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Some Fixed Point Theorems for (CAB)-contractive Mappings and Related Results

Arslan Hojat Ansari, Maher Berzig, and Sumit Chandok*

Abstract. In this paper, we introduced the concept of (CAB)-contractive mappings and provide sufficient conditions for the existence and uniqueness of a fixed point for such class of generalized nonlinear contractive mappings in metric spaces and several interesting corollaries are deduced. Also, as application, we obtain some results on coupled fixed points, fixed point on metric spaces endowed with N-transitive binary relation and fixed point for cyclic mappings. The proved results generalize and extend various well-known results in the literature.

1. Introduction and Preliminaries

Fixed point theory is one of the traditional branch of nonlinear analysis. The importance of fixed point theory has been increasing rapidly over the time as this theory provide useful tools for proving the existence and uniqueness of the solutions to various mathematical models (integral and partial differential equations, variational inequalities etc). Also, it has a broad range of application potential in various fields such as engineering, economics, computer science, and many others.

It is well known that the contractive-type conditions are very indispensable in the study of fixed point theory and Banach's fixed point theorem [1] for contraction mappings is one of the pivotal result in analysis. This theorem that has been extended and generalized by various authors (see, e.g., [2],[7],[8],[9],[15],[17],[28]) and has many applications in mathematics and other related disciplines as well. In [26], Samet and Turinici extended and generalized the Banach contraction principle to spaces endowed with an arbitrary binary relation, and they unified many known results. Recently, there have been so many exciting developments in the field of existence of fixed point in partially ordered metric spaces and fixed point for cyclic mappings. For more details, we refer the reader to the Bhaskar et al. [6], Berzig

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*Corresponding author.

et al. [4], Chandok et al. [10, 11, 13, 12, 14], Karapinar et al. [18], Kim et al. [20], Nieto et al. [22, 23], O'Regan et al. [24], Ran et al. [25], Samet et al. [27], and Turinici [29].

In this paper, we introduced the concept of (CAB)-contractive mappings, which generalize well exist nonlinear contractive type mappings. These classes of mappings are used to obtain some fixed point theorems in metric spaces by generalizing and extending some well-known results. Moreover, several interesting corollaries are deduced for coupled fixed point, fixed point on metric spaces endowed with N-transitive binary relation and fixed point for cyclic mappings. Finally, we prove that some existing results in the literature are particular cases from our main theorems.

To begin with, first we give some definitions and notations which will be used in the sequel.

Definition 1.1 (see [19])**.** A function $\psi : [0, \infty) \to [0, \infty)$ is called altering distance function if the following properties are satisfied:

(a1) ψ is continuous and non-decreasing;
(a2) $\psi(t) = 0$ if and only if $t = 0$.

We denote Ψ the set of all altering distance functions.

Definition 1.2. The pair of functions (ψ, ϕ) is a pair of *generalized altering distance* where $\psi, \phi : [0, +\infty) \to [0, +\infty)$ if the following hypotheses hold:

(a1) ψ is continuous and non-decreasing;
(a2) $\lim_{n\to\infty} \phi(t_n) = 0 \Rightarrow \lim_{n\to\infty} t_n = 0$.

Definition 1.3. A mapping $h : [0, +\infty) \to [0, +\infty)$ is an A-*class* function if $h(t) \geq t, \forall t \geq 0$.
We denote $\mathcal{A}$ the set of all $\mathcal{A}$-class functions.

Example 1.1. The following functions $h : [0, +\infty) \to [0, +\infty)$ are elements of $\mathcal{A}$:

(1) $h(t) = a^t - 1$, $a > 1$, $t \in [0, +\infty)$;
(2) $h(t) = mt$, $m \geq 1$, $t \in [0, +\infty)$.

Definition 1.4 ([3])**.** Let X be a set, and let $\mathcal{R}$ be a binary relation on X. A mapping $T : X \to X$ is an $\mathcal{R}$-preserving mapping if $x, y \in X : x\mathcal{R}y \Rightarrow Tx\mathcal{R}Ty$.

In the sequel, let $\mathbb{N}$ denote the set of all non-negative integers, let $\mathbb{R}$ denote the set of all real numbers.

Definition 1.5 ([3])**.** Let $N \in \mathbb{N}$. $\mathcal{R}$ is N-transitive on X if $x_0, x_1, \ldots, x_{N+1} \in X : x_i\mathcal{R}x_{i+1}$ for all $i = \{0, 1, \ldots, N\} \Rightarrow x_0\mathcal{R}x_{N+1}$.

The following remark is a consequence of the previous definition.

Remarks 1.1. Let $N \in \mathbb{N}$. We have:

(i) If $\mathcal{R}$ is transitive, then it is N-transitive for all $N \in \mathbb{N}$;

(ii) If $\mathcal{R}$ is N-transitive, then it is k N-transitive for all $k \in \mathbb{N}$.

Definition 1.6 ([5])**.** Let (X, d) be a metric space and $\mathcal{R}_1, \mathcal{R}_2$ two binary relations on X. A metric space (X, d) is $(\mathcal{R}_1, \mathcal{R}_2)$-regular if for every sequence $\{x_n\}$ in X such that $x_n \to x \in X$ as $n \to +\infty$, and $x_n\mathcal{R}_1x_{n+1}, x_n\mathcal{R}_2x_{n+1}$ for all $n \in \mathbb{N}$, there exists a subsequence $\{x_{n(k)}\}$ such that $x_{n(k)}\mathcal{R}_1x, x_{n(k)}\mathcal{R}_2x$ for all $k \in \mathbb{N}$.

Definition 1.7 ([3])**.** A subset D of X is $(\mathcal{R}_1, \mathcal{R}_2)$-directed if for all $x, y \in D$, there exists $z \in X$ such that $(x\mathcal{R}_1z) \wedge (y\mathcal{R}_1z)$ and $(x\mathcal{R}_2z) \wedge (y\mathcal{R}_2z)$.

Definition 1.8. Let X be a set and $\alpha, \beta : X \times X \to [0, +\infty)$ are two mappings. We define two binary relations $\mathcal{R}_1$ and $\mathcal{R}_2$ on X by

$$x\mathcal{R}_1y \iff \alpha(x, y) \leq 1 \quad \text{and} \quad x\mathcal{R}_2y \iff \beta(x, y) \geq 1.$$

for all $x, y \in X$.

2. Main Results

Definition 2.1. A mapping $f : [0, \infty)^4 \to \mathbb{R}$ is a 1-1-upclass function if the following conditions hold for all $u, v, s, t \in [0, \infty)$

(1) $f(1, 1, s, t)$ is continuous;
(2) $0 \leq u \leq 1, v \geq 1 \Rightarrow f(u, v, s, t) \leq f(1, 1, s, t) \leq s$;
(3) $f(1, 1, s, t) = s \Rightarrow s = 0$ or $t = 0$.

We denote $\mathcal{C}$ the set of all 1-1-upclass functions.

Example 2.1. The following functions $f : [0, \infty)^4 \to \mathbb{R}$ are elements of $\mathcal{C}$ for all $u, v, s, t \in [0, \infty)$:

(1) $f(u, v, s, t) = us - vt,\ f(1, 1, s, t) = s \Rightarrow t = 0$;
(2) $f(u, v, s, t) = \dfrac{us - vt}{1 + vt},\ f(1, 1, s, t) = s \Rightarrow t = 0$;
(3) $f(u, v, s, t) = \dfrac{us}{1 + vt},\ f(1, 1, s, t) = s \Rightarrow s = 0$ or $t = 0$;
(4) $f_a(u, v, s, t) = \log_a \dfrac{ut + a^{us}}{1 + vt},\ a > 1,\ f_a(1, 1, s, t) = s \Rightarrow s = 0$ or $t = 0$;
(5) $f(u, v, s, t) = \ln \dfrac{u + e^{us}}{1 + v},\ f(1, 1, s, 1) = s \Rightarrow s = 0$;
(6) $f_a(u, v, s, t) = (us + a)^{\frac{1}{1+vt}} - a,\ a > 1,\ f_a(1, 1, s, t) = s \Rightarrow t = 0$;
(7) $f_a(u, v, s, t) = us \log_{a+vt} a,\ a > 1,\ f_a(1, 1, s, t) = s \Rightarrow s = 0$ or $t = 0$

Definition 2.2. Let (X, d) be a metric space. A mapping $T : X \to X$ is (CAB)-contractive mapping if there exists a pair of generalized altering function (ψ,ϕ), $h \in \mathcal{A}$ and $f \in \mathcal{C}$ such that

$$h(\psi(d(Tx,Ty))) \leq f(\alpha(x,y),\beta(x,y),\psi(d(x,y)),\phi(d(x,y))), \quad \text{for all } x,y \in X, \tag{1}$$

where $\alpha,\beta : X \times X \to [0,+\infty)$.

If $f(u,v,s,t) = us - vt$ and $h(t) = t$, we obtain

Definition 2.3 ([5]). Let (X,d) be a metric space. A mapping $T : X \to X$ is $(\alpha\psi,\beta\phi)$-contractive mappings if there exists a pair of generalized distance (ψ,ϕ) such that

$$\psi(d(Tx,Ty)) \leq \alpha(x,y)\psi(d(x,y)) - \beta(x,y)\phi(d(x,y)), \quad \text{for all } x,y \in X,$$

where $\alpha,\beta : X \times X \to [0,+\infty)$.

Now we are ready to state our first main result.

Theorem 2.1. *Let (X,d) be a complete metric space, $N \in \mathbb{N}\backslash\{0\}$, and $T : X \to X$ be an (CAB)-contractive mapping satisfying the following conditions:*

(A1) $\mathcal{R}_i$ is N-transitive for $i = 1,2$;
(A2) T is $\mathcal{R}_i$-preserving for $i = 1,2$;
(A3) there exists $x_0 \in X$ such that $x_0\mathcal{R}_iTx_0$ for $i = 1,2$;
(A4) T is continuous.

Then, T has a fixed point, that is, there exists $x^* \in X$ such that $Tx^* = x^*$.

Proof. Let $x_0 \in X$ such that $x_0\mathcal{R}_iTx_0$ for $i = 1,2$. Define the sequence $\{x_n\}$ in X by $x_{n+1} = Tx_n$ for all $n \geq 0$.

If $x_n = x_{n+1}$ for some $n \geq 0$, then $x^* = x_n$ is a fixed point T. Assume that $x_n \neq x_{n+1}$ for all $n \geq 0$. From (A2) and (A3), we have

$$x_0\mathcal{R}_1Tx_0 \Rightarrow \alpha(x_0,Tx_0) = \alpha(x_0,x_1) \leq 1 \Rightarrow \alpha(Tx_0,Tx_1) = \alpha(x_1,x_2) \leq 1.$$

Similarly, we have

$$x_0\mathcal{R}_2Tx_0 \Rightarrow \beta(x_0,Tx_0) = \beta(x_0,x_1) \geq 1 \Rightarrow \beta(Tx_0,Tx_1) = \beta(x_1,x_2) \geq 1.$$

Using mathematical induction, and (A2) it follows that

$$\alpha(x_n,x_{n+1}) \leq 1 \text{ for all } n \geq 0, \tag{2}$$

and, similarly, we have

$$\beta(x_n,x_{n+1}) \geq 1 \text{ for all } n \geq 0, \tag{3}$$

Substituting $x = x_n$ and $y = x_{n+1}$ in (1), we obtain

$$\begin{aligned}\psi(d(Tx_n,Tx_{n+1})) &\leq h(\psi(d(Tx_n,Tx_{n+1}))) \\ &\leq f(\alpha(x_n,x_{n+1}),\beta(x_n,x_{n+1}),\psi(d(x_n,x_{n+1})),\phi(d(x_n,x_{n+1})))\end{aligned}$$

So, by (2) and (3) it follows that

$$\psi(d(x_{n+1},x_{n+2})) \leq f(1,1,\psi(d(x_n,x_{n+1})),\phi(d(x_n,x_{n+1}))) \leq \psi(d(x_n,x_{n+1})). \tag{4}$$

Using monotone property of ψ, we have

$$d(x_{n+1}, x_{n+2}) \leq d(x_n, x_{n+1}),$$

for every $n \geq 1$. Hence the sequence $\{d(x_n, x_{n+1})\}$ is a decreasing sequence. So for the nonnegative decreasing sequence $\{d(x_n, x_{n+1})\}$, there exists some $r \geq 0$, such that

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = r. \tag{5}$$

Assume that $r > 0$. On letting $n \to \infty$ in (4), so by using (5) and the continuity of ψ and f , we obtain

$$\psi(r) \leq f(1, 1, \psi(r), \phi(r)) \leq \psi(r), \tag{6}$$

thus $f(1, 1, \psi(r), \phi(r)) = \psi(r)$. Now, by using Definition 2.1, we get that either $\psi(r) = 0$ or $\phi(r) = 0$, in both cases it follows that $r = 0$, which implies

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = 0. \tag{7}$$

On the other hand, by (2) and (A1), we obtain

$$\alpha(x_m, x_{m+kN+1}) \leq 1 \text{ for all } m, k \geq 0. \tag{8}$$

Similarly, by (3) and (A1), we obtain

$$\beta(x_m, x_{m+kN+1}) \geq 1 \text{ for all } m, k \geq 0. \tag{9}$$

Now, for some $m, k \geq 0$, substituting $x = x_m$ and $y = x_{m^*}$ in (1), where $m^* := m + kN + 1$, we get

$$\begin{aligned}\psi(d(Tx_m, Tx_{m^*})) &\leq h(\psi(d(Tx_m, Tx_{m^*}))) \\ &\leq f(\alpha(x_m, x_{m^*}), \beta(x_m, x_{m^*}), \psi(d(x_m, x_{m^*})), \phi(d(x_m, x_{m^*}))).\end{aligned}$$

So, using (8) and (9), we have

$$\psi(d(x_{m+1}, x_{m^*+1})) \leq f(1, 1, \psi(d(x_m, x_{m^*})), \phi(d(x_m, x_{m^*}))) \leq \psi(d(x_m, x_{m^*})). \tag{10}$$

Using monotone property of ψ, we have

$$d(x_{m+1}, x_{m^*+1}) \leq d(x_m, x_{m^*}).$$

Hence the sequence $\{d(x_m, x_{m^*})\}$ is a decreasing sequence. So for the nonnegative decreasing sequence $\{d(x_m, x_{m^*})\}$, there exists some $s \geq 0$, such that

$$\lim_{n\to\infty} d(x_m, x_{m^*}) = s. \tag{11}$$

Assume that $s > 0$. On letting $n \to \infty$ in (4), so by using (11) and and the continuity of f, we obtain

$$\psi(s) \leq f(1, 1, \psi(s), \phi(s)) \leq \psi(s), \tag{12}$$

which $f(1,1,\psi(s),\phi(s))=\psi(s)$, again using Definition 2.1, we get $\psi(s)=0$ or $\phi(s)=0$, which implies that $s=0$ and so

$$\lim_{n\to\infty} d(x_m,x_{m^*})=0. \tag{13}$$

Now, we shall prove that $\{x_n\}$ is a Cauchy sequence. If possible, let $\{x_n\}$ be not a Cauchy sequence. Then there exists $\delta>0$ for which we can find subsequences $\{x_{n_k}\}$ and $\{x_{m_k}\}$ of $\{x_n\}$ with $n_k>m_k>k$ such that

$$d(x_{n_k},x_{m_k})\geq\delta. \tag{14}$$

Further, corresponding to m_k, we can choose n_k in such a way that it is the smallest integer with $n_k>m_k$ and satisfying (14). Therefore, we have

$$d(x_{n_k-1},x_{m_k})<\delta. \tag{15}$$

Using (14), (15) and triangle inequality, we have

$$\begin{aligned}0<\delta\leq d(x_{n_k},x_{m_k})\leq\\ \leq d(x_{n_k},x_{n_k-1})+d(x_{n_k-1},x_{m_k})<\delta+d(x_{n_k},x_{n_k-1}).\end{aligned} \tag{16}$$

On letting $k\to\infty$ and using (7), in (16), we have

$$\lim_{k\to\infty} d(x_{n_k},x_{m_k})=\delta. \tag{17}$$

Furthermore, for each $k\geq 0$, there exist $\mu_k,\eta_k>0$ such that $m_k^*:=m_k+N\mu_k+1=n_k+\eta_k$. Hence, by (14) we have

$$\begin{aligned}\delta\leq d(x_{m_k},x_{m_k^*})&\leq d(x_{m_k},x_{n_k-1})+\sum_{n_k-1}^{m_k^*-1} d(x_i,x_{i+1})\\ &<\delta+\sum_{n_k-1}^{m_k^*-1} d(x_i,x_{i+1})\end{aligned} \tag{18}$$

Again, letting $k\to\infty$ and using (7), we get

$$\lim_{k\to\infty} d(x_{m_k},x_{m_k^*})=\delta. \tag{19}$$

Also, consider

$$|d(x_{m_k},x_{m_k^*})-d(x_{m_k-1},x_{m_k^*-1})|\leq d(x_{m_k-1},x_{m_k})+d(x_{m_k^*},x_{m_k^*-1}).$$

On letting $k\to\infty$ in the above inequality and using (7), (13), (19), we get

$$\lim_{k\to\infty} d(x_{m_k-1},x_{m_k^*-1})=\delta. \tag{20}$$

Now, by setting $x=x_{m_k-1}$ and $y=x_{m_k^*-1}$ in (1), we obtain

$$\begin{aligned}\psi(d(x_{m_k},x_{m_k^*}))&\leq h(\psi(d(Tx_{m_k-1},Tx_{m_k^*-1})))\\ &\leq f\Big(\alpha(x_{m_k-1},x_{m_k^*-1}),\beta(x_{m_k-1},x_{m_k^*-1}),\\ &\qquad\psi(d(x_{m_k-1},x_{m_k^*-1})),\phi(d(x_{m_k-1},x_{m_k^*-1}))\Big).\end{aligned}$$

Now, using (8) and (9), we get

$$\psi(d(x_{m_k}, x_{m_k^*})) \leq f(1, 1, \psi(d(x_{m_k-1}, x_{m_k^*-1})), \phi(d(x_{m_k-1}, x_{m_k^*-1}))) \leq$$
$$\leq \psi(d(x_{m_k-1}, x_{m_k^*-1})).$$

On letting $k \to \infty$ in the above equation, using (19), (20), and continuity of ψ and f, we obtain

$$f(1, 1, \psi(\delta), \phi(\delta)) = \psi(\delta)$$

which implies either $\psi(\delta) = 0$ or $\phi(\delta) = 0$, so we get $\delta = 0$. This shows that $\{x_n\}$ is a Cauchy sequence. As (X, d) is complete metric space, there exists $x^* \in X$ such that $\lim_{n\to\infty} x_n = x^*$.

Since T is continuous, we get

$$\lim_{n\to\infty} Tx_n = Tx^*.$$

Since $x_{n+1} = Tx_n$, we have also

$$\lim_{n\to\infty} Tx_n = x^*.$$

By the uniqueness of the limit, we get $Tx^* = x^*$, that is, x^* is a fixed point of T. □

Theorem 2.2. *In Theorem 2.1, if we replace the continuity of T by the $(\mathcal{R}_1, \mathcal{R}_2)$-regularity of the metric space (X, d), then T has a fixed point $x^* \in X$.*

Proof. Following the lines of the proof of Theorem 2.1, we get that $\{x_n\}$ is a Cauchy sequence. Since (X, d) is a complete metric space, then there exists $x^* \in X$ such that $x_n \to x^*$. Furthermore, the sequence $\{x_n\}$ satisfies (2) and (3), that is, $x_n \mathcal{R}_1 x_{n+1}$, and $x_n \mathcal{R}_2 x_{n+1}$ for all $n \in \mathbb{N}$.

Now, since (X, d) is $(\mathcal{R}_1, \mathcal{R}_2)$-regular, then there exists a subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $x_{n(k)} \mathcal{R}_1 x^*$, that is, $\alpha(x_{n(k)}, x^*) \leq 1$ and $x_{n(k)} \mathcal{R}_2 x^*$, that is, $\beta(x_{n(k)}, x^*) \geq 1$, for all k. By setting $x = x_{n(k)}$ and $y = x^*$, in (1), we obtain

$$\psi(d(Tx_{n(k)}, Tx^*)) \leq h(\psi(d(Tx_{n(k)}, Tx^*)))$$
$$\leq f(\alpha(x_{n(k)} x^*), \beta(x_{n(k)}, x^*), \psi(d(x_{n(k)}, x^*)), \phi(d(x_{n(k)}, x^*))),$$

which implies that

$$\psi(d(x_{n(k)+1}, Tx^*)) \leq f(1, 1, \psi(d(x_{n(k)}, x^*)), \phi(d(x_{n(k)}, x^*)))$$
$$\leq \psi(d(x_{n(k)}, x^*)).$$

Hence using the monotone property of ψ, we obtain

$$d(x_{n(k)+1}, Tx^*) \leq d(x_{n(k)}, x^*).$$

On letting $k \to \infty$, we get $d(x^*, Tx^*) = 0$, that is, $x^* = Tx^*$. □

Theorem 2.3. *Adding to the hypotheses of Theorem 2.1 (respectively, Theorem 2.2) that X is $(\mathcal{R}_1, \mathcal{R}_2)$-directed, we obtain uniqueness of the fixed point of T.*

Proof. Suppose that x^* and y^* are two fixed points of T. Since X is $(\mathcal{R}_1, \mathcal{R}_2)$-directed, there exists $z \in X$ such that

$$\alpha(x^*, z) \leq 1, \quad \alpha(y^*, z) \leq 1 \tag{21}$$

and

$$\beta(x^*, z) \geq 1, \quad \beta(y^*, z) \geq 1 \tag{22}$$

Since T is $\mathcal{R}_i$-preserving for $i = 1, 2$, from (21) and (22), we get

$$\alpha(x^*, T^n z) \leq 1, \quad \alpha(y^*, T^n z) \leq 1, \quad \forall n \geq 0; \tag{23}$$

and

$$\beta(x^*, T^n z) \geq 1, \quad \beta(y^*, T^n z) \geq 1, \quad \forall n \geq 0. \tag{24}$$

Using (23), (24) and (1), we have

$$\begin{aligned}
\psi(d(x^*, T^{n+1}z)) &= \psi(d(Tx^*, T(T^n z))) \\
&\leq h(\psi(d(Tx^*, T(T^n z)))) \\
&\leq f(\alpha(x^*, T^n z), \beta(x^*, T^n z), \psi(d(x^*, T^n z)), \phi(d(x^*, T^n z))) \\
&\leq f(1, 1, \psi(d(x^*, T^n z)), \phi(d(x^*, T^n z))) \\
&\leq \psi(d(x^*, T^n z)).
\end{aligned}$$

So, we get

$$\begin{aligned}
\psi(d(x^*, T^{n+1}z)) &\leq f(1, 1, \psi(d(x^*, T^n z)), \phi(d(x^*, T^n z))) \\
&\leq \psi(d(x^*, T^n z)).
\end{aligned} \tag{25}$$

Using the monotone property of ψ, we have for each $n \geq 0$,

$$d(x^*, T^{n+1}z) \leq d(x^*, T^n z).$$

It follows that $\{d(x^*, T^n z)\}$ is monotone decreasing and consequently, there exists $r \geq 0$ such that $d(x^*, T^n z) \to r$. On letting $n \to \infty$, in (25) and using the continuity of ψ and f, we obtain

$$f(1, 1, \psi(r), \phi(r)) = \psi(r),$$

which implies either $\psi(r) = 0$ or $\phi(r) = 0$, then $d(x^*, T^n z) \to 0$, as $n \to \infty$.

Similarly, we obtain $d(y^*, T^n z) \to 0$, as $n \to \infty$. By the uniqueness of limit, we have $x^* = y^*$. □

3. CONSEQUENCES

In this section, we derive some consequences from our main results.

3.1. **Coupled fixed point results in complete metric spaces.**

Definition 3.1 (see [16]). Let $F : X \times X \to X$ be a given mapping. We say that $(x, y) \in X \times X$ is a coupled fixed point of F if $F(x, y) = x$ and $F(y, x) = y$.

Lemma 3.1. *A pair (x, y) is a coupled fixed point of F if and only if (x, y) is a fixed point of T where $T : X \times X \to X \times X$ is given by*

$$T(x, y) = (F(x, y), F(y, x)) \quad \text{for all } (x, y) \in X \times X. \tag{26}$$

Definition 3.2. Let (X, d) be a metric space and $F : X \times X \to X$ be a given mapping. A mapping F is an (CAB)-type contractive mapping if there exists a pair of generalized altering function (ψ, ϕ), $h \in \mathcal{A}$ and $f \in \mathcal{C}$ such that for all $x, y, u, v \in X$

$$h(\psi(d(F(x, y), F(u, v)))) \leq f\Big(\alpha((x, y), (u, v)), \beta((x, y), (u, v)), \psi(\max\{d(x, u), d(y, v)\}), \phi(\max\{d(x, u), d(y, v)\})\Big),$$

where $\alpha, \beta : X^2 \times X^2 \to [0, +\infty)$.

Definition 3.3. Let X be a set, and $\mathcal{S}_1, \mathcal{S}_2$ be two binary relations on $X \times X$ defined by

$$(x, y), (u, v) \in X \times X : (x, y)\mathcal{S}_1(u, v) \Rightarrow \alpha((x, y), (u, v)) \leq 1$$

and

$$(x, y), (u, v) \in X \times X : (x, y)\mathcal{S}_2(u, v) \Rightarrow \beta((x, y), (u, v)) \geq 1.$$

Definition 3.4. Let (X, d) be a metric space. We say that $(X \times X, d)$ is $(\mathcal{S}_1, \mathcal{S}_2)$-biregular if for all sequences $\{x_n, y_n\}$ in $X \times X$ such that $x_n \to x \in X$, $y_n \to y \in X$ as $n \to \infty$, and $(x_n, y_n)\mathcal{S}_i(x_{n+1}, y_{n+1})$, $(y_{n+1}, x_{n+1})\mathcal{S}_i(y_n, x_n)$ for $i = 1, 2$, and for all $n \in \mathbb{N}$, there exists a subsequence $\{x_{n(k)}, y_{n(k)}\}$ such that $(x_{n(k)}, y_{n(k)})\mathcal{S}_i(x, y)$, $(y, x)\mathcal{S}_i(y_{n(k)}, x_{n(k)})$ for $i = 1, 2$ and for all $k \in \mathbb{N}$.

Definition 3.5. We say that $X \times X$ is $(\mathcal{S}_1, \mathcal{S}_2)$-bidirected if for all $(x, y), (u, v) \in X \times X$, there exists $(z_1, z_2) \in X \times X$ such that $((x, y)\mathcal{S}_i(z_1, z_2)) \wedge ((z_2, z_1)\mathcal{S}_i(y, x))$ and $((u, v)\mathcal{S}_i(z_1, z_2)) \wedge ((z_2, z_1)\mathcal{S}_i(v, u))$ for $i = 1, 2$.

Corollary 3.1. *Let (X, d) be a complete metric space and $F : X \times X \to X$ be an (CAB)-type contractive mapping satisfying the following conditions:*

(i) *$\mathcal{S}_i$ is N-transitive for $i = 1, 2$ $(N > 0)$;*

(ii) *For all $(x, y), (u, v) \in X \times X$, we have*

$$(x, y)\mathcal{S}_i(u, v) \Rightarrow (F(x, y), F(y, x))\mathcal{S}_i(F(u, v), F(v, u)) \text{ for } i = 1, 2;$$

(iii) *There exists $(x_0, y_0) \in X \times X$ such that*

$$\begin{matrix} (x_0, y_0)\mathcal{S}_i(F(x_0, y_0), F(y_0, x_0)), \\ (F(y_0, x_0), F(x_0, y_0))\mathcal{S}_i(y_0, x_0) \end{matrix} \qquad \text{for } i = 1, 2;$$

(iv) *F is continuous, or $(X \times X, d)$ is $(\mathcal{S}_1, \mathcal{S}_2)$-biregular.*

Then, F has a coupled fixed point $(x^*, y^*) \in X \times X$. Moreover, if $X \times X$ is $(\mathcal{S}_1, \mathcal{S}_2)$-bidirected, then we have the uniqueness of the coupled fixed point.

Proof. By Lemma 3.1, a pair (x, y) is a coupled fixed point of F if and only if (x, y) is a fixed point of T. Now, consider the complete metric space (Y, δ), where $Y = X \times X$ and $\delta((x, y), (u, v)) = \max\{d(x, u), d(y, v)\}$ for all $(x, y), (u, v) \in X \times X$. Hence

$$\begin{aligned}\psi(d(F(x,y),F(u,v)) &\leq h(\psi(d(F(x,y),F(u,v))))\\ &\leq f\Big(\alpha((x,y),(u,v)),\beta((x,y),(u,v)),\\ &\qquad \psi(\delta((x,u),(y,v))),\phi(\delta((x,u),(y,v)))\Big),\end{aligned}$$

and

$$\begin{aligned}\psi(d(F(v,u),F(y,x))) &\leq h(\psi(d(F(v,u),F(y,x))))\\ &\leq f\Big(\alpha((v,u),(y,x)),\beta((v,u),(y,x)),\\ &\qquad \psi(\delta((v,y),(u,x))),\phi(\delta((v,y),(u,x)))\Big).\end{aligned}$$

Since $\psi : [0, \infty) \to [0, \infty)$ is nondecreasing, then

$$\psi(\max\{r, s\}) = \max\{\psi(r), \psi(s)\} \quad \text{for all } r, s \in [0, \infty).$$

Hence, for all $\xi := (\xi_1, \xi_2)$, $\eta := (\eta_1, \eta_2) \in X \times X$, we have

$$\begin{aligned}\psi(\delta(T\xi,T\eta)) &\leq h(\psi(\delta(T\xi,T\eta)))\\ &\leq f(a(\xi,\eta),b(\xi,\eta),\psi(\delta(\xi,\eta)),\phi(\delta(\xi,\eta))),\end{aligned}$$

where $a, b : Y \times Y \to [0, +\infty)$ are the functions defined by

$$a((\xi_1,\xi_2),(\eta_1,\eta_2)) = \max\{\alpha((\xi_1,\xi_2),(\eta_1,\eta_2)),\alpha((\eta_2,\eta_1),(\xi_2,\xi_1))\}$$

and

$$b((\xi_1,\xi_2),(\eta_1,\eta_2)) = \min\{\beta((\xi_1,\xi_2),(\eta_1,\eta_2)),\beta((\eta_2,\eta_1),(\xi_2,\xi_1))\}$$

and $T : Y \to Y$ is given by (26). We shall prove that T is (a, b, ψ, ϕ, h, f)-contractive mapping.

Define two binary relations $\mathcal{R}_1$ and $\mathcal{R}_2$ by $\xi \mathcal{R}_1 \eta \Leftrightarrow a(\xi, \eta) \leq 1$ and $\xi \mathcal{R}_2 \eta \Leftrightarrow b(\xi, \eta) \geq 1$ for all $\xi, \eta \in X \times X$.

First, we claim that $\mathcal{R}_j$ for $j = 1, 2$, are N-transitive. Let $(x_i, y_i) \in X \times X$ for all $i \in \{0, \ldots, N\}$, such that $(x_i, y_i)\mathcal{R}_j(x_{i+1}, y_{i+1})$ for $j = 1, 2$, that is, $a((x_i, y_i), (x_{i+1}, y_{i+1})) \leq 1$ and $b((x_i, y_i), (x_{i+1}, y_{i+1})) \geq 1$ for all $i \in \{0, \ldots, N\}$.

By definitions of a and b, it follows that

$$\left.\begin{aligned}\alpha((x_i,y_i),(x_{i+1},y_{i+1})) \leq 1\\ \alpha((y_{i+1},x_{i+1}),(y_i,x_i)) \leq 1\end{aligned}\right. \quad \text{for all } i \in \{0,\ldots,N\},$$

and

$$\begin{matrix} \beta((x_i, y_i), (x_{i+1}, y_{i+1})) \geq 1 \\ \beta((y_{i+1}, x_{i+1}), (y_i, x_i)) \geq 1 \end{matrix} \quad \text{for all } i \in \{0, \dots, N\},$$

or

$$\begin{matrix} (x_0, y_0)\mathcal{S}_j(x_{i+1}, y_{i+1}) \\ (x_{i+1}, y_{i+1})\mathcal{S}_j(x_i, y_i) \end{matrix} \quad \text{for } j = 1, 2 \text{ and for all } i \in \{0, \dots, N\}.$$

Hence by (i), we have

$$(x_0, y_0)\mathcal{S}_j(x_{N+1}, y_{N+1}) \quad \text{and} \quad (x_{N+1}, y_{N+1})\mathcal{S}_j(x_0, y_0) \quad \text{for } j = 1, 2,$$

that is,

$$\begin{aligned} &\alpha((x_0, y_0), (x_{N+1}, y_{N+1})) \leq 1; \quad \alpha((y_{N+1}, x_{N+1}), (y_0, x_0)) \leq 1; \\ &\beta((x_0, y_0), (x_{N+1}, y_{N+1})) \geq 1; \quad \beta((y_{N+1}, x_{N+1}), (y_0, x_0)) \geq 1; \end{aligned}$$

or

$$(x_0, y_0)\mathcal{R}_j(x_{N+1}, y_{N+1}) \quad \text{for } j = 1, 2.$$

Then our claim holds.

Let $\xi = (\xi_1, \xi_2)$, $\eta = (\eta_1, \eta_2) \in Y$ such that $a(\xi, \eta) \leq 1$ and $b(\xi, \eta) \geq 1$. Using condition (ii), we obtain immediately that $a(T\xi, T\eta) \leq 1$ and $b(T\xi, T\eta) \geq 1$. Then T is $\mathcal{R}_j$-preserving for $j = 1, 2$. Moreover, from condition (iii), we know that there exists $(x_0, y_0) \in Y$ such that $(x_0, y_0)\mathcal{R}_j T(x_0, y_0)$ for $j = 1, 2$. If F is continuous, then T also is continuous. Then all the hypotheses of Theorem 2.1 are satisfied.

If $(X \times X, d)$ is $(\mathcal{S}_1, \mathcal{S}_2)$-biregular, then we easily have that $(X \times X, d)$ is $(\mathcal{R}_1, \mathcal{R}_2)$-regular. Hence, Theorem 2.2 yields the result. We deduce the existence of a fixed point of T that gives us from (26) the existence of a coupled fixed point of F. Now, since $X \times X$ is $(\mathcal{S}_1, \mathcal{S}_2)$-bidirected, one can easily derive that $X \times X$ is $(\mathcal{R}_1, \mathcal{R}_2)$-directed by regarding Lemma 3.1 and Definition 3.5. Finally, by using Theorem 2.3, we obtain the uniqueness of the fixed point of T, that is, the uniqueness of the coupled fixed point of F. □

3.2. **Fixed point results on metric spaces endowed with N-transitive binary relation.** In this subsection, we establish a fixed point theorem on metric space endowed with N-transitive binary relation $\mathcal{S}$.

Corollary 3.2. *Let X be a non-empty set endowed with a binary relation $\mathcal{S}$. Suppose that there is a metric d on X such that (X, d) is complete metric space. Suppose there exists a pair of generalized altering distance (ψ, ϕ), $h \in \mathcal{A}$ and $\widetilde{f} \in \mathcal{C}$ such that $T : X \to X$ satisfies the following contraction*

$$h(\psi(d(Tx, Ty))) \leq \widetilde{f}(1, 1, \psi(d(x, y)), \phi(d(x, y))), \text{ for all } x\mathcal{S}y.$$

Suppose also that the following conditions hold:

(i) *$\mathcal{S}$ is N-transitive ($N > 0$);*
(ii) *T is a $\mathcal{S}$-preserving mapping;*

(iii) *there exists* $x_0 \in X$ *such that* $x_0 \mathcal{S} T x_0$*;*
(iv) T *is continuous or* (X, d) *is* $\mathcal{S}$*-regular.*

Then T *has a fixed point. Moreover, if* X *is* $\mathcal{S}$*-directed, we have the uniqueness of the fixed point.*

Proof. Define the mappings $\alpha, \beta : X \times X \to [0, +\infty)$ by

$$\alpha(x, y) = \begin{cases} 1, & \text{if } x\mathcal{S}y, \\ 2 + h(\psi(d(Tx, Ty))), & \text{otherwise} \end{cases}$$

and

$$\beta(x, y) = \begin{cases} 1, & \text{if } x\mathcal{S}y, \\ 0, & \text{otherwise} \end{cases}$$

Now by using Definition 1.8, the conclusion follows directly from Theorems 2.1-2.3 where f is given by

$$f(u, v, s, t) = \begin{cases} \widetilde{f}(u, v, s, t), & \text{if } u \leq 1; \\ u, & \text{otherwise.} \end{cases} \qquad \square$$

3.3. **Fixed point results for cyclic contractive mappings.** In [21], Kirk et al. generalized the Banach contraction principle and obtained some new fixed point results for cyclic type contractive mappings. On the similar lines we obtained some new results for cycllic type contraction mappings in this section.

Let us define the binary relations $\mathcal{R}_1$ and $\mathcal{R}_2$.

Definition 3.6. Let X be a nonempty set and A_i, $i \in \{1, \ldots, N\}$ be nonempty closed subsets of X. We define two binary relations $\mathcal{R}_k$ for $k = 1, 2$ by

$$x, y \in X : x\mathcal{R}_k y \Leftrightarrow (x, y) \in \Gamma := \bigcup_{i=1}^{N} (A_i \times A_{i+1}) \text{ with } A_{N+1} := A_1.$$

Corollary 3.3. *For* $i \in \{1, \ldots, N\}$*, let* A_i *be nonempty closed subsets of a complete metric space* (X, d)*, and let* $T : X \to X$ *be a given mapping. Suppose that the following conditions hold:*

(i) $T(A_i) \subseteq A_{i+1}$ *for all* $i \in \{1, \ldots, N\}$ *with* $A_{N+1} := A_1$*;*
(ii) *there exist a pair of generalized altering distance* (ψ, ϕ)*,* $h \in \mathcal{A}$ *and* $\widetilde{f} \in \mathcal{C}$ *such that*

$$h(\psi(d(Tx, Ty))) \leq \widetilde{f}(1, 1, \psi(d(x, y)), \phi(d(x, y))), \text{ for all } (x, y) \in \Gamma.$$

Then T *has a unique fixed point in* $\cap_{i=1}^{N} A_i$*.*

Proof. Let $Y := \bigcup_{i=1}^{N} A_i$. For all $i \in \{1, \dots, N\}$, we have by assumption that each A_i is nonempty closed subset of the complete metric space X, which implies that (Y, d) is complete. Define the mappings $\alpha, \beta : Y \times Y \to [0, +\infty)$ by

$$\alpha(x, y) = \begin{cases} 1, & \text{if } (x, y) \in \Gamma, \\ 2 + h(\psi(d(Tx, Ty))), & \text{otherwise;} \end{cases}$$

and

$$\beta(x, y) = \begin{cases} 1, & \text{if } (x, y) \in \Gamma, \\ 0, & \text{otherwise.} \end{cases}$$

Hence, Definition 3.6 is equivalent to Definition 1.8.

We start by checking that $\mathcal{R}_1$ and $\mathcal{R}_2$ are N-transitive. Indeed, let $x_0, \dots, x_{N+1} \in Y$ such that $x_k \mathcal{R}_1 x_{k+1}$ and $x_k \mathcal{R}_2 x_{k+1}$ for all $\in \{0, \dots, N\}$, that is, $\alpha(x_k, x_{k+1}) \leq 1$ and $\beta(x_k, x_{k+1}) \geq 1$, for all $k \in \{0, \dots, N\}$ such that $x_0 \in A_i$, $x_1 \in A_{i+1}$, $\dots$, $x_k \in A_{i+k}$,$\dots$,$x_{N+1} \in A_{i+N+1} = A_{i+1}$, which implies that $(x_0, x_{N+1}) \in A_i \times A_{i+1} \subseteq \Gamma$. Hence, we obtain $\alpha(x_0, x_{N+1}) \leq 1$ and $\beta(x_0, x_{N+1}) \geq 1$, that is, $x_0 \mathcal{R}_1 x_{N+1}$ and $x_0 \mathcal{R}_2 x_{N+1}$, which implies that $\mathcal{R}_1$ and $\mathcal{R}_2$ are N-transitive.

Next, from (ii) and the definition of α and β, we can write

$$h(\psi(d(Tx, Ty))) \leq f(\alpha(x, y), \beta(x, y), \psi(d(x, y)), \phi(d(x, y))), \text{ for all } x, y \in Y$$

where f is given by

$$f(u, v, s, t) = \begin{cases} \widetilde{f}(u, v, s, t), & \text{if } u \leq 1 \\ u, & \text{otherwise.} \end{cases}$$

Thus, T is (CAB)-contractive mapping.

We claim next that T is $\mathcal{R}_1$-preserving and $\mathcal{R}_2$-preserving. Indeed, let $x, y \in Y$ such that $x \mathcal{R}_1 y$ and $x \mathcal{R}_2 y$, that is, $\alpha(x, y) \leq 1$ and $\beta(x, y) \geq 1$; hence, there exists $i \in \{1, \dots, N\}$ such that $x \in A_i$, $y \in A_{i+1}$. Thus, $(Tx, Ty) \in A_{i+1} \times A_{i+2} \subseteq \Gamma$, then $\alpha(Tx, Ty) \leq 1$ and $\beta(Tx, Ty) \geq 1$, that is, $Tx \mathcal{R}_1 Ty$ and $Tx \mathcal{R}_2 Ty$. Hence, our claim holds.

Also, from (i), for any $x_0 A_i$ for all $i \in \{1, \dots, N\}$, we have $(x_0, Tx_0) \in A_i \times A_{i+1}$, which implies that $\alpha(x_0, Tx_0) \leq 1$ and $\beta(x_0, Tx_0) \geq 1$, that is, $x_0 \mathcal{R}_1 Tx_0$ and $x_0 \mathcal{R}_2 Tx_0$.

Now, we claim that Y is $(\mathcal{R}_1, \mathcal{R}_2)$-regular. Let $\{x_n\}$ be a sequence in Y such that $x_n \to x \in Y$ as $n \to \infty$, and $x_n \mathcal{R}_1 x_{n+1}$, $x_n \mathcal{R}_2 x_{n+1}$ for all n, that is, $\alpha(x_n, x_{n+1}) \leq 1$, $\beta(x_n, x_{n+1}) \geq 1$ for all n. It follows that there exist$i, j \in \{1, \dots, N\}$ such that $x_n \in A_{i+n}$ for all $n \in \mathbb{N}$ and $x \in A_j$, so $x_{(j-i-1+N)+kN} \in A_{j-1+(k+1)N} = A_{j-1}$ for all $k \in \mathbb{N}$.

By letting $n(k) := (j - i - 1 + N) + kN$ for all $k \in \mathbb{N}$, we conclude that the subsequence $\{x_{n(k)}\}$ satisfies $(x_{n(k)}, x) \in A_{j-1} \times A_j \subseteq \Gamma$ for all $k \in \mathbb{N}$, hence $\alpha(x_{n(k)}, x) \leq 1$ and $\beta(x_{n(k)}, x) \geq 1$ for all k, that is, $x_{n(k)}\mathcal{R}_1 x$ and $x_{n(k)}\mathcal{R}_2 x$, which proves our claim.

Hence, all the hypotheses of Theorem 2.2 are satisfied on (Y, d), and we deduce that T has a fixed point $x^* \in Y$. Since $x^* \in A_i$ for some $i \in \{1, \ldots, N\}$ and $x^* = Tx^* \in A_{i+1}$ for all $i \in \{1, \ldots, N\}$, then $x^* \cap_{i=1}^{N} A_i$.

Moreover, it is easy to check that X is $(\mathcal{R}_1, \mathcal{R}_2)$-directed. Indeed, let $x, y \in Y$ with $x \in A_i$, $y \in A_j$, $i, j \in \{1, \ldots, N\}$. For $z = x^* \in Y$, we have $((\alpha(x, z) \leq 1) \wedge (\alpha(y, z) \leq 1))$ and $((\beta(x, z) \geq 1) \wedge (\beta(y, z) \geq 1))$. Thus, X is $(\mathcal{R}_1, \mathcal{R}_2)$-directed.

Finally, the uniqueness follows by Theorem 2.3. □

4. SOME RELATED RESULTS

As a consequence of our results some fixed point theorems of Berzig and Karapınar in [5] can be derived from our main results by taking $f(x, y, z, t) = xz - yt$.

Corollary 4.1. *Theorem 2.1 from [5] is a particular case of Theorem 2.1.*

Corollary 4.2. *Theorem 2.2 from [5] is a particular case of Theorem 2.2.*

Corollary 4.3. *Theorem 2.3 from [5] is a particular case of Theorem 2.3.*

Moreover, corollaries in [5] on coupled fixed point, fixed point on partially ordered metric spaces and fixed point for cyclic mappings can be derived from our results by taking $\widetilde{f}(1, 1, z, t) = z - t$. So, we have

Corollary 4.4. *Corollary 3.1 from [5] is a particular case of Corollary 3.1.*

Corollary 4.5. *Corollary 3.2 from [5] is a particular case of Corollary 3.2.*

Corollary 4.6. *Corollary 3.3 from [5] is a particular case of Corollary 3.3.*

COMPETING INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this article.

REFERENCES

[1] S. Banach, *Sur les opérations dans les ensembles abstraits et leur application aux équations integrales*, Fund. Math., 3(1922), 133–181.

[2] M. Berzig, *Coincidence and common fixed point results on metric spaces endowed with an arbitrary binary relation and applications*, J. Fixed Point Theory Appl. 12(2013), 221–238.

[3] M. Berzig and M-D. Rus, *Fixed point theorems for α-contractive mappings of Meir-Keeler type and applications*, Nonlinear Anal. Model. Control. 19 (2), 178–198.

[4] M. Berzig, E. Karapınar, and A. Roldán. *Discussion on generalized-$(\alpha\psi, \beta\varphi)$-contractive mappings via generalized altering distance function and related fixed point theorems*, Abstr. Appl. Anal. 2013 (2013) Article ID 634371.

[5] M. Berzig and E. Karapınar, *Fixed point results for* $(\alpha\psi, \beta\phi)$*-contractive mappings for a generalized altering distance*, Fixed Point Theory Appl. 2013, 2013:205.

[6] T. G. Bhaskar and V. Lakshmikantham, *Fixed point theorems in partially ordered metric spaces and applications*, Nonlinear Anal. 65 (2006), 1379–1393.

[7] D. W. Boyd and J. S. W. Wong, On nonlinear contractions, Proc. American Math. Soc. 20(1969), 458–464.

[8] S. Chandok, *Some common fixed point theorems for generalized nonlinear contractive mappings*, Computers Math. Appl. 62 (10)(2011), 3692–3699.

[9] S. Chandok, *On common fixed points for generalized contractive type mappings in ordered metric spaces*, Proc. Jangjeon Math. Soc. 16(2013), 327–333.

[10] S. Chandok, B. S. Choudhury and N. Metiya, *Some fixed point results in ordered metric spaces for rational type expressions with auxiliary functions*, J. Egyptian Math Soc. 23(1)(2015), 95-101 doi 10.1016/j.joems.2014.02.002.

[11] S. Chandok and S. Dinu, *Common fixed points for weak* ψ*-contractive mappings in ordered metric spaces with applications*, Abs. Appl. Anal. 2013(2013) Article ID 879084.

[12] S. Chandok, E. Karapinar and M.S. Khan, *Existence and uniqueness of common coupled fixed point results via auxiliary functions*, Bull. Iranian Math. Soc. 40(1) (2014), 199–215.

[13] S. Chandok, T.D. Narang and M.A. Taoudi, *Some common fixed point results in partially ordered metric spaces for generalized rational type contraction mappings*, Veitnam J. Math. 41(3) (2013), 323–331.

[14] S. Chandok and M. Postolache, *Fixed point theorem for weakly Chatterjea type cyclic contractions*, Fixed Point Theory Appl. 2013(2013):28.

[15] P. N. Dutta and B. S. Choudhury, *A generalisation of contraction principle in metric spaces*, Fixed Point Theory Appl. vol. 2008 (2008) Article ID 406368.

[16] D. Guo and V. Lakshmikantham, *Coupled fixed points of nonlinear operators with applications*, Nonlinear Anal. 11(5)(1987), 623-632.

[17] E. Karapınar, *Fixed point theory for cyclic weak* ϕ*-contraction*, Appl. Math. Lett. 24 (2011), 822–825.

[18] E. Karapınar and K. Sadarangani, *Fixed point theory for cyclic* $(\phi\text{-}\psi)$*-contractions*, Fixed Point Theory Appl. 2011, 69:2011.

[19] M. S. Khan, M. Swaleh, and S. Sessa, *Fixed point theorems by altering distances between the points*, Bull. Australian Math. Soc. 30(1984), 1–9.

[20] J. K. Kim and S. Chandok, *Coupled common fixed point theorems for generalized nonlinear contraction mappings with the mixed monotone property in partially ordered metric spaces*, Fixed Point Theory Appl. 2013, 2013:307.

[21] W. A. Kirk, P. S. Srinivasan and P. Veeramani, *Fixed points for mappings satisfying cyclical contractive conditions*, Fixed Point Theory 4(1) (2003), 79–89.

[22] J. J. Nieto and R. R. Lopez, *Contractive mapping theorems in partially ordered sets and applications to ordinary differential equations*, Order 22 (2005), 223–239.

[23] J. J. Nieto and R. R. Lopez, *Existence and uniqueness of fixed point in partially ordered sets and applications to ordinary differential equations*, Acta Math. Sin. (Engl. Ser.) 23 (2007), 2205–2212.

[24] D. O' Regan and A. Petrusel, *Fixed point theorems for generalized contractions in ordered metric spaces*, J. Math. Anal. Appl. 341 (2008), 1241–1252.

[25] A. C. M. Ran and M. C. B. Reurings, *A fixed point theorem in partially ordered sets and some applications to matrix equations*, Proc. Amer. Math. Soc. 132 (2004), 1435–1443.

[26] B. Samet and M. Turinici, *Fixed point theorems on a metric space endowed with an arbitrary binary relation and applications*, Commun. Math. Anal. 13(2)(2012), 82–97.

[27] B. Samet, C. Vetro and P. Vetro, *Fixed point theorem for α-ψ-contractive type mappings*, Nonlinear Anal. 75(2012), 2154–2165.

[28] B. E. Rhoades, *Some theorems on weakly contractive maps*, Nonlinear Anal. 47(2001), 2683–2693.

[29] M. Turinici, *Contractive maps in locally transitive relational metric spaces*, The Scientific World Journal, 2014 (2014), Article ID 169358, 10 pages.

ARSLAN HOJAT ANSARI
DEPARTMENT OF MATHEMATICS
COLLEGE OF BASIC SCIENCES
KARAJ BRANCH, ISLAMIC AZAD UNIVERSITY
ALBORZ
IRAN
E-mail address: aminansari7@yahoo.com

MAHER BERZIG
DEPARTMENT OF MATHEMATICS
TUNIS COLLEGE OF SCIENCES AND TECHNIQUES
5 AVENUE TAHA HUSSEIN
TUNIS UNIVERSITY
TUNISIA
E-mail address: maher.berzig@gmail.com

SUMIT CHANDOK
SCHOOL OF MATHEMATICS
THAPAR UNIVERSITY
PATIALA-147004
INDIA
E-mail address: chansok.s@gmail.com
sumit.chandok@thapar.edu