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APPLICATION OF AN EXPANDED MIN–MAX THEOREM AND TABLES OF DECISION MAKING FOR SOLVING MULTI–CRITERION CONFLICT SITUATIONS

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Abstract. This paper proves that it is possible to define optimal strategy for multi-criterion conflict situation using Min-max theorem of Tasković in conjunction with tables of decision making.

1. Introductory notes

Majority of practical multi-criterion conflict problems give themselves to Modelling to multi-criterion games. An optimal solution can be obtained using Tasković's Min-max theorem and tables of decision making. Tables of decision making are considered to be a language or means to define, analyse and solve a problem of decision making.

A general form of a decision making table is as shown in the Figure 1.

Conditions	(Not) meeting conditions
Actions (procedures)	(Not) activating procedures

Figure 1. General form of a decison making table

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2. Fundamentals of Min-max theorem for solving multicriterion games

For the problem involving two players with many criteria, however, the following stands true [1 and 3]:

$$\begin{aligned}
&\mu_{R_1\cdot R_2\cdots R_n}(x,y;z_1,z_2,\dots,z_n)=\\
&=\min_{x,y,z_1,\dots,z_n\in[0,1]}\max\left[\mu_{R_1}(x,y;z_1),\dots,\mu_{R_n}(x,y;z_n)\right.,\\
&\quad\left.,g(\mu_{R_1}(x,y;z_1),\dots,\mu_{R_n}(x,y;z_n))\right]=\\
&=\max_{x,y,z_1,\dots,z_n\in[0,1]}\min\left[\mu_{R_1}(x,y;z_1),\dots,\mu_{R_n}(x,y;z_n)\right.,\\
&\quad\left.,g(\mu_{R_1}(x,y;z_1),\dots,\mu_{R_n}(x,y;z_n))\right].
\end{aligned}$$

With multicriteria conflict situations, the function $g(X)$ is defined through priority order vector I and priority value vector V:

$$g(X)=g(x_1,x_2,\dots,x_k),\qquad g(x_1,x_2,\dots,x_k)=\Lambda,$$

where Λ is a vector of weight coefficient.

This vector is calculated on the basis of priority vector: $V(v_1,v_2,\dots,v_n)$, for $v_i\in[0,1]$, form the area $\mathbf{R}^n$ and determined in accordance with priority order vector: $I(i_1,i_2,\dots,i_n)$.

There e_q,e_{q+1} denote the importance of criteria. Some equations stand thrue as follows:

$$V_q=\frac{e_q}{e_{q+1}},\quad 0\le\lambda_q<1,\quad q\in[1,2,\dots,k],$$

$$\sum_{q=1}^{k}\lambda_q=\lambda_1+\lambda_2+\dots+\lambda_k=1,\quad \text{for } \lambda_q\ge\lambda_{q+1}.$$

The vectors Λ and V are related as follows:

$$v_q=\left(\frac{\lambda_q}{\lambda_{q+1}}\right).$$

This relationship is the basis for determinatin of the function $g(X)$ and the vector Λ. It is shown by the following expression:

$$\lambda_q=\frac{\prod_{i=q}^{k}v_i}{\sum_{q=1}^{k}\prod_{j=q}^{k}v_i}.$$

The given expression can be derived as follows:

$$v_1 = \lambda_1/\lambda_2, \quad v_2 = \lambda_2/\lambda_3, \ldots, v_{k-1} = \lambda_{k-1}/\lambda_k, \quad v_k = 1$$

resulting in:

$$\prod_{i=1}^{q-1} v_i = v_1 \cdot v_2 \cdot \ldots \cdot v_{q-1} = \frac{\lambda_1 \cdot \lambda_2 \cdot \ldots \cdot \lambda_{q-1}}{\lambda_2 \cdot \ldots, \cdot \lambda_{q-1} \cdot \lambda_q} = \frac{\lambda_1}{\lambda_q},$$

$$1 + \sum_{i=q}^{k} \frac{1}{\prod_{i=1}^{q-1} v_i} = \frac{\sum_{q=1}^{k} \prod_{i=q}^{k} v_i}{\prod_{i=q}^{k} v_i},$$

resulting in $\lambda_1 = \prod_{i=q}^{k} v_i \Big/ \sum_{q=1}^{k} \prod_{i=q}^{k} v_i$, the relationship between Λ and V.

3. Example

Types of emergency situations (US) (A) to be solved are traffic accidents (SbN) involving passenger cars (a_1), SbN with lorries (a_2), SbN with military vehicles (a_3) and SbN involving other vehicles (a_4). Conditions (S) under which SbN_s take place are activites in sevice (s_1) and out of service (s_2). To make a choice of the most suitable solution, we consider a situation from three major aspects – criterion (cause) such as lower concentration (z_1) behaviour of a person at work (z_2), ownership (z_3).

Order of priority we set first: $i = (i_1, i_2, i_3)$.

We allocate a payment matrix for each criterion (z_1, z_2, z_3) in accordance with possibilities of actions A and conditions S and order of priority.

For criterion z_1:

A/S	a_1	a_2	a_3	a_4
s_1	1	2	5	3
s_2	2	8	0	1

For criterion z_2:

A/S	a_1	a_2	a_3	a_4
s_1	4	3	1	8
s_2	1	5	2	2

For criterion z_3:

A/S	a_1	a_2	a_3	a_4
s_1	4	4	2	1
s_2	2	1	2	4

Step 1: To solve games according to each criterion

For criterion z_1:

	a_1	X				X			
A	a_2		X				X		
	a_3			X				X	
	a_4				X				X
S	s_1	X	X	X	X				
	s_2					X	X	X	X
	z_1	X	X	X	X	X	X	X	X
K	z_2								
	z_3								
$V(A_i, S_j, K_z)$		1	2	5	3	2	8	0	1
$A_{1/2}$		0,4300				0,5700			
$S_{1/2}$		0,7800				0,2200			
$V(A,S,K) = \mu_R(a,s;z_1) = 1,6700$									

For criterion z_2:

	a_1	X				X			
A	a_2		X				X		
	a_3			X				X	
	a_4				X				X
S	s_1	X	X	X	X				
	s_2					X	X	X	X
	z_1								
K	z_2	X	X	X	X	X	X	X	X
	z_3								
$V(A_i, S_j, K_z)$		4	3	1	8	1	5	2	2
$A_{1/2}$		0,1250				0,8750			
$S_{1/2}$		0,8333				0,1667			
$V(A,S,K) = \mu_R(a,s;z_2) = 1,7500$									

For criterion z_3:

	a_1	X				X			
A	a_2		X				X		
	a_3			X				X	
	a_4				X				X
S	s_1	X	X	X	X				
	s_2					X	X	X	X
	z_1								
K	z_2								
	z_3	X	X	X	X	X	X	X	X
$V(A_i, S_j, K_z)$		4	4	2	1	2	1	2	4
$A_{1/2}$		0,7550				0,2450			
$S_{1/2}$		0,6500				0,3500			
$V(A,S,K) = \mu_R(a,s;z_3) = 2,0600$									

Adding matrixes using unique criterion:

$$\begin{bmatrix} 1,6060 & 2,2460 & 4,2480 & 37240 \\ 1,8480 & 72440 & 0,4080 & 1,2860 \end{bmatrix}$$

	a_1	X				X			
A	a_2		X				X		
	a_3			X				X	
	a_4				X				X
S	s_1	X	X	X	X				
	s_2					X	X	X	X
	z_1	X	X	X	X	X	X	X	X
K	z_2	X	X	X	X	X	X	X	X
	z_3	X	X	X	X	X	X	X	X
$V(A_i, S_j, K_z)$		2	2	4	4	2	7	0	1
$A_{1/2}$		0,2000				0,8000			
$S_{1/2}$		0,8600				0,1400			
$V(A,S,K) = \mu_R(a,s;z_1,z_2,z_3) = 1,7712$									

Step 2: To link criteria to define function $g(x)$. The $g(x)$ function is defined as described above:

Priority vectors $V(5,4,1)$

$$V_i = \frac{e_{q_i}}{e_q} \qquad \lambda_q = \frac{\prod_{i=q}^{k} V_i}{\sum_{q=1}^{k} \prod_{i=q}^{k} V_i}$$

$\lambda_1 = 10/13 = 0,8000$
$\lambda_2 = 4/13 = 0,1600$
$\lambda_3 = 1/13 = 0,0400$

Step 3: To reduce problems to a matrix:

$$\begin{bmatrix} 1 & 2 & 5 & 3 \\ 2 & 8 & 0 & 1 \end{bmatrix} \cdot \lambda_1 = \begin{bmatrix} 0,80 & 1,60 & 4,00 & 1,28 \\ 1,60 & 6,40 & 0,00 & 0,80 \end{bmatrix}$$
$$\begin{bmatrix} 4 & 3 & 1 & 8 \\ 1 & 5 & 2 & 2 \end{bmatrix} \cdot \lambda_2 = \begin{bmatrix} 0,64 & 0,48 & 0,16 & 1,28 \\ 0,16 & 0,80 & 0,32 & 0,32 \end{bmatrix}$$
$$\begin{bmatrix} 4 & 4 & 2 & 1 \\ 2 & 1 & 2 & 4 \end{bmatrix} \cdot \lambda_3 = \begin{bmatrix} 0,16 & 0,16 & 0,08 & 0,04 \\ 0,08 & 0,04 & 0,08 & 0,16 \end{bmatrix}$$

4. Conclusion

An application of Professor's Tasković Min-max theorem (YUJOR, 1993) has been shown:

$$\begin{aligned} \xi = & \max_{x\in X, y\in Y} \min\,[x,y,g(x,y)] = \\ = & \min_{x\in X, y\in Y} \max\,[x,y,g(x,y)] \end{aligned}$$

in solving multi-criterion conflict situations combined with tables of decision making. They are considered to be a highly efficient language to define, analyse and solve problems of decision making.

5. References

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