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A Common Fixed Point Theorem for Weakly Compatible Multi-Valued Mappings Satisfying Strongly Tangential Property

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Abstract. In this paper we prove a common fixed point theorem for two weakly compatible pairs of single and set-valued mappings which satisfying contractive condition of integral type in metric space by using the concept of strongly tangential property, our results generalize and extend some previous results.

1. Introduction

The concept of compatibility was been introduced and used by G. Jungck [8] to prove the existence of a common fixed point, this notion generalizes the weakly commuting, further there are various type of compatibility, compatibility of type (A),of type (B),of type (C) and of type (P) for two self mappings f and g of metric space (X, d) was introduced respectively in [10], [18],[17] and [16] as follows: the pair$\{f, g\}$ is compatible of type (A) if

$$\lim_{n\to\infty} d(fgx_n, g^2x_n) = 0 \quad \text{and} \quad \lim_{n\to\infty} d(gfx_n, f^2x_n) = 0,$$

f and g are compatible of type (B) if

$$\lim_{n\to\infty} d(fgx_n, g^2x_n) \leq \frac{1}{2}\Big[\lim_{n\to\infty} d(fgx_n, ft) + \lim_{n\to\infty} d(ft, f^2x_n)\Big] \quad \text{and}$$
$$\lim_{n\to\infty} d(gfx_n, f^2x_n) \leq \frac{1}{2}\Big[\lim_{n\to\infty} d(gfx_n, gt) + \lim_{n\to\infty} d(gt, g^2x_n)\Big],$$

they are compatible of type (C) if

$$\lim_{n\to\infty} d(fgx_n, g^2x_n) \leq \frac{1}{3}\Big[\lim_{n\to\infty} d(fgx_n, ft)+\lim_{n\to\infty} d(ft, f^2x_n)+\lim_{n\to\infty} d(ft, g^2x_n)\Big]$$

and

$$\lim_{n\to\infty} d(gfx_n, f^2x_n) \leq \frac{1}{3}\Big[\lim_{n\to\infty} d(gfx_n, gt)+\lim_{n\to\infty} d(gt, g^2x_n)+\lim_{n\to\infty} d(gt, f^2x_n)\Big],$$

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and said to be compatible of type (P) if

$$\lim_{n\to\infty} d(f^2x_n, g^2x_n) = 0,$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to+\infty} Sx_n = t$, for some $t \in X$.
Let, $f : X \to X$ and $S : X \to B(X)$ single and set valued mappings respectively, a point $x \in X$ is said to be a coincidence point of$f$ and S if $fx \in Sx$, it called a fixed point of S if $x \in Sx$ and a stationary(or absolutely) fixed point of S if $Sx = \{x\}$.

In 1996 , Jungck[11] introduced a concept which generalizes the all above type of compatibility and it is weaker than them: two self mappings of metric space (X, d) into itself are to be weakly compatible if they are commute at their coincidence points, i.e if $fu = gu$ for some $u \in X$, then $fgu = gfu$.
Let (X, d) be a metric space, $CB(X)$ is the set of all non-empty bounded closed subsets of X. For all $A, B \in CB(X)$ the metric of Hausdorff defined by:
$H : CB(X) \times CB(X) \to \mathbb{R}_+$ such that

$$H(A, B) = \max\Big\{\sup_{a\in A} d(a, B), \sup_{b\in B} d(b, A)\Big\},$$

where $d(a, B) = \inf_{b\in B} d(a, b)$ and $(CB(X), H)$ is a metric space.
For all $a \in A$, we have

$$d(a, B) \leq H(A, B).$$

2. Preliminaries

H. Kaneko and S. Sessa [13] extended the concept of compatibility to the setting of single and set-valued maps as follows: Let $f : X \to X$ and $S : X \to CB(X)$ two single and set-valued mappings, the pair $\{f, S\}$ is to be compatible if for all $x \in X, fSx \in CB(X)$ and

$$\lim_{n\to\infty} H(fSx_n, Sfx_n) = 0,$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = M \in CB(X)$ and $\lim_{n\to\infty} fx_n = t \in M$. Jungck and Rhoades [8] generalised the concept of weak compatibility to setting of single and set valued mappings:

Definition 2.1. Two single mapping $f : X \to X$ and set valued mapping $S : X \to CB(X)$ of metric space (X, d) are said to be weakly compatible if they commute at their coincidence point, i.e if $fu \in Su$ for some $u \in X$, then $fSu = Sfu$.

Example 2.1. Let $X = [0, \infty)$, and d the euclidian metric, we define two mappings f, S as follows:

$$fx = \frac{X+2}{3}, \qquad Sx = \begin{cases} \{1\}, & 0 \leq x \leq 2, \\ [2,4] & x > 2. \end{cases}$$

It is clear that the point $x = 1$ is coincidence point of f and S,i.e $f(1) = 1 \in \{1\} = S(1)$ and we have $fS(1) = \{1\} = Sf(1)$, then $\{f, S\}$ is weakly compatible.

Recently, Al-Thagafi and Shahzad [4] introduced the notion of occasionally weakly compatible maps in metric spaces:
Two self mappings f and g of a metric space (X, d) are to be occasionally weakly compatible (owc) if and only if there is a point $u \in X$ such that $fu = gu$ and $fgu = gfu$.
Notice that the weak compatibility implies occasional weakly compatibility, the converse may be not.
Later, Abbas and Rhoades [1] extended the occasionally weakly compatible mappings to the setting of single and set-valued mappings:

Definition 2.2. Two mappings $f : X \to X$ and $S : X \to CB(X)$ are said to be owc if and only if there exists some point u in X such that $fu \in Su$ and $fSz \subseteq Sfz$.

Example 2.2. Let $X = [0, \infty)$ and d is the euclidian metric, we define f and S as follows:

$$fx = x + 1, \quad Sx = [0, 2x + 1],$$

we have $f(1) = 2 \in [0, 3] = S(1)$, and $fS(1) = [1, 4] \subseteq [0, 5] = Sf(1)$, then f and S are owc.

Pathak and Shahzad [16] introduced the concept of tangential property as follows:
Let $f, g : X \to X$ two self mappings of metric space (X, d), a point $z \in X$ is said to be a weak tangent point to (f, g) if there exist two sequences $\{x_n\}, \{y_n\}$ in X such

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = z$$

for some $z \in X$.
In 2011, W. Sintunavarat and P. Kumam [25] extended the last notion for single and multi valued maps:

Definition 2.3. Let $f, g : X \to X$ be single mappings and $S, T : X \to B(X)$ two multi-valued mappings on metric space (X, d), the pair $\{f, g\}$ is said to be tangential with respect to $\{S, T\}$ if there exists two sequences $\{x_n\}, \{y_n\}$

in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = A,$$
$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = z \in A.$$

S. Chauhan, M. Imdad, E. Karapinar and B. Fisher [6] introduced a generalization to the last notion by adding another condition as follows:

Definition 2.4. Let $f, g : X \to X$ be single valued mappings and $S, T : X \to CB(X)$ two multi-valued mappings on metric space (X, d), the pair $\{f, g\}$ is said to be strongly tangential with respect to $\{S, T\}$ if there exists two sequences $\{x_n\}, \{y_n\}$ in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = M,$$
$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = z \in M,$$

and $z \in f(X) \cap g(X)$.

Example 2.3. Let $([0, 4]$ and d the euclidian metric, we define f, g, S and T by:

$$fx = \begin{cases} x+1, & 0 \leq x \leq 2, \\ 2, & 2 < x \leq 4, \end{cases} \qquad gx = \begin{cases} 2x+1, & 0 \leq x \leq 2, \\ 1, & 2 < x \leq 4, \end{cases}$$

$$gx = \begin{cases} [0, x+1], & 0 \leq x \leq 2, \\ [2, x], & 2 < x \leq 4, \end{cases} \quad Tx = \begin{cases} [0, 2x+1], & 0 \leq x \leq 2, \\ [2, 4], & 2 < x \leq 4. \end{cases}$$

We have $f(X) = [1, 3]$ and g$(X) = [0, 5]$, then $f(X) \cap g(X) = [1, 3]$. Consider two sequences $\{x_n\}, \{y_n\}$ which defined for all $n \geq 1$ by:

$$x_n = 2 - \frac{1}{n}, \quad y_n = 1 + \frac{1}{n}.$$

Clearly that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} T(y_n = [0, 3]$ and $\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} gy_n = 3 \in [0, 3]$, also $3 \in [1, 3) = f(X) \cap g(X)$, then $\{f, g\}$ is strongly tangential with respect to $\{S, T\}$.

If in Definition 2.4 we have $S = T$ and $f = g$ we get to the following definition:

Definition 2.5. Let $f : X \to X$ and $S : X \to B(X)$ two mappings on metric space (X, d), f is said to be strongly tangential with respect to S if

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Sy_n = M$$

and $z \in f(X)$, whenever $\{x_n\}, \{y_n\}$ two sequences in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} fy_n = z \in M.$$

Example 2.4. Let ([0, 2] with the euclidian metric, f and S defined by:

$$fx = \begin{cases} 1-x, & 0 \leq x \leq 1, \\ x, & 1 < x \leq 2, \end{cases} \qquad Sx = \begin{cases} [0, x+1], & 0 \leq x \leq 1, \\ [x-1, x], & 1 < x \leq 2. \end{cases}$$

Consider two sequence $\{x_n\}, \{y_n\}$ which defined for all $n \geq 1$ by: $x_n = \frac{1}{n}$, $y_n = 1 + \frac{1}{n}$, we have:

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} fy_n = 1,$$

$$S(x_n) = \left[0, 1 + \frac{1}{n}\right] \to [0,1] \quad \text{as} \quad n \to \infty,$$

and

$$\lim_{n\to\infty} Sy_n = [0,1],$$

also $1 \in [0,2] = f(X)$, then f is strongly tangential to respect S.

Let Φ be the set of all upper semi continuous functions $\phi : \mathbb{R}_+^5 \to \mathbb{R}_+$ satisfying the conditions:

(ϕ_1): ϕ is non decreasing in each coordinate variable.
(ϕ_2): For any $t > 0$,

$$\psi(t) = \max\Big(\phi(0,t,0,0,t), \phi(0,0,t,t,0), \phi(t,0,0,t,t)\Big) < t.$$

The aim of this paper is to prove the existence of a common fixed point for weakly compatible single and set valued mappings in metric space, which satisfying a contractive condition of integral type by using the strongly tangential property, our results generalize and extend some previous results.

3. MAIN RESULTS

Theorem 3.1. *Let $f, g : X \to X$, be single valued mappings and $S, T : X \to CB(X)$ multi-valued mappings of metric space (X, d) such for all x, y in X we have:*

(1)

$$\int_0^{H(Sx,Ty)} \varphi(t) \leq \phi\Big(\int_0^{d(fx,gy)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(fx,Sx)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(gy,Ty)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(fx,Ty)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(gy,Sx)} \varphi(t)\,\mathrm{d}\,t\Big),$$

where $\phi \in \Phi$ and $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesgue-integrable function which is summable on each compact subset of $\mathbb{R}^+$, non-negative, and such that for each $\varepsilon > 0$, $\int_0^\varepsilon \varphi(t)dt > 0$. Suppose that the two pairs $\{f, S\}, \{g, T\}$ are weakly compatible and $\{f, g\}$ is strongly tangential with respect to $\{S, T\}$, then f, g, S and T have a unique common fixed point in X.

Proof. Suppose $\{f, g\}$ is strongly tangential with respect to $\{S, T\}$, then there exists two sequences $\{x_n\}, \{y_n\}$ such

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = M, \qquad \lim_{n\to\infty} fx_n = \lim_{n\to\infty} gy_n = z \in M,$$

and $z \in f(X) \cap g(X)$, then there exists $u, v \in X$ such $z = fu = gv$, now we claim $z \in Su$, if not by using (1) we get

$$\int_0^{H(Su,Ty_n)} \varphi(t)\,\mathrm{d}\,t \le \phi\Big(\int_0^{d(fu,gy_n)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(fu,Su)} \varphi(t)\,\mathrm{d}\,t,$$
$$\int_0^{d(gy_n,Ty_n)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(fu,Ty_n)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(gy_n,Su)} \varphi(t)\,\mathrm{d}\,t\Big)$$

letting $n \to \infty$ and since $d(Su, z) \le H(Su, M)$, we get

$$\begin{aligned}\int_0^{d(Su,z)} \varphi(t) &\le \int_0^{H(Su,M)} \varphi(t)\\ &\le \phi\left(0, \int_0^{d(z,Su)} \varphi(t)\,\mathrm{d}\,t, 0, 0, \int_0^{d(z,Su)} \varphi(t)\,\mathrm{d}\,t\right)\\ &\le \psi\left(\int_0^{d(Su,z)} \varphi(t)\right) < \int_0^{d(Su,z)} \varphi(t),\end{aligned}$$

which is a contradiction with (ϕ_2), then $d(z, Su) = 0$ and so $z \in Su$.

We claim $z = gv \in Tv$, if not and using (1) we get:

$$\int_0^{H(Sx_n,Tv)} \varphi(t) \le \phi\Big(\int_0^{d(fx_n,gv)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(fx_n,Sx_n)} \varphi(t)\,\mathrm{d}\,t,$$
$$\int_0^{d(gv,Tv)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(fx_n,Tv)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(gv,Sx_n)} \varphi(t)\,\mathrm{d}\,t\Big)$$

letting $n \to \infty$ and since $d(z, Tv) \le H(M, Tv)$, we get

$$\begin{aligned}\int_0^{d(z,Tv)} \varphi(t)\,\mathrm{d}\,t &\le \int_0^{H(M,Tv)} \varphi(t)\,\mathrm{d}\,t\\ &\le \phi\Big(0, 0, \int_0^{d(z,Tv)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(z,Tv)} \varphi(t)dt, 0\Big)\\ &\le \psi\Big(\int_0^{d(z,Tv)} \varphi(t)\,\mathrm{d}\,t\Big) < \int_0^{d(z,Tv)} \varphi(t)\,\mathrm{d}\,t,\end{aligned}$$

which is a contradiction, then $z \in Tv$.

Since $\{f, S\}$ is weakly compatible and $fu \in Su$, then $fSu = Sfu$ and so $fz \in Sz$, as well as $\{g, T\}$ we obtain $gz \in Tz$.

Now, we claim $z = fz$, if not by using (1) we get:

$$\int_0^{H(Sz,Ty_n)} \varphi(t)\,\mathrm{d}\,t \leq \phi\Big(\int_0^{d(fz,gy_n)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(fz,Sz)} \varphi(t)\,\mathrm{d}\,t,$$
$$\int_0^{d(gy_n,Ty_n)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(fz,Ty_n)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(gy_n,Sz)} \varphi(t)\,\mathrm{d}\,t\Big),$$

letting $n \to \infty$, since $d(z, fz) \leq H(M, Sz)$ and applying the triangle inequality we get $d(fz, M) \leq d(fz, z) + d(z, M) = d(fz, z)$, then:

$$\begin{aligned}\int_0^{d(z,fz)} \varphi(t)\,\mathrm{d}\,t &\leq \int_0^{H(Sz,M)} \varphi(t)\,\mathrm{d}\,t \\ &\leq \phi\Big(\int_0^{d(z,fz)} \varphi(t)\,\mathrm{d}\,t, 0, 0, \int_0^{d(fz,M)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(fz,M)} \varphi(t)\,\mathrm{d}\,t\Big) \\ &\leq \phi\Big(\int_0^{d(z,fz)} \varphi(t)\,\mathrm{d}\,t, 0, 0, \int_0^{d(fz,z)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(fz,z)} \varphi(t)\,\mathrm{d}\,t\Big) \\ &\leq \psi\Big(\int_0^{d(z,fz)} \varphi(t)\,\mathrm{d}\,t\Big) < \int_0^{d(z,fz)} \varphi(t)\,\mathrm{d}\,t,\end{aligned}$$

which is a contradiction, then $z = fz \in Sz$ which implies that z is common fixed point of f and S.

Similarly, we claim $z = gz$, if not by using (1) we get:

$$\int_0^{H(Sx_n,Tz)} \varphi(t)\,\mathrm{d}\,t \leq \phi\Big(\int_0^{d(fx_n,gz)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(fx_n,Sx_n)} \varphi(t)\,\mathrm{d}\,t,$$
$$\int_0^{d(gz,Tz)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(fx_n,Tz)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(gz,Sx_n)} \varphi(t)\,\mathrm{d}\,t\Big)$$

letting $n \to \infty$, since $d(z, gz) \leq H(M, Tz)$ and $d(gz, M) \leq d(gz, z)$, we get

$$\begin{aligned}\int_0^{d(z,gz)} \varphi(t)\,\mathrm{d}\,t &\leq \int_0^{H(Tz,M)} \varphi(t)\,\mathrm{d}\,t \\ &\leq \phi\Big(\int_0^{d(z,gz)} \varphi(t)\,\mathrm{d}\,t, 0, 0, \int_0^{d(z,Tz)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(gz,M)} \varphi(t)\,\mathrm{d}\,t\Big) \\ &\leq \phi\Big(\int_0^{d(z,gz)} \varphi(t)\,\mathrm{d}\,t, 0, 0, \int_0^{d(z,gz)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(gz,z)} \varphi(t)\,\mathrm{d}\,t\Big) \\ &\leq \psi\Big(\int_0^{d(z,gz)} \varphi(t)\,\mathrm{d}\,t\Big) < \int_0^{d(z,gz)} \varphi(t)\,\mathrm{d}\,t,\end{aligned}$$

which is a contradicts (ϕ_2), then $z = gz \in Tz$, consequently z is common fixed point of f, g, S and T.

For the uniqueness, suppose there is an other point w satisfying $w = fw = gw \in Sw = T$, if $w \neq z$ by using (1) we get:

$$\begin{aligned}\int_0^{d(z,w)} \varphi(t)\,\mathrm{d}\,t = \int_0^{\delta(Sz,Tw)} \varphi(t)\,\mathrm{d}\,t &\le \phi\Big(\int_0^{d(fz,gw)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(fz,Sz)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(gw,Tw)} \varphi(t)\,\mathrm{d}\,t,\\ &\qquad \int_0^{d(fz,Tw)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(gw,Sz)} \varphi(t)\,\mathrm{d}\,t\Big)\\ &\le \phi\Big(\int_0^{d(z,w)} \varphi(t)\,\mathrm{d}\,t, 0, 0, \int_0^{d(z,w)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(z,w)} \varphi(t)\,\mathrm{d}\,t\Big)\\ &\le \psi(\int_0^{d(z,w)} \varphi(t)\,\mathrm{d}\,t) < \int_0^{d(z,w)} \varphi(t)\,\mathrm{d}\,t),\end{aligned}$$

which is a contradiction, then $z = w$. □

If $S = T$ and $f = g$, we obtain the following corollary:

Corollary 3.1. *Let $f : X \to X$, and $S : X \to CB(X)$ be single and set valued mappings of metric space (X, d) such:*

$$\begin{aligned}\int_0^{H(Sx,Sy)} \varphi(t) \le \phi\Big(&\int_0^{d(fx,fy)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(fx,Sx)} \varphi(t)\,\mathrm{d}\,t,\\ &\int_0^{d(fy,Sy)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(fx,Sy)} \varphi(t)\,\mathrm{d}\,t, \int_0^{d(fy,Sx)} \varphi(t)\,\mathrm{d}\,t\Big),\end{aligned}$$

where $\phi \in \Phi$ and $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesgue-integrable function which is summable on each compact subset of $\mathbb{R}^+$, non-negative, and such that for each $\varepsilon > 0, \int_0^{\varepsilon} \varphi(t)dt > 0$, if f is strongly tangential with respect to S and $\{f, S\}$ is weakly compatible, then f and S have a unique common fixed point.

Corollary 3.2. *Let $f, g : X \to X$, and $S, T : X \to CB(X)$ be single and set valued mappings of metric space (X, d) such:*

$$\begin{aligned}\Big(\int_0^{H(Sx,Ty)} \varphi(t)\,\mathrm{d}\,t\Big)^p \le a\Big(&\int_0^{d(fx,gy)} \varphi(t)\,\mathrm{d}\,t\Big)^p\\ &+ b\Big(\int_0^{d(fx,Sx)} \varphi(t)\,\mathrm{d}\,t\Big)^p + c\Big(\int_0^{d(gy,Ty)} \varphi(t)\,\mathrm{d}\,t\Big)^p,\end{aligned}$$

where $\phi \in \Phi$ and $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a Lebesgue-integrable function which is summable on each compact subset of $\mathbb{R}^+$, non-negative, and such that for each $\varepsilon > 0$, $\int_0^{\varepsilon} \varphi(t)\,\mathrm{d}\,t > 0$. and a, b, c are nonnegative real numbers such $a + b + b < 1$ and $p \in \mathbb{N}^\star$, if $\{f, g\}$ is strongly tangential with respect to $\{S, T\}$ and the two pairs $\{f, g\}, \{S, T\}$ are weakly compatible, then f, g, S and T have a unique common fixed point.

Proof. It suffices to show that the function

$$\phi(t_1, t_2, t_3, t_4, t_5) = (at_1^p + bt_2^p + ct_3^p)^{\frac{1}{p}},$$

where a, b and c are non negative real numbers such $a+b+c < 1$ and $p \in \mathbb{N}^*$, clearly that $\phi \in \Phi$ and so all hypothesis of Theorem 3.1 satisfied. $\square$

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