initial work on double series is found in *Apostol [1]* and double sequence spaces is found in *Hardy [5]*, *Subramanian et al. [10-12]*, and many others. Later on investigated by some initial work on triple sequence spaces is found in *Sahiner et al. [9]* , *Esi et al. [2-4]*, *Subramanian et al. [13-19]* and many others.

Let (x_{mnk}) be a triple sequence of real or complex numbers. Then the series $\sum_{m,n,k=1}^{\infty} x_{mnk}$ is called a triple series. The triple series $\sum_{m,n,k=1}^{\infty} x_{mnk}$ is said to be convergent if and only if the triple sequence (S_{mnk}) is convergent, where

$$S_{mnk} = \sum_{i,j,q=1}^{m,n,k} x_{ijq}(m, n, k = 1, 2, 3, \dots).$$

A sequence $x = (x_{mnk})$ is said to be triple analytic if

$$\sup_{m,n,k} |x_{mnk}|^{\frac{1}{m+n+k}} < \infty.$$

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The vector space of all triple analytic sequences are usually denoted by Λ^3. A sequence $x = (x_{mnk})$ is called triple entire sequence if

$$|x_{mnk}|^{\frac{1}{m+n+k}} \to 0 \text{ as } m, n, k \to \infty.$$

The vector space of all triple entire sequences are usually denoted by Γ^3. The space Λ^3 and Γ^2 is a metric space with the metric

$$d(x, y) = \sup_{m,n,k} \left\{ |x_{mnk} - y_{mnk}|^{\frac{1}{m+n+k}} : m, n, k : 1, 2, 3, \ldots \right\}, \quad (1)$$

for all $x = \{x_{mnk}\}$ and $y = \{y_{mnk}\}\, in\, \Gamma^3$.

Let $\phi = \{\text{finite sequences}\}$.

Consider a double sequence $x = (x_{mnk})$. The $(m, n, k)^{th}$ section $x^{[m,n,k]}$ of the sequence is defined by $x^{[m,n,k]} = \sum_{i,j,q=0}^{m,n,k} x_{ijq}\delta_{ijq}$ for all $m, n, k \in \mathbb{N}$,

$$\delta_{mnk} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & \cdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \\ 0 & 0 & \cdots & 1 & 0 & \cdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots \end{bmatrix}$$

with 1 in the $(m, n, k)^{th}$ position and zero other wise.

A sequence $x = (x_{mnk})$ is said to be triple gai sequence if

$$((m + n + k)! \, |x_{mnk}|)^{\frac{1}{m+n+k}} \to 0 \text{ as } m, n, k \to \infty.$$

The triple gai sequences will be denoted by χ^3.

Consider a triple sequence $x = (x_{mnk})$. The $(m, n, k)^{th}$ section $x^{[m,n,k]}$ of the sequence is defined by $x^{[m,n,k]} = \sum_{i,j,q=0}^{m,n,k} x_{ijq}\Im_{ijq}$ for all $m, n, k \in \mathbb{N}$; where $\Im_{ijq}$ denotes the triple sequence whose only non zero term is a $\frac{1}{(i+j+k)!}$ in the $(i, j, k)^{th}$ place for each $i, j, k \in \mathbb{N}$.

An FK-space(or a metric space)X is said to have AK property if $(\Im_{mnk})$ is a Schauder basis for X, or equivalently $x^{[m,n,k]} \to x$.

An FDK-space is a triple sequence space endowed with a complete metrizable; locally convex topology under which the coordinate mappings are continuous.

$$\delta_{mnk} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & \cdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \\ 0 & 0 & \cdots & 1 & 0 & \cdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots \end{bmatrix}$$

with 1 in the $(m, n, k)^{th}$ position and zero other wise.

A sequence $x = (x_{mnk})$ is said to be triple gai sequence if $((m + n + k)! \, |x_{mnk}|)^{\frac{1}{m+n+k}} \to 0$ as $m, n, k \to \infty$. The triple gai sequences will be denoted by χ^3.

Consider a triple sequence $x = (x_{mnk})$. The $(m,n,k)^{th}$ section $x^{[m,n,k]}$ of the sequence is defined by $x^{[m,n,k]} = \sum_{i,j,q=0}^{m,n,k} x_{ijq} \Im_{ijq}$ for all $m,n,k \in \mathbb{N}$; where $\Im_{ijq}$ denotes the triple sequence whose only non zero term is a $\frac{1}{(i+j+k)!}$ in the $(i,j,k)^{th}$ place for each $i,j,k \in \mathbb{N}$.

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If X is a sequence space, we give the following definitions:

(i) X' is continuous dual of X;

(ii) $X^{\alpha} = \left\{ a = (a_{mnk}) : \sum_{m,n,k=1}^{\infty} |a_{mnk} x_{mnk}| < \infty, \text{ for each } x \in X \right\}$;

(iii) $X^{\beta} = \left\{ a = (a_{mnk}) : \sum_{m,n,k=1}^{\infty} a_{mnk} x_{mnk} \text{ is convergent, for each } x \in X \right\}$;

(iv) $X^{\gamma} = \left\{ a = (a_{mn}) : \sup_{m,n \geq 1} \left| \sum_{m,n,k=1}^{M,N,K} a_{mnk} x_{mnk} \right| < \infty, \text{ for each } x \in X \right\}$;

(v) Let X be an FK-space $\supset \phi$; then $X^f = \left\{ f(\Im_{mnk}) : f \in X' \right\}$;

(vi) $X^{\delta} = \left\{ a = (a_{mnk}) : \sup_{m,n,k} |a_{mnk} x_{mnk}|^{1/m+n+k} < \infty, \text{ for each } x \in X \right\}$;

X^{α}, X^{β}, X^{γ} are called α - (or Köthe-Toeplitz) dual of X, β-(or generalized-Köthe-Toeplitz) dual of X, γ-dual of X, δ-dual of X respectively. X^{α} is defined by Gupta and Kamptan [10]. It is clear that $X^{\alpha} \subset X^{\beta}$ and $X^{\alpha} \subset X^{\gamma}$, but $X^{\alpha} \subset X^{\gamma}$ does not hold.

2. Definitions and Preliminaries

A sequence $x = (x_{mnk})$is said to be triple analytic if $\sup_{mnk} |x_{mnk}|^{\frac{1}{m+n+k}} < \infty$. The vector space of all triple analytic sequences is usually denoted by Λ^3. A sequence $x = (x_{mnk})$ is called triple entire sequence if $|x_{mnk}|^{\frac{1}{m+n+k}} \to 0$ as $m,n,k \to \infty$. The vector space of triple entire sequences is usually denoted by Γ^3. A sequence $x = (x_{mnk})$ is called triple gai sequence if $((m+n+k)! \, |x_{mnk}|)^{\frac{1}{m+n+k}} \to 0$ as $m,n,k \to \infty$. The vector space of triple gai sequences is usually denoted by χ^3. The space χ^3 is a metric space with the metric

$$d(x,y) = \sup_{m,n,k} \left\{ ((m+n+k)! \, |x_{mnk} - y_{mnk}|)^{\frac{1}{m+n+k}}, \quad m,n,k : 1,2,3,\ldots \right\} \tag{2}$$

for all $x = \{x_{mnk}\}$ and $y = \{y_{mnk}\}$ in χ^3.

Let w^3 denote the set of all complex double sequences $x = (x_{mnk})_{m,n,k=1}^{\infty}$ and $M : [0,\infty) \to [0,\infty)$ be an Orlicz function. Given a triple sequence,

$x \in w^3$. Define the sets

$$\chi^3_M = \Big\{x \in w^3 : \left(M\left(\frac{((m+n+k)!\,|x_{mnk}|)^{\frac{1}{m+n+k}}}{\rho}\right)\right) \to 0, \text{ as } m,n,k \to \infty \text{ for some } \rho > 0\Big\}$$

and

$$\Lambda^3_M = \Big\{x \in w^3 : \sup_{m,n,k \geq 1} \left(M\left(\frac{|x_{mnk}|^{\frac{1}{m+n+k}}}{\rho}\right)\right) < \infty \text{ for some } \rho > 0\Big\}.$$

The space Λ^3_M is a metric space with the metric

$$d(x,y) = \inf\left\{\rho > 0 : \sup_{m,n,k \geq 1} \left(M\left(\frac{|x_{mnk} - y_{mnk}|}{\rho}\right)\right)^{\frac{1}{m+n+k}} \leq 1\right\}$$

The space χ^3_M is a metric space with the metric

$$\widetilde{d}(x,y) = \inf\left\{\rho > 0 : \sup_{m,n,k \geq 1} \left(M\left(\frac{(m+n+k)!|x_{mnk} - y_{mnk}|}{\rho}\right)\right)^{\frac{1}{m+n+k}} \leq 1\right\}.$$

This paper is a study of the characterization and general properties of gai sequences via triple Orlicz space of χ^3_M of χ^3 establishing some inclusion relations.

3. MAIN RESULTS

Proposition 3.1. *If M is a Orlicz function, then χ^3_M is a linear set over the set of complex numbers $\mathbb{C}$.*

Proof. It is trivial. Therefore, the proof is omitted. □

Proposition 3.2. $\left(\chi^3_M\right)^\delta \overset{\subset}{\neq} \Lambda^3_M$

Proof. Let $y \in \delta-$ dual of χ^3_M. Then $\left(M\left(\frac{|x_{mnk}y_{mnk}|}{\rho}\right)\right) \leq M^{m+n+k}$ for some constant $M > 0$ and for each $x \in \chi^3_M$. Therefore, $\left(M\left(\frac{|y_{mnk}|}{\rho}\right)\right) \leq M^{m+n+k}$ for each m,n,k by taking $x = (\Im_{mnk})$. This implies that $y \in \Lambda^3_M$. Thus,

$$\left(\chi^3_M\right)^\delta \subset \Lambda^3_M. \tag{3}$$

We now choose $M = \text{id}$ and define the triple sequences (y_{mnk}) and (x_{mnk}) by $(y_{mnk}) = 1$ for all m,n and k, and by

$(m+2)!x_{m11} = 2^{(m+2)^2}$ and $(m+n+k)!x_{mnk} = 0 (n,k \geq 2)$ for all $m = 1,2,\ldots$

Obviously, $y \in \Lambda^3_M$ and since $(m+n+k)!x_{mnk} = 0$ for all $m,n,k \geq 0$, $(m+n+k)!(x_{mnk})$ converges to zero. Hence, $x \in \chi^3_M$. But

$$((m+2)!\,|a_{m11}x_{m11}|)^{\frac{1}{m+n+k}} = 2^{m+2} \to \infty \text{ as } m \to \infty,$$

hence

$$x \notin \left(\chi_M^3\right)^\delta . \tag{4}$$

From (3) and (4), we are granted $\left(\chi_M^3\right)^\delta \overset{\subset}{\neq} \Lambda_M^3$.

This completes the proof. □

Proposition 3.3. *The dual space of* χ_M^3 *is* Λ_M^3. *In other words* $(\chi_M^3)^* = \Lambda_M^3$.

Proof. We recall that

$$\Im_{mnk} = \begin{pmatrix} 0 & 0 & & \cdots & 0 & 0 & \cdots \\ 0 & 0 & & \cdots & 0 & 0 & \cdots \\ \vdots & \vdots & & \ddots & \vdots & \vdots & \ddots \\ 0 & 0 & \cdots & \frac{1}{(m+n+k)!} & 0 & \cdots & \\ 0 & 0 & & \cdots & 0 & 0 & \cdots \end{pmatrix}$$

with $\frac{1}{(m+n+k)!}$ in the $(m,n,k)th$ position and zero's else where, with

$$x = \Im_{mnk},$$

$$\left\{ M\left(\frac{((m+n+k)!\,|x_{mnk}|)^{\frac{1}{m+n+k}}}{\rho}\right)\right\} =$$

$$\begin{pmatrix} M(0^{1/3}/\rho)\vdots & M(0^{1/1+n+k}/\rho) & \\ \vdots & \ddots & \vdots \\ M(0^{1/m+4}/\rho) & \underset{(m,n)^{th}}{M((\frac{1}{(m+n+k)!})^{\frac{1}{m+n+k}}/\rho)} & M(0^{1/m+n++k+2}/\rho) \\ M(0^{1/m+4}/\rho) & \cdots & M(0^{1/m+n+k+4}/\rho) \end{pmatrix} =$$

$$\begin{pmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \underset{(m,n,k)^{th}}{M\left(\left(\frac{1}{(m+n+k)!}\right)^{\frac{1}{m+n+k}}/\rho\right)} & 0 \\ 0 & \cdots & 0 \end{pmatrix},$$

which is a triple gai sequence. Hence, $\Im_{mnk} \in \chi_M^3$, $f(x) = \sum_{m,n,k=1}^{\infty} x_{mnk}y_{mnk}$ with $x \in \chi_M^3$ and $f \in (\chi_M^3)^*$, where $(\chi_M^3)^*$ is the dual space of χ_M^3.

Take $x = (x_{mnk}) = \Im_{mnk} \in \chi_M^3$. Then,

$$|y_{mnk}| \leq \|f\|\, d(\Im_{mnk}, 0) < \infty \quad \forall m,n,k. \tag{5}$$

Thus, (y_{mnk}) is a bounded sequence and hence an triple analytic sequence. In other words, $y \in \Lambda_M^3$. Therefore $(\chi_M^3)^* = \Lambda_M^3$. This completes the proof. □

Proposition 3.4. $\left(\Lambda^3_M\right)^\beta \overset{\subset}{\neq} \chi^3_M$

Proof. Step 1: Let $(x_{mnk}) \in \left(\Lambda^3_M\right)^\beta$,

$$\sum_{m,n,k=1}^{\infty} |x_{mnk}y_{mnk}| < \infty \forall \left(y_{mnk}\right) \in \Lambda^3_M. \tag{6}$$

Let us assume that $(x_{mnk}) \notin \chi^3_M$. Then, there exists a strictly increasing sequence of positive integers $(m_p + n_p + k_p)$ such that

(7)
$$\left(M\left(\frac{(m_p+n_p+k_p)!\left|x_{(m_p+n_p+k_p)}\right|}{\rho}\right)\right) > \frac{1}{2^{(m_p+n_p+k_p)}}, \quad (p = 1, 2, 3, \dots).$$

Let

$$(m_p+n_p+k_p)!y_{(m_p+n_p+k_p)} = 2^{(m_p+n_p+k_p)} \text{ for } (p = 1, 2, 3, \dots),$$
$$y_{mnk} = 0 \text{ otherwise.}$$

Then $(y_{mnk}) \in \Lambda^3_M$. However,

$$\sum_{m,n,k=1}^{\infty}\left(M\left(\frac{|x_{mnk}y_{mnk}|}{\rho}\right)\right) =$$
$$= \sum_{p=1}^{\infty}\left(M\left(\frac{(m_p+n_p+k_p)!\left|x_{(m_pn_pk_p)}y_{(m_pn_pk_p)}\right|}{\rho}\right)\right) >$$
$$> 1 + 1 \dots$$

We know that the infinite series $1 + 1 + 1 + \dots$ diverges. Now we choose M=id, where id is the identity and hence $\sum_{m,n,k=1}^{\infty}\left(M\left(|x_{mnk}y_{mnk}|/\rho\right)\right)$ diverges. This contradicts (6). Hence $(x_{mnk}) \in \chi^3_M$. Therefore,

$$\left(\Lambda^3_M\right)^\beta \subset \chi^3_M. \tag{8}$$

If we now choose $M = \text{id}$, where id is the identity and $y_{1nk} = x_{1nk} = 1$ and $y_{mnk} = x_{mnk} = 0$ $(m > 1)$ for all n, k, then obviously $x \in \chi^3_M$ and $y \in \Lambda^3_M$, but $\sum_{m,n,k=1}^{\infty} x_{mnk}y_{mnk} = \infty$. Hence,

$$y \notin \left(\Lambda^3_M\right)^\beta. \tag{9}$$

From (8) and (9), we are granted $\left(\Lambda^3_M\right)^\beta \overset{\subset}{\neq} \chi^3_M$.
This completes the proof. □

Definition 3.5. Let $p = (p_{mnk})$ be a triple sequence of positive real numbers. Then

$$\chi^3_M(p) = \left\{x = (x_{mnk}) : \left(M\left(\frac{((m+n+k)!|x_{mnk}|)^{\frac{1}{m+n+k}}}{\rho}\right)\right)^{p_{mnk}} \to 0, \quad (m,n,k \to \infty)\right\} \tag{10}$$

for some $\rho > 0$. Suppose that p_{mnk} is a constant for all m,n,k then $\chi^3_M(p) = \chi^3_M$.

Proposition 3.6. *Let $0 \leq p_{mnk} \leq q_{mnk}$ for all $m,n,k \in \mathbb{N}$ and let $\left\{\frac{q_{mnk}}{p_{mnk}}\right\}$ be bounded. Then $\chi^3_M(q) \subset \chi^3_M(p)$.*

Proof. Let

$$x \in \chi^3_M(q), \tag{11}$$

then

$$\left(M\left(\frac{((m+n+k)!\,|x_{mnk}|)^{\frac{1}{m+n+k}}}{\rho}\right)\right)^{q_{mnk}} \to 0, \text{ as } m,n,k \to \infty. \tag{12}$$

Let $t_{mnk} = (M(((m+n+k)!\,|x_{mnk}|)\,1/m+n+k/\rho))^{q_{mnk}}$, and let $\gamma_{mnk} = p_{mnk}/q_{mnk}$. Since $p_{mnk} \leq q_{mnk}$, we have $0 \leq \gamma_{mnk} \leq 1$. Let $0 < \gamma < \gamma_{mnk}$, then

$$\begin{aligned} u_{mnk} &= \begin{cases} t_{mnk}, & \text{if } (t_{mnk} \geq 1) \\ 0, & \text{if } (t_{mnk} < 1) \end{cases} \\ v_{mnk} &= \begin{cases} 0, & \text{if } (t_{mnk} \geq 1) \\ t_{mnk}, & \text{if } (t_{mnk} < 1) \end{cases} \\ t_{mnk} &= u_{mnk} + v_{mnk}, \quad t^{\gamma_{mnk}}_{mnk} = u^{\gamma_{mnk}}_{mnk} + v^{\gamma_{mnk}}_{mnk}. \end{aligned} \tag{13}$$

Now, it follows that

$$u^{\gamma_{mnk}}_{mnk} \leq u_{mnk} \leq t_{mnk}, \quad v^{\gamma_{mnk}}_{mnk} \leq u^{\gamma}_{mnk}. \tag{14}$$

Since $t^{\gamma_{mnk}}_{mnk} = u^{\gamma_{mnk}}_{mnk} + v^{\gamma_{mnk}}_{mnk}$, we have $t^{\gamma_{mnk}}_{mnk} \leq t_{mnk} + v^{\gamma}_{mnk}$. Thus,

$$\begin{aligned} &\left(M\left(((m+n+k)!\,|x_{mnk}|)^{1/m+n+k}/\rho\right)^{q_{mnk}}\right)^{\gamma_{mnk}} \\ &\leq \left(M\left(((m+n+k)!\,|x_{mnk}|)^{1/m+n+k}/\rho\right)\right)^{q_{mnk}}, \\ &\left(M\left(((m+n+k)!\,|x_{mnk}|)^{1/m+n+k}/\rho\right)^{q_{mnk}}\right)^{p_{mnk}/q_{mnk}} \\ &\leq \left(M\left(((m+n+k)!\,|x_{mnk}|)^{1/m+n+k}/\rho\right)\right)^{q_{mnk}}, \end{aligned} \tag{15}$$

which yields

$$\left(M\left(((m+n+k)!\,|x_{mnk}|)^{1/m+n+k}/\rho\right)\right)^{p_{mnk}}$$

$$\leq \left(M\left(\left((m+n+k)!\left|x_{mnk}\right|\right)^{1/m+n+k}/\rho\right)\right)^{q_{mnk}}.$$

However,

$$\left(M\left(\left((m+n+k)!\left|x_{mnk}\right|\right)^{1/m+n+k}/\rho\right)\right)^{q_{mnk}} \to 0 \quad (\text{by}(12)).$$

Thus,

$$\left(M\left(\left((m+n+k)!\left|x_{mnk}\right|\right)^{1/m+n+k}/\rho\right)\right)^{p_{mnk}} \to 0 \text{ as } m,n,k \to \infty.$$

Hence,

$$x \in \chi^3_M(p). \tag{16}$$

Hence (11) and (16), we are granted

$$\chi^3_M(q) \subset \chi^3_M(p). \tag{17}$$

This completes the proof. □

Proposition 3.7. *(a) Let* $0 < \inf p_{mnk} \leq p_{mnk} \leq 1$*, then* $\chi^3_M(p) \subset \chi^3_M$*.*
(b) If $1 \leq p_{mnk} \leq \sup_{mnk} < \infty$*, then* $\chi^3_M \subset \chi^3(p)$*.*

Proof. The above statements are special cases of Proposition 3.6. Therefore, it can be proved by similar arguments. □

Proposition 3.8. *If* $0 < p_{mnk} \leq q_{mnk} < \infty$ *for each* m,n,k *then* $\chi^3_M(p) \subseteq \chi^3(q)$*.*

Proof. Let $x \in \chi^3_M(p)$, then

$$\left(M\left(\left((m+n+k)!\left|x_{mnk}\right|\right)^{1/m+n+k}\right)\right)^{p_{mnk}} \to 0, \text{ as } m,n,k \to \infty. \tag{18}$$

This implies that $\left(M\left(\left((m+n+k)!\left|x_{mnk}\right|\right)^{1/m+n+k}/\rho\right)\right) \leq 1$ for sufficiently large m,n,k. Since M is non-decreasing, we get

$$\begin{aligned} &\left(M\left(\left((m+n+k)!\left|x_{mnk}\right|\right)^{1/m+n+k}/\rho\right)\right)^{q_{mnk}} \\ &\leq \left(M\left(\left((m+n+k)!\left|x_{mnk}\right|\right)^{1/m+n+k}/\rho\right)\right)^{p_{mnk}}, \end{aligned} \tag{19}$$

then $\left(M\left(\left((m+n+k)!\left|x_{mnk}\right|\right)^{1/m+n+k}/\rho\right)\right)^{q_{mnk}} \to 0$ as $m,n,k \to \infty$ (by using (18)).

Let $x \in \chi^3_M(q)$. Hence, $\chi^3_M(p) \subseteq \chi^3(q)$. This completes the proof. □

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N. SUBRAMANIAN
DEPARTMENT OF MATHEMATICS
SASTRA UNIVERSITY
THANJAVUR-613 401
INDIA
E-mail address: nsmaths@yahoo.com

A. ESI
ADIYAMAN UNIVERSITY
ADIYAMAN
TURKEY
E-mail address: aesi23@hotmail.com