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Well-Posedness of Fixed Point Problem for a Multifunction Satisfying an Implicit Relation

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Abstract. The notion of well-posedness of a fixed point problem for a single valued mapping has generated much interest to a several mathematicians, for examples, F.S. De Blassi and J. Myjak (1989), S. Reich and A. J. Zaslavski (2001), B.K. Lahiri and P. Das (2005) and V. Popa (2006 and 2008). In this paper we extend the notion of well-posedness known for single valued mappings to the case of multifunctions. We establish the well-posedness of fixed point problem for a multifunction satisfying an implicit relation in orbitally complete metric spaces.

1. Introduction

Throughout this paper, $\mathbb{N}$ will be the set of non negative integers. Let (X, d) be a metric space and $B(X)$ the set of all nonempty bounded sets of X. As in [6], [7] and [8], we define the functions $\delta(A, B)$ and $D(A, B)$ by

$$\delta(A, B) := \sup\{d(a, b) : a \in A,\ b \in B\},$$
$$D(A, B) := \inf\{d(a, b) : a \in A,\ b \in B\}.$$

If A consists of single point "a", we write $\delta(A, B) = \delta(a, B)$.
If B consists of single point "b", we write $\delta(A, B) = \delta(A, b)$.

It follows immediately from the definition of $\delta(A, B)$ that

$$\delta(A, B) = \delta(B, A), \quad \forall A, B \in B(X),$$

and

$$\delta(A, B) \le \delta(A, C) + \delta(C, B), \quad \forall A, B, C \in B(X).$$

Definition 1.1. A sequence $\{A_n\}$ of nonempty subsets of X is said to converge to a subset A of X if:

(i) Each point $a \in A$ is the limit of a convergent sequence $\{a_n\}$, where $a_n \in A_n$, for all $n \in \mathbb{N}$.

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(ii) For arbitrary $\epsilon > 0$ there exists an integer $m > 0$ such that $A_n \subset A(\epsilon)$, where
$$A(\epsilon) := \{x \in X : \exists a \in A : d(x,a) < \epsilon\}.$$

The set A is said to be the limit of the sequence $\{A_n\}$.

Lemma 1.1 (Fisher [6]). *If $\{A_n\}$ and $\{B_n\}$ are two sequences in $B(X)$ converging to the sets A and B respectively in $B(X)$, then the sequence $\{\delta(A_n, B_n)\}$ converges to $\delta(A,B)$.*

Lemma 1.2 (Fisher and Sessa [8]). *Let $\{A_n\}$ be a sequence in $B(X)$ and $y \in X$ such that $\lim_{n\to\infty} \delta(A_n, y) = 0$. Then the sequence $\{A_n\}$ converges to $\{y\}$ in $B(X)$.*

Definition 1.2. Let $F : X \to B(X)$ be a multifunction.

a) A point $x \in X$ is a fixed point of F if $x \in Fx$.
b) A point $x \in X$ is a strict fixed point of F if $\{x\} = Fx$.

The importance of orbits of points under self-mappings in metric spaces is well recognized. In many early papers dealing with fixed point theory, the orbits were used to investigate fixed points. (See for example [5], [3] and others).

We recall the following definition (see for instance [2], [3] and others).

Definition 1.3. Let $f : (X,d) \to (X,d)$. If for any $x \in X$, every Cauchy sequence of the orbit $O(f,x) := \{x, fx, f^2x, \ldots\}$ is convergent in X, then the metric space is said to be f-orbitally complete.

Remark 1.1. *Every complete metric space is f-orbitally complete for any f. An orbitally complete space may not be complete metric space (see [15]).*

Let $F : X \to B(X)$ and $x_0 \in X$. An orbit of F at point x_0, is a sequence $\{x_n\}$ given by
$$O(F, x_0) := \{x_n : x_{n+1} \in F(x_n),\ n = 0, 1, 2, \ldots\}.$$

Definition 1.4. Let (X,d) be a metric space. Let $F : X \to B(X)$ be a multifunction. (X,d) is called to be F-orbitally complete, if for all $x \in X$, every Cauchy subsequence of the orbit $O(F,x)$ converges to a point in X.

The notion of well-posedness of a fixed point problem has evoked much interest to several mathematicians (see for example [14], [4], [9], [12], [13] and [1]).

Definition 1.5. Let (X,d) be a metric space and $f : (X,d) \to (X,d)$ be a mapping. The fixed point problem of f is said to be well posed if:

(i) f has a unique fixed point z in X,
(ii) for any sequence $\{x_n\}$ of points in X such that $\lim_{n\to\infty} d(Tx_n, x_n) = 0$, we have $\lim_{n\to\infty} d(x_n, z) = 0$.

We extend Definition 1.5 for multifunctions.

Definition 1.6. Let (X, d) be a metric space and $F : X \to B(X)$ be a multifunction. The fixed point problem of F is said to be well-posed if:

(i) F has a unique strict fixed point z in X,
(ii) for any sequence $\{x_n\}$ of points in X such that $\lim_{n\to\infty} \delta(Fx_n, x_n) = 0$, we have $\lim_{n\to\infty} d(x_n, z) = 0$.

The study of fixed point for mappings satisfying an implicit relation is initiated and studied in [10] and [11].

In this paper we prove a general fixed point theorem for multifunctions satisfying an implicit relation in orbitally complete metric spaces and that fixed point problem is well-posed generalizing some results from [1] and [9].

2. Implicit relations

Let $\phi(t_1, \ldots, t_6) : \mathbb{R}^6 \to \mathbb{R}$ be a continuous function. We define the following properties:

(ϕ_1): ϕ is non-increasing in the variables t_2, t_5 and t_6 and non-decreasing in the variable t_1.
(ϕ_2): There exists a real number $h \in (0,1)$ such that for every $u \geq 0$, $v \geq 0$ with $\phi(u, v, v, u, u+v, 0) \leq 0$, we have $u \leq hv$.
(ϕ_3): $\phi(t, t, 0, 0, t, t) > 0$, for every $t > 0$.
(ϕ_p): There exists $p \in (0,1)$ such that for every $u \geq 0$, $v \geq 0$, $w \geq 0$ with $\phi(u, v, 0, w, u, v) \leq 0$, we have $u \leq p \max\{v, w\}$.

Example 2.1. $\phi(t_1, \ldots, t_6) = t_1 - c \max\{t_2, t_3, t_4, \frac{1}{2}(t_5 + t_6)\}$, where $c \in (0,1)$.

(ϕ_1): Obviously.
(ϕ_2): For all $u, v \geq 0$, we have

$$\phi(u, v, v, u, u+v, 0) = u - c \max\{u, v, \frac{1}{2}(u+v)\} = u - c \max\{u, v\}. \tag{2.1}$$

Suppose that $\phi(u, v, v, u, u+v, 0) \leq 0$ and that $u > v$. Then, from (2.1), we get $u(1-c) \leq 0$, a contradiction. Therefore $u \leq v$, which yields (by (2.1)) that $u \leq cv$. Thus (ϕ_2) is true with $h := c \in (0,1)$.

(ϕ_3): $\phi(t, t, 0, 0, t, t) = t(1-c) > 0$ for all $t > 0$.
(ϕ_p): For all $u, v, w \geq 0$, we have

$$\phi(u, v, 0, w, u, v) = u - c \max\{v, w, \frac{1}{2}(u+v)\}.$$

Suppose that $\phi(u, v, 0, w, u, v) \leq 0$, with $u > 0$ and $u \geq \max\{v, w\}$. Then we have $u(1-c) \leq 0$, a contradiction. Hence, $0 < u \leq \max\{v, w\}$, which implies that $\frac{1}{2}(u+v) \leq \max\{v, w\}$. Thus, we get $u \leq c \max\{v, w\}$. If $u = 0$, then $u \leq c \max\{v, w\}$. This shows that (ϕ_p) is true with $p := c \in (0,1)$.

Example 2.2. $\phi(t_1, \ldots, t_6) = t_1 - a_1t_2 - a_2t_3 - a_3t_4 - a_4t_5 - a_5t_6$, where $a_i \geq 0$ for $i = 1, 2, \ldots, 5$, $a_1 + a_3 + a_5 > 0$, $0 < a_1 + a_3 + a_4 + a_5 < 1$ and $0 < a_1 + a_2 + a_3 + 2a_4 < 1$.

(ϕ_1): Obviously.

(ϕ_2): For all $u, v \geq 0$, we have

$$\phi(u, v, v, u, u + v, 0) = u(1 - a_3 - a_4) - v(a_1 + a_2 + a_4). \tag{2.2}$$

If $\phi(u, v, v, u, u + v, 0) \leq 0$, then $u \leq hv$, where $h := \frac{a_1+a_2+a_4}{1-a_3-a_4}$. By assumptions, we have $h \in (0, 1)$.

(ϕ_3): $\phi(t, t, 0, 0, t, t) = t(1 - a_1 - a_4 - a_5) > 0$ for all $t > 0$.

(ϕ_p): For all $u, v, w \geq 0$, we have

$$\phi(u, v, 0, w, u, v) = u(1 - a_4) - v(a_1 + a_5) - a_3w.$$

Suppose that $\phi(u, v, 0, w, u, v) \leq 0$, then

$$u(1 - a_4) \leq v(a_1 + a_5) + a_3w \leq (a_1 + a_3 + a_5) \max\{v, w\}.$$

Thus $u \leq p \max\{v, w\}$, where $p := \frac{a_1+a_3+a_5}{1-a_4}$. By assumptions, we have $p \in (0, 1)$.

Example 2.3. $\phi(t_1, \ldots, t_6) = t_1^2 - at_2t_3 - bt_3t_4 - ct_5t_6$, where $a > 0$, $b, c \geq 0$, $a + b < 1$ and $a + c < 1$.

(ϕ_1): Obviously.

(ϕ_2): For all $u, v \geq 0$, we have

$$\phi(u, v, v, u, u + v, 0) = u^2 - av^2 - buv. \tag{2.3}$$

Let $v > 0$ and $f(t) = t^2 - bt - a$, where $t = \frac{u}{v}$. We observe that $f(0) = -a < 0$ and $f(1) = 1 - (a + b) > 0$. Then there exists $h \in (0, 1)$ suc that $f(h) = 0$. Since the other root of the equation $f(t) = 0$ is strictly negative, then the inequality $f(t) \leq 0$ $(t \geq 0)$ implies that $t \leq h$. Thus, if $\phi(u, v, v, u, u+v, 0) \leq 0$ with $v > 0$, then we have $u \leq h\,v$. If $v = 0$, then from (2.3) we get $u = 0$. Therefore $u \leq hv$.

(ϕ_3): $\phi(t, t, 0, 0, t, t) = t^2(1 - c) > 0$ for all $t > 0$.

(ϕ_p): For all $u, v, w \geq 0$, we have

$$\phi(u, v, 0, w, u, v) = u^2 - cuv.$$

Suppose that $\phi(u, v, 0, w, u, v) \leq 0$ and $u > 0$. Then we obtain $u \leq cv \leq \max\{v, w\}$. Thus $u \leq p \max\{v, w\}$, where $p := c \in (0, 1)$. If $u = 0$, then $u \leq p \max\{v, w\}$. This shows that (ϕ_p) is satisfied.

Example 2.4. $\phi(t_1, \ldots, t_6) = t_1 - at_2 - bt_3 - ct_4 - d \min\{t_5, t_6\}$, where $a, b, c \geq 0$, $0 < a + b \leq a + b + c < 1$ and $0 < a + c + d < 1$.

(ϕ_1): Obviously.

(ϕ_2): For all $u, v \geq 0$, we have

$$\phi(u, v, v, u, u+v, 0) = u(1-c) - v(a+b). \tag{2.4}$$

Suppose that $\phi(u, v, v, u, u+v, 0) \leq 0$. Then $u \leq hv$, where $h := \frac{a+b}{1-c} \in (0, 1)$. Hence ($\phi_2$) is satisfied.

(ϕ_u): $\phi(t, t, 0, 0, t, t) = t(1-a-d) > 0$ for all $t > 0$.

(ϕ_p): For all $u, v, w \geq 0$, we have

$$\phi(u, v, 0, w, u, v) = u - av - cw - d\min\{u, v\}.$$

If $\phi(u, v, 0, w, u, v) \leq 0$ and $u > \max\{u, v\}$, then we obtain $u(1 - a - c - d) \leq 0$, a contradiction. Hence $u \leq \max\{v, w\}$, and then $u \leq p\max\{v, w\}$, where $p := a + c + d \in (0, 1)$. This proves that ($\phi_p$) is satisfied.

Example 2.5. $\phi(t_1, \ldots, t_6) = t_1 - c\max\{t_2, t_3, \sqrt{t_4 t_6}, \sqrt{t_5 t_6}\}$, where $0 < c < 1$.

(ϕ_1): Obviously.

(ϕ_2): For all $u, v \geq 0$, we have

$$\phi(u, v, v, u, u+v, 0) = u - cv. \tag{2.5}$$

If $\phi(u, v, v, u, u+v, 0) \leq 0$, then $u \leq hv$, where $h := c \in (0, 1)$.

(ϕ_3): $\phi(t, t, 0, 0, t, t) = t(1-c) > 0$ for all $t > 0$. Because $c \in (0, 1)$.

(ϕ_p): For all $u, v, w \geq 0$, we have

$$\phi(u, v, 0, w, u, v) = u - c\max\{v, \sqrt{wv}, \sqrt{uv}\}.$$

If $\phi(u, v, 0, w, u, v) \leq 0$ and $u > \max\{v, w\}$, then we obtain $u(1-c) \leq 0$, a contradiction. Hence $u \leq \max\{v, w\}$ and therefore we have $u \leq p\max\{v, w\}$, where $p := c \in (0, 1)$. This proves that (ϕ_p) is satisfied.

Remark 2.1. *There exists $\phi : \mathbb{R}^6 \to \mathbb{R}$ increasing in variables t_3, t_4 which satisfies properties (ϕ_1), (ϕ_2), (ϕ_3) and (ϕ_p).*

Example 2.6. $\phi(t_1, \ldots, t_6) = t_1^2 - at_2^2 - b\frac{t_5 t_6}{1+t_3+t_4}$, where $a > 0$, $b \geq 0$ and $0 < a + b < 1$.

(ϕ_1): Obviously.

(ϕ_2): For all $u, v \geq 0$, we have

$$\phi(u, v, v, u, u+v, 0) = u^2 - av^2.$$

If $\phi(u, v, v, u, u+v, 0) \leq 0$, then $u \leq hv$, where $h := a \in (0, 1)$. Hence (ϕ_2) is satisfied.

(ϕ_3): $\phi(t, t, 0, 0, t, t) = t^2(1-a-b) > 0$ for all $t > 0$.

(ϕ_p): For all $u, v, w \geq 0$, we have

$$\phi(u, v, 0, w, u, v) = u^2 - av^2 - b\frac{uv}{1+w}.$$

If $\phi(u, v, 0, w, u, v) \leq 0$, then $u^2 - av^2 - buv \leq 0$. As in the proof of (ϕ_2) in Example 2.3, we obtain $u \leq p \max\{v, w\}$, for some $p \in (0, 1)$. This proves that (ϕ_p) is satisfied.

3. Main results

Theorem 3.1. *Let (X, d) be a metric space and $F : X \to B(X)$ a mulifunction such that*

$$\phi\Big(\delta(Fx, Fy), d(x, y), \delta(x, Fx), \delta(y, Fy), D(x, Fy), D(y, Fx)\Big) \leq 0, \tag{3.1}$$

for all $x, y \in X$, where ϕ satisfies property (ϕ_3), then F has at most one strict fixed point in X.

Proof. Suppose that z and y are strict fixed points of F with $z \neq y$. Then $\{z\} = Fz$ and $\{y\} = Fy$. By (3.1) we obtain

$$\phi\big(\delta(Fz, Fy), d(z, y), \delta(z, Fz), \delta(y, Fy), D(z, Fy), D(y, Fz)\big) =$$
$$= \phi\big(d(z, y), d(z, y), 0, 0, d(z, y), d(z, y)\big) \leq 0$$

a contradiction of (ϕ_3). □

Theorem 3.2. *Let (X, d) be a metric space and $F : X \to B(X)$ a mulifunction such that*

$$\phi\left(\delta(Fx, Fy), d(x, y), \delta(x, Fx), \delta(y, Fy), D(x, Fy), D(y, Fx)\right) \leq 0, \tag{3.1}$$

for all $x, y \in X$, where ϕ satisfies properties (ϕ_1), (ϕ_2) and (ϕ_3). Then F has an unique fixed point in X which is strict fixed point for F.

Proof. Let x_0 be any arbitrary point in X and consider the orbit of F at x_0 given by the sequence $\{x_n\}$ such that $x_{n+1} \in Fx_n$ for all integers $n = 0, 1, 2, \ldots$. Then by (3.1), we have

$$\phi\Big(\delta(Fx_n, Fx_{n+1}), d(x_n, x_{n+1}), \delta(x_n, Fx_n),$$
$$\delta(x_{n+1}, Fx_{n+1}), D(x_n, Fx_{n+1}), D(x_{n+1}, Fx_n)\Big) \leq 0$$

Since $D(x_{n+1}, Fx_n) = 0$, $\delta(Fx_n, Fx_{n+1}) \geq \delta(x_{n+1}, Fx_{n+1})$ and ϕ in non-decreasing in the variable t_1 then we have

$$\phi\left(\delta(x_{n+1}, Fx_{n+1}), d(x_n, x_{n+1}), \delta(x_n, Fx_n), \delta(x_{n+1}, Fx_{n+1}), D(x_n, Fx_{n+1}), 0\right) \leq 0.$$

Since $d(x_n, x_{n+1}) \leq \delta(x_n, Fx_n)$, $D(x_n, Fx_{n+1}) \leq d(x_n, x_{n+1}) + \delta(x_{n+1}, Fx_{n+1})$ and ϕ is non-increasing in the variables t_2 and t_5 then we get

$$\phi\Big(\delta(x_{n+1}, Fx_{n+1}), \delta(x_n, Fx_n), \delta(x_n, Fx_n),$$
$$\delta(x_{n+1}, Fx_{n+1}), \delta(x_n, Fx_n) + \delta(x_{n+1}, Fx_{n+1}), 0\Big) \leq 0.$$

By property (ϕ_2), we have $\delta(x_{n+1}, Fx_{n+1}) \leq h\, \delta(x_n, Fx_n)$ and so

$$\delta(x_n, Fx_n) \leq h^n \delta(x_0, Fx_0), \quad \forall n \geq 0. \tag{3.2}$$

(3.2) shows that the sequence $\{\delta(x_n, Fx_n)\}$ is a strongly Cauchy sequence (that is $\sum_{n\geq 0}\delta(x_n, Fx_n)$ converges). Since $d(x_n, x_{n+1}) \leq \delta(x_n, Fx_n)$, then the sequence $\{d(x_n, x_{n+1})\}$ is also a strongly Cauchy sequence. It follows that $\{x_n\}$ is a Cauchy sequence in the orbit $O(F, x_0)$. Since (X, d) is F-orbitally complete, the sequence $\{x_n\}$ is convergent to a point $z \in X$. We prove that $\{z\} = Fz$. For each positive integer n, we have

$$\delta(Fx_n, z) \leq \delta(Fx_n, x_n) + d(x_n, z).$$

By (3.2) we obtain that $\lim_{n\to\infty}\delta(Fx_n, z) = 0$. Then by Lemma 1.2, the sequence $\{Fx_n\}$ converges to the set $\{z\}$ in $B(X)$. By the inequality (3.1) for $x := x_n$ and $y := z$, we obtain

$$\phi\left(\delta(Fx_n, Fz), d(x_n, z), \delta(x_n, Fx_n), \delta(z, Fz), D(x_n, Fz), D(z, Fx_n)\right) \leq 0,$$

which (since ϕ is non-increasing in variables t_5, t_6) implies

$$\phi\left(\delta(Fx_n, Fz), d(x_n, z), \delta(x_n, Fx_n), \delta(z, Fz), \delta(x_n, Fz), \delta(z, Fx_n)\right) \leq 0.$$

Letting n tend to infinity we obtain

$$\phi\left(\delta(z, Fz), 0, 0, \delta(z, Fz), \delta(z, Fz), 0)\right) \leq 0.$$

By property (ϕ_2), we obtain $\delta(z, Fz) = 0$, i.e., $\{z\} = Fz$. Therefore z is a strict fixed point for F. By Theorem 3.1, z is the unique stricte fixed point for F. This completes the proof. □

If F is single-valued, Then the proof of Theorem 3.1 does not need the assumption (ϕ_1). So we recapture Theorem 3.1 of [1].

Corollary 3.1 (Theorem 3.1 [1]). *Let (X, d) be a metric space and let $T : X \to X$ be a self-mapping. Suppose that (X, d) is T-orbitally complete and that T satisfies the inequality*

$$\phi\left(d(Tx, Ty), d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\right) \leq 0, \tag{3.3}$$

for all $x, y \in X$, where ϕ satisfies properties (ϕ_2) and (ϕ_3). Then T has a unique fixed point in X.

Theorem 3.3. *Let (X, d) be a metric space and $F : X \to B(X)$ a muli-function such that*

$$\phi\left(\delta(Fx, Fy), d(x, y), \delta(x, Fx), \delta(y, Fy), D(x, Fy), D(y, Fx)\right) \leq 0, \tag{3.1}$$

for all $x, y \in X$, where ϕ satisfies properties (ϕ_1), (ϕ_2), (ϕ_3) and (ϕ_p). Then the fixed point problem for F is well posed.

Proof. By Theorem 3.2, F has a unique strict fixed point z. Let $\{x_n\}$ be a sequence in X such that

$$\lim_{n\to\infty}\delta(x_n, Fx_n) = 0.$$

By inequality (3.1) we obtain

$$\phi\left(\delta(Fz, Fx_n), d(z, x_n), \delta(z, Fz), \delta(x_n, Fx_n), D(z, Fx_n), D(x_n, Fz)\right) \leq 0.$$

Since $\{z\} = Fz$, the previous inequality is equivalent to the following

$$\phi\left(\delta(z, Fx_n), d(z, x_n), 0, \delta(x_n, Fx_n), D(z, Fx_n), d(x_n, z)\right) \leq 0.$$

Since ϕ is non-increasing in the variable t_5, then we have

$$\phi\left(\delta(z, Fx_n), d(z, x_n), 0, \delta(x_n, Fx_n), \delta(z, Fx_n), d(x_n, z)\right) \leq 0.$$

By (ϕ_p), we have

$$\delta(z, Fx_n) \leq p \max\{d(z, x_n), \delta(x_n, Fx_n)\}.$$

On the other hand, we have

$$d(z, x_n) \leq \delta(z, Fx_n) + \delta(Fx_n, x_n) \leq p[d(z, x_n), \delta(x_n, Fx_n)] + \delta(Fx_n, x_n),$$

which implies that

$$d(z, x_n) \leq \frac{1+p}{1-p}\,\delta(x_n, Fx_n) \quad \to 0 \quad \text{as} \quad n \to \infty.$$

Hence $\lim_{n\to\infty} d(z, x_n) = 0$ and the fixed point problem of F is well-posed. □

As a consequence, we have the following result.

Corollary 3.2 (Theorem 3.2 [1]). *Let (X, d) be a metric space and let $T : X \to X$ be a self-mapping. Suppose that (X, d) is T-orbitally complete and that T satisfies the inequality*

$$\phi\left(d(Tx, Ty), d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\right) \leq 0, \tag{3.3}$$

for all $x, y \in X$, where ϕ satisfies properties (ϕ_2), (ϕ_3) and (ϕ_p). Then the fixed point problem of T is well-posed.

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