Mathematica Moravica
Vol. **15 - 1** (2011), 7–16

Mappings with a Common Fixed Point in Generalized D^*-Metric Spaces

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Abstract. The purpose of this paper is to establish a common fixed point theorem in a generalized D^*-metric space. Our results unify, generalize and complement the comparable results from the current literature.

1. Introduction and Preliminaries

Dhage [2] introduced the notion of generalized metric spaces (D-metric spaces) in 1992. He proved the existence of a unique fixed point of a self-map satisfying a contractive condition in complete and bounded D-metric spaces. In a subsequent series of papers Dhage attempted to develop topological structures in such spaces (see, for instance [3], [4], and [5]). He claimed that D-metric provide a generalization of ordinary metric functions and went on to present several fixed point results. In 2004, Mustafa and Sims [9] demonstrated that the claims concerning the fundamental topological structure of D-metric spaces are incorrect and introduced more appropriate notion of D-metric spaces. In 2007, Sedghi, Shobe and Zhoh [7] introduced the notion of D^*-metric spaces which is a modification of the definition of D-metric spaces and proved a common fixed point theorem for a class of mappings in complete D^*-metric spaces.

Huang and Zhang [6] introduced the concept of a cone metric space, replacing the set of real numbers by an ordered Banach space and obtained some fixed point theorems for mappings satisfying different contractive conditions.

Let E be a real Banach space and P a subset of E. The set P is called a cone if and only if

(i) P is closed, non-empty and $P \neq \{0\}$;
(ii) $a, b \in R$, $a, b \geq 0$, $x, y \in P$ imply that $ax + by \in P$;
(iii) $P \cap (-P) = \{0\}$.

For a given cone $P \subset E$ we define a partial ordering $\leq$ with respect to P by $x \leq y$ if and only if $y - x \in P$. We write $x < y$ to indicate that $x \to y$ but

2000 *Mathematics Subject Classification.* Primary: 47H10, 54H25.
Key words and phrases. Generalized D^*-metric space, normal cones, fixed point.

$x \neq y$, while $x << y$ will stand for $y - x \in \operatorname{int} P$ the interior of the set P. Let E be a Banach space and $P \subset E$ a cone. The cone P is called normal if there is a number $K > 0$ such that

$$0 \leq x \leq y \quad \text{implies} \quad \|x\| \leq K\|y\| \quad \text{for all} \quad x, y \in E. \tag{1.1}$$

The least positive number K satisfying the above inequality is called the normal constant of P. In the following we always suppose that E is a Banach space, P is a cone in E with $\operatorname{int} P \neq \phi$ and $\leq$ is partial ordering with respect to P. Recently, Aage and Salunke introduced the notion of a generalized D^*-metric space by replacing R by a real Banach space in D^*-metric space for all x, y, z, w in X and proved some fixed point theorems in complete generalized D^*- metric spaces.

The following definitions and some basic results in generalized D^*-metric spaces are due to [1].

Definition 1.1. Let X be a nonempty set. A generalized metric (or D^*-metric) on X is a function $D^* : X^3 \to E$ that satisfies the following conditions for each $x, y, z, w \in X$:

(1) $D^*(x, y, z) \geq 0$,
(2) $D^*(x, y, z) = 0$ if and only if $x = y = z$,
(3) $D^*(x, y, z) = D^*(p\{x, y, z\})$ (symmetry) where p is a permutation function,
(4) $D^*(x, y, z) \leq D^*(x, y, w) + D^*(w, z, z)$.

The pair (X, D^*) is called a generalized metric (or D^*-metric) space.

Proposition 1.1. *If (X, D^*) be a generalized D^*-metric space, for all $x, y \in X$, then we have $D^*(x, x, y) = D^*(x, y, y)$.*

Definition 1.2. Let (X, D^*) be a generalized D^*-metric space. Let $\{x_n\}$ be a sequence with x a point in X. If for every $c \in E$ with $0 << c$ there is N such that for all $m, n > N$, $D^*(x_m, x_n, x) << c$, then $\{x_n\}$ is said to be convergent and $\{x_n\}$ converges to x and x is the limit of $\{x_n\}$. We denote this by $x_n \to x$ as $n \to \infty$.

Definition 1.3. Let (X, D^*) be a generalized D^*-metric space, P be a normal cone with normal constant K. Let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ converges to x if and only if $D^*(x_m, x_n, x) \to 0$ as $m, n \to \infty$.

Lemma 1.1. *Let (X, D^*) be generalized D^*-metric space, then the following are equivalent:*

(i) *$\{x_n\}$ is D^*-convergent to x;*
(ii) *$D^*(x_n, x_n, x) \to 0$ (as $n \to \infty$);*
(iii) *$D^*(x_n, x, x) \to 0$ (as $n \to \infty$).*

Lemma 1.2. *Let (X, D^*) be a generalized D^*-metric space, P be a normal cone with normal constant K. Let $\{x_n\}$ be a sequence in X. If $\{x_n\}$ converges to x and $\{x_n\}$ converges to y, then $x = y$. That is the limit of $\{x_n\}$, if exists, is unique.*

Definition 1.4. Let (X, D^*) be a generalized D^*-metric space, $\{x_n\}$ be a sequence in X. If for any $c \in E$ with $0 << c$,there is N such that for all $m, n, l > N$, $D^*(x_m, x_n, x_l) << c$, then $\{x_n\}$ is called a Cauchy sequence in X.

Definition 1.5. Let (X, D^*) be a generalized D^*-metric space. If every Cauchy sequence in X is convergent in X, then X is called a complete generalized D^*-metric space.

Lemma 1.3. *Let (X, D^*) be generalized D^*-metric space, $\{x_n\}$ be a sequence in X. if $\{x_n\}$ converges to x, then $\{x_n\}$ is a Cauchy sequence.*

Lemma 1.4. *Let (X, D^*) be a generalized D^*-metric space, P be a normal cone with normal constant K. Let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if $D^*(x_m, x_n, x_l) \to 0$ as $m, n, l \to \infty$.*

Lemma 1.5. *Let (X, D^*) be a generalized D^*-metric space, P be a normal cone with normal constant K. Let $\{x_n\}$, $\{y_n\}$ and $\{z_n\}$ be three sequences in X and let $x_n \to x$, $y_n \to y$ and $z_n \to z$ as $n \to \infty$. Then $D^*(x_n, y_n, z_n) \to D^*(x, y, z)$ as $n \to \infty$.*

2. MAIN RESULTS

Theorem 2.1. *Let(X, D^*) be a generalized D^*-metric space, P be a normal cone with normal constant K and let $S, T : X \to X$, be two mappings which satisfies the following conditions:*

(i) $T(X) \subset S(X)$,
(ii) $T(X)$ *or* $S(X)$ *is* D^**-complete, and*
(iii) *inequality:*

$$\begin{aligned} D^*(Tx, Ty, Tz) \leq h \max\Big\{ & D^*(Sx, Sy, Sz), \\ & D^*(Sx, Tx, Tx), D^*(Sy, Ty, Ty), D^*(Sx, Ty, Tz), \\ & D^*(Sy, Tz, Tx), D^*(Sz, Tz, Tz), D^*(Sz, Tx, Ty)\Big\} \end{aligned} \tag{2.1}$$

for all $x, y, z \in X$, where $0 \leq h < 1$.

Then S and T have a unique coincident point in X. Moreover if S and T are weakly compatible, then S and T have a unique common fixed point.

Proof. Let $x_0 \in X$ be a arbitrary, there exists $x_1 \in X$ such that$Tx_0 = Sx_1$, in this way we have a sequence$\{Sx_n\}$ with $Tx_{n-1} = Sx_n$. Then from the inequality (2.1), we have

$$\begin{aligned} D^*(Tx_{n-1}, Tx_n, Tx_n) &\le h \max\Big\{ D^*(Sx_{n-1}, Sx_n, Sx_n), \\ &\quad D^*(Sx_{n-1}, Tx_{n-1}, Tx_{n-1}), D^*(Sx_n, Tx_n, Tx_n), \\ &\quad D^*(Sx_n, Tx_n, Tx_n), D^*(Sx_{n-1}, Tx_n, Tx_n), \\ &\quad D^*(Sx_n, Tx_{n-1}, Tx_n), D^*(Sx_n, Tx_{n-1}, Tx_n) \Big\} \\ &\le h \max\Big\{ D^*(Sx_{n-1}, Sx_n, Sx_n), D^*(Sx_{n-1}, Sx_n, Sx_n), \\ &\quad D^*(Sx_n, Sx_{n+1}, Sx_{n+1}), D^*(Sx_n, Sx_{n+1}, Sx_{n+1}), \\ &\quad D^*(Sx_{n-1}, Sx_{n+1}, Sx_{n+1}), D^*(Sx_n, Sx_n, Sx_{n+1}), \\ &\quad D^*(Sx_n, Sx_n, Sx_{n+1}) \Big\} \\ &\le hD^*(Sx_{n-1}, Sx_n, Sx_n), \end{aligned}$$

where $0 \le h < 1$. By repeated application of above inequality we have

$$D^*(Sx_n, Sx_{n+1}, Sx_{n+1}) \le h^n D^*(Sx_0, Sx_1, Sx_1). \tag{2.2}$$

Then, for all $n, m \in N, n < m$ we have by repeated use of rectangle inequality and equality (2.2) that

$$\begin{aligned} D^*(Sx_n, Sx_m, Sx_m) &\le D^*(Sx_n, Sx_n, Sx_{n+1}) + D^*(Sx_{n+1}, Sx_{n+1}, Sx_{n+2}) + \\ &\quad D^*(Sx_{n+2}, Sx_{n+2}, Sx_{n+3}) + \cdots + D^*(Sx_{m-1}, Sx_{m-1}, Sx_m) \\ &\le D^*(Sx_n, Sx_{n+1}, Sx_{n+1}) + D^*(Sx_{n+1}, Sx_{n+2}, Sx_{n+2}) + \cdots + \\ &\quad D^*(Sx_{m-1}, Sx_m, Sx_m) \\ &\le (h^n + h^{n+1} + \cdots + h^{m-1}) D^*(Sx_0, Sx_1, Sx_1) \\ &\le \frac{h^n}{1-h} D^*(Sx_0, Sx_1, Sx_1) \end{aligned}$$

and so

$$\|D^*(Sx_n, Sx_m, Sx_m)\| \le \frac{h^n}{1-h} K \|D^*(Sx_0, Sx_1, Sx_1)\|.$$

This implies that $D^*(Sx_n, Sx_m, Sx_m) \to 0$, as $n, m \to \infty$, since

$$\frac{h^n}{1-h} K \|D^*(Sx_0, Sx_1, Sx_1)\| \to 0, \quad \text{as} \quad n, m \to \infty, \quad \text{for} \quad n, m, l \in N,$$

$$D^*(Sx_n, Sx_m, Sx_l) \le D^*(Sx_n, Sx_m, Sx_m) + D^*(Sx_m, Sx_l, Sx_l),$$

from (1.1), we have,

$$\|D^*(Sx_n, Sx_m, Sx_l)\| \le K\big[\|D^*(Sx_n, Sx_m, Sx_m)\| + \|D^*(Sx_m, Sx_l, Sx_l)\|\big].$$

Taking the limit as $n, m, l \to \infty$, we get $D^*(Sx_n, Sx_m, Sx_l) \to 0$. So $\{Sx_n\} = \{Tx_{n-1}\}$ is a D^*-Cauchy sequence. Since $S(X)$ is a D^*-complete, there exists $u \in S(X)$ such that $\{Sx_n\} \to u$ as $n \to \infty$. Then there exists $p \in X$ such that $Sp = u$. If $T(x)$ is D^*-complete, there exists $u \in T(X)$

such that $\{Tx_{n-1}\} \to u$ and since $T(X) \subset S(X)$, we have $u \in S(X)$. Then there exists $p \in X$ such that $Sp = u$.

We claim that $Tp = u$,

$$\begin{aligned}
D^*(Tp,u,u) &\leq D^*(Tp,Tp,Tx_n) + D^*(Tx_n,u,u) \\
&\leq h \max\Big\{D^*(Sp,Sp,Sx_n), D^*(Sp,Tp,Tp), D^*(Sp,Tp,Tp), \\
&\quad D^*(Sx_n,Tx_n,Tx_n), D^*(Sp,Tx_n,Tp), D^*(Sp,Tp,Tx_n), \\
&\quad D^*(Sx_n,Tp,Tp)\Big\} + D^*(Tx_n,u,u) \\
&\leq h \max\Big\{D^*(u,u,Sx_n), D^*(u,Tp,Tp), D^*(u,Tp,Tp), \\
&\quad D^*(Sx_n,Sx_{n+1},Sx_{n+1}), D^*(u,Sx_{n+1},Tp), D^*(u,Tp,Sx_{n+1}), \\
&\quad D^*(Sx_n,Tp,Tp)\Big\} + D^*(Tx_n,u,u)
\end{aligned}$$

$$\begin{aligned}
D^*(Tp,u,u) \leq h \max\Big\{&D^*(u,u,Sx_n), D^*(u,Tp,Tp), \\
&D^*(Sx_n,Sx_{n+1},Sx_{n+1}), D^*(u,Sx_{n+1},Tp), \\
&D^*(Sx_n,Tp,Tp)\Big\} + D^*(Sx_{n+1},u,u)
\end{aligned}$$

and so,

$$\begin{aligned}
\|D^*(Tp,Tp,u)\| \leq Kh \max\{&\|D^*(u,u,Sx_n)\|, \|D^*(u,Tp,Tp)\|, \\
&\|D^*(Sx_n,Sx_{n+1},Sx_{n+1})\|, \|D^*(u,Sx_{n+1},Tp)\|, \\
&\|D^*(Sx_n,u,Tp)\|\} + \|D^*(Sx_{n+1},u,u)\|.
\end{aligned}$$

As $n \to \infty$, the right hand side tends to zero. Hence $\|D^*(Tp,Tp,u)\| = 0$ and $Tp = u$, i.e., $Tp = Sp$ and p is a coincident point of S and T. Now we show that S and T have a unique coincident point. For this, assume that there exists a point q in X such that $Sq = Tq$. Now,

$$\begin{aligned}
D^*(Tp,Tp,Tq) &\leq h \max\Big\{D^*(Sp,Sp,Sq), D^*(Sp,Tp,Tp), D^*(Sp,Sp,Tp), \\
&\quad D^*(Sq,Tq,Tq), D^*(Sp,Tp,Tq), D^*(Sp,Tq,Tp), D^*(Sq,Tp,Tp)\Big\} \\
&\leq h \max\Big\{D^*(Sp,Sp,Sq), 0, 0, 0, D^*(Sp,Tq,Tp), D^*(Sq,Tp,Tp)\Big\} \\
&\leq h \max\Big\{D^*(Tp,Tp,Tq), 0, 0, 0, D^*(Tp,Tq,Tp), D^*(Tq,Tp,Tp)\Big\} \\
&= hD^*(Tp,Tp,Tq),
\end{aligned}$$

and so we have $D^*(Tp,Tp,Tq) \leq hD^*(Tp,Tp,Tq)$, i.e., $(h-1)D^*(Tp,Tp,Tq) \in P$. However, $(h-1)D^*(Tp,Tp,Tq) \in -P$, since $h-1<0$ and hence $(h-1)D^*(Tp,Tp,Tq) = 0$. This implies that $D^*(Tp,Tp,Tq) = 0$, i.e., $Tp = Tq$. Thus p is the unique coincident point of S and T. So S and T have a unique common fixed point. □

Corollary 2.1. *Let(X, D^*) be a generalized D^*-metric space, P be a normal cone with normal constant K and let $T : X \to X$, be a mapping which satisfies the following conditions:*

$$D^*(Tx, Ty, Tz) \leq h \max\Big\{D^*(x, y, z), D^*(x, Tx, Tx), D^*(y, Ty, Ty), D^*(x, Ty, Ty), D^*(y, Tx, Tx), D^*(z, Tz, Tz), D^*(z, Ty, Ty)\Big\}$$

for all $x, y, z \in X$, where $0 \leq h < 1$. Then T has a unique fixed point in X.

Theorem 2.2. *Let (X, D^*) be a generalized D^*-metric space, P be a normal cone with normal constant K and let $S, T : X \to X$, be two mappings which satisfies the following conditions*

(i) *$T(X) \subset S(X)$,*
(ii) *$T(X)$ or $S(X)$ is D^*-complete, and*
(iii) *inequality*

$$D^*(Tx, Ty, Tz) \leq h \max\{D^*(Sx, Sy, Sz), D^*(Sx, Tx, Tx), D^*(Sy, Ty, Ty)\} \tag{2.3}$$

for all $x, y, z \in X$, where $0 \leq h < \frac{1}{2}$.

Then S and T have a unique coincident point in X.

Proof. Let $x_0 \in X$ be arbitrary, there exists $x_1 \in X$ such that $Tx_0 = Sx_1$, in this way we have a sequence $\{Sx_n\}$ with $Tx_{n-1} = Sx_n$. Then from the inequality (2.3), we have

$$\begin{aligned} D^*(Sx_n, Sx_{n+1}, Sx_{n+1}) &= D^*(Tx_{n-1}, Tx_n, Tx_n) \\ &\leq h \max\{D^*(Sx_{n-1}, Sx_n, Sx_n), D^*(Sx_{n-1}, Tx_{n-1}, Tx_{n-1}), \\ &\qquad D^*(Sx_n, Tx_n, Tx_n)\} \\ &\leq h \max\{D^*(Sx_{n-1}, Sx_n, Sx_n), D^*(Sx_{n-1}, Sx_n, Sx_n), \\ &\qquad D^*(Sx_n, Sx_{n+1}, Sx_{n+1})\} \\ &\leq hD^*(Sx_{n-1}, Sx_n, Sx_n). \end{aligned}$$

This implies that

$$D^*(Sx_n, Sx_{n+1}, Sx_{n+1}) \leq hD^*(Sx_{n-1}, Sx_n, Sx_n)$$

where $0 \leq h < \frac{1}{2}$. By repeated application of above inequality we have

$$D^*(Sx_n, Sx_{n+1}, Sx_{n+1}) \leq h^n D^*(Sx_0, Sx_1, Sx_1).$$

Then, for all $n, m \in N, n < m$ we have by repeated use of rectangle inequality

$$\begin{aligned}
D^*(Sx_n, Sx_m, Sx_m) &\leq D^*(Sx_n, Sx_n, Sx_{n+1}) + D^*(Sx_{n+1}, Sx_{n+1}, Sx_{n+2}) + \\
&\quad D^*(Sx_{n+2}, Sx_{n+2}, Sx_{n+3}) + \cdots + D^*(Sx_{m-1}, Sx_{m-1}, Sx_m) \\
&\leq D^*(Sx_n, Sx_{n+1}, Sx_{n+1}) + D^*(Sx_{n+1}, Sx_{n+2}, Sx_{n+2}) + \cdots + \\
&\quad D^*(Sx_{m-1}, Sx_m, Sx_m) \\
&\leq (h^n + h^{n+1} + \cdots + h^{m-1}) D^*(Sx_0, Sx_1, Sx_1).
\end{aligned}$$

From (1.1), we have

$$D^*(Sx_n, Sx_m, Sx_m) \leq \frac{h^n}{1-h} D^*(Sx_0, Sx_1, Sx_1)$$

and so,

$$\|D^*(Sx_n, Sx_m, Sx_m)\| \leq \frac{h^n}{1-h} K \|D^*(Sx_0, Sx_1, Sx_1)\|,$$

which implies that $D^*(Sx_n, Sx_m, Sx_m) \to 0$, as $n, m \to \infty$, since

$$\frac{h^n}{1-h} K \|D^*(Sx_0, Sx_1, Sx_1)\| \to 0,$$

as $n, m \to \infty$.

Since $0 \leq h < \frac{1}{2}$, $\{Sx_n\}$ is D^*-Cauchy sequence. By the completeness of $S(X)$, there exists $u \in S(X)$ such that $\{Sx_n\}$ is D^*-convergent to u. Then there is $p \in X$, such that $Sp = u$. If $T(X)$ is complete, then there exist $u \in T(X)$ such that $Sx_n \to u$, as $T(X) \subset S(X)$, we have $u \in S(X)$. Then there exist $p \in X$ such that $Sp = u$.

We claim that $Tp = u$.

$$\begin{aligned}
D^*(Tp, u, u) &= D^*(Tp, Tp, u) \\
&\leq D^*(Tp, Tp, Tx_n) + D^*(Tx_n, u, u) \\
&\leq h \max\big\{D^*(Sp, Sp, Sx_n), D^*(Sp, Tp, Tp), \\
&\qquad D^*(Sp, Tp, Tp)\big\} + D^*(Tx_n, u, u) \\
&\leq h \max\big\{D^*(Sp, Tp, Tp), D^*(Sp, Sp, Sx_n)\big\} + D^*(Tx_n, u, u)
\end{aligned}$$

$$\begin{aligned}
\|D^*(Tp, Tp, u)\| \leq Kh \max\big\{&\|D^*(Sp, Tp, Tp)\|, \|D^*(Sp, Sp, Sx_n)\|\big\} \\
&+ \|D^*(Sx_{n+1}, u, u)\|.
\end{aligned}$$

Hence,

$$\|D^*(Tp, Tp, u)\| \leq Kh \max\{\|D^*(u, Tp, Tp)\|, 0\} + \|D^*(u, u, u)\|.$$

The right hand side tends to zero as $n \to \infty$. Hence $\|D^*(Tp, Tp, u)\| = 0$ and $Tp = u$. Hence $Tp = Sp$ and p is a coincident point of S and T.

Now we show that S and T have a unique coincident point. For this, assume that there exists a point q in X such that $Sq = Tq$. Now

$$\begin{aligned} D^*(Tp, Tp, Tq) &\leq h \max\{D^*(Sp, Sp, Sq), D^*(Sp, Tp, Tp), D^*(Sp, Sp, Tp), \\ &\leq h \max\{0, 0, D^*(Tp, Tp, Tq)\}. \end{aligned}$$

This implies $(h-1)D^*(Tp, Tp, Tq) \in P$ and $(h-1)D^*(Tp, Tp, Tq) \in -P$ since $0 \leq h < \frac{1}{2}$. As $P \cap -P = \{0\}$, we have $(h-1)D^*(Tp, Tp, Tq) = 0$, i.e., $D^*(Tp, Tp, Tq) = 0$. Hence $Tp = Tq$. Also $Sp = Sq$, since $Tp = Sp$. Hence p is the unique coincident point of S and T. So p is a unique common fixed point of S and T in X. □

Corollary 2.2. *Let (X, D^*) be a generalized D^*-metric space, P be a normal cone with normal constant K and let $T : X \to X$, be a mapping which satisfies the following conditions*

$$D^*(Tx, Ty, Tz) \leq h \max\{D^*(x, y, z), D^*(x, Tx, Tx), D^*(y, Ty, Ty)\}$$

for all $x, y, z \in X$, where $0 \leq h < 1$. Then T has a unique fixed point in X.

Example 2.1. Let (X, D^*) be a complete D^*-metric space, where $X = (0, 1]$ and $D^*(x, y, z) = |x - y| + |y - z| + |z - x|$. Define self-maps S and T on X as follows: $Sx = \frac{x+1}{2}$ and $Tx = \frac{x+5}{6}$, for all $x \in X$. For any nonzero $x \in X$ we have

$$STx = S\left(\frac{x+5}{6}\right) = \frac{x+11}{6}, \qquad TSx = T\left(\frac{x+1}{2}\right) = \frac{x+11}{6}.$$

Since $STx = TSx$ and S, T are weakly compatible on X.

Now

$$\begin{aligned} D^*(STx, TSx, TSx) &= \left|\frac{x+11}{12} - \frac{x+11}{12}\right| + \left|\frac{x+11}{12} - \frac{x+11}{12}\right| + \\ &\quad + \left|\frac{x+11}{12} - \frac{x+11}{12}\right| = 0, \\ D^*(Sx, Tx, Tx) &= \left|\frac{x+1}{2} - \frac{x+5}{6}\right| + \left|\frac{x+5}{6} - \frac{x+5}{6}\right| + \\ &\quad + \left|\frac{x+1}{2} - \frac{x+5}{6}\right| = \frac{2x-2}{3}. \end{aligned}$$

We see that

$$D^*(STx, TSx, TSx) \leq D^*(Sx, Tx, Tx),$$

and so $\{A, S\}$ are weakly commuting pairs.

$$\begin{aligned}D^*(Tx, Ty, Tz) &= D^*\left(\frac{x+5}{6}, \frac{y+5}{6}, \frac{z+5}{6}\right)\\ &= \left|\frac{x+5}{6} - \frac{y+5}{6}\right| + \left|\frac{y+5}{6} - \frac{z+5}{6}\right| + \left|\frac{x+5}{6} - \frac{z+5}{6}\right|\\ &= \frac{(x-y-z)}{3}\end{aligned}$$

$$h\max\Big\{D^*(Sx, Sy, Sz), D^*(Sx, Tx, Tx), D^*(Sy, Ty, Ty)\Big\} =$$
$$h\max\Big\{(x-y-z), \frac{2x-2}{3}, \frac{2y-2}{3}\Big\}$$

for all $x, y, z \in X$, $h \in (0, \frac{1}{2}]$, Theorem 2.2 is satisfied. So 1 is the unique common fixed point for S and T.

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