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Central Operation of the (n,m)-Group

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Abstract. In this paper we have defined a central operation of the (n,m)-group, as a mapping α of the set Q^{n-2m} into the set Q^m, such that for every $a_1^{n-2m}, b_1^{n-2m} \in Q$ and for every $x_1^m \in Q^m$ the following equality holds:

$$A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, x_1^m\right) = A\left(x_1^m, \alpha\left(b_1^{n-2m}\right), b_1^{n-2m}\right).$$

This is a generalization of the notion of a central operation of the n-group, i.e. of the central element of a binary group. The notion of the central operation of the n-group was defined by Janez Ušan in [4]. Furthermore, in this paper we have proved some claims which hold for the central operation of the (n,m)-group.

1. Notion and Example

Definition 1.1. Let Q be a nonempty set and let A be a mapping of the set Q^n into the set Q^m. Then, we say that (Q,A) is an (n,m)-groupoid.

Definition 1.2 ([1])**.** Let $n \geqslant m+1$ and let (Q,A) be an (n,m)-groupoid. We say that (Q,A) is an (n,m)-semigroup iff for every $i,j \in \{1,\ldots,n-m+1\}$, $i<j$ and for every $x_1^{2n-m} \in Q$ the following equality holds:

$$A\left(x_1^{i-1}, A\left(x_i^{i+n-1}\right), x_{i+n}^{2n-m}\right) = A\left(x_1^{j-1}, A\left(x_j^{j+n-1}\right), x_{j+n}^{2n-m}\right) \tag{1}$$

Remark. The equality (1) is called an $\langle i,j\rangle$-associative law.

Definition 1.3 ([1])**.** Let $n \geqslant m+1$ and let (Q,A) be an (n,m)-groupoid. We say that (Q,A) is an (n,m)-group iff the following statements hold:

a) (Q,A) is an (n,m)-semigroup, and
b) for every $i \in \{1,\ldots,n-m+1\}$ and for every $a_1^n \in Q$, there is exactly one $x_1^m \in Q^m$ such that the following equality holds:

$$A\left(a_1^{i-1}, x_1^m, a_i^{n-m}\right) = a_{n-m+1}^n. \tag{2}$$

An important notion in the theory of the (n,m)-group is $\{1,n-m+1\}$-neutral operation. This notion was introduced by Janez Ušan in 1989 and it is a generalization of the notion of a neutral element in binary structures.

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Definition 1.4 ([3])**.** Let $n \geqslant 2m$ and let (Q, A) be an (n, m)-groupoid. Also, let e_L, e_R and e be mappings of the set Q^{n-2m} into the set Q^m. Then:

(i) e_L is a left $\{1, n-m+1\}$-neutral operation of the (n, m)-groupoid (Q, A) iff for every $x_1^m \in Q$ and for every $a_1^{n-2m} \in Q$ the following equality holds
$$A\left(e_L\left(a_1^{n-2m}\right), a_1^{n-2m}, x_1^m\right) = x_1^m;$$

(ii) e_R is a right $\{1, n-m+1\}$-neutral operation of the (n, m)-groupoid (Q, A) iff for every $x_1^m \in Q$ and for every $a_1^{n-2m} \in Q$ the following equality holds
$$A\left(x_1^m, a_1^{n-2m}, e_R\left(a_1^{n-2m}\right)\right) = x_1^m;$$

(iii) e is a $\{1, n-m+1\}$-neutral operation of the (n, m)-groupoid (Q, A) iff for every $x_1^m \in Q$ and for every $a_1^{n-2m} \in Q$ the following equalities hold
$$A\left(e\left(a_1^{n-2m}\right), a_1^{n-2m}, x_1^m\right) = x_1^m$$
and
$$A\left(x_1^m, a_1^{n-2m}, e\left(a_1^{n-2m}\right)\right) = x_1^m.$$

Remark. For $(n, m) = (2, 1)$, the definition 1.4. is a definition of a neutral element in binary groupoid.

In an (n, m)-group, for its $\{1, n-m+1\}$-neutral operation, the following proposition holds.

Proposition 1.5 ([2])**.** *Let (Q, A) be an (n, m)-group, $n \geqslant 2m$ and e its $\{1, n-m+1\}$-neutral operation. Then for every $x_1^m \in Q^m$, for every $a_1^{n-2m} \in Q$ and for every $i \in \{1, \ldots, n-2m+1\}$ the following equalities hold:*
$$A\left(x_1^m, a_i^{n-2m}, e\left(a_1^{n-2m}\right), a_1^{i-1}\right) = x_1^m,$$
$$A\left(a_i^{n-2m}, e\left(a_1^{n-2m}\right), a_1^{i-1}, x_1^m\right) = x_1^m.$$

One more important notion in the theory of the binary structures is a central element. The next definition give the generalization of this notion.

Definition 1.6. Let (Q, A) be an (n, m)-group, $n \geqslant 2m$ and let α be a mapping of the set Q^{n-2m} into the set Q^m. We say that α is a central operation of the (n, m)-group (Q, A) iff for every $a_1^{n-2m}, b_1^{n-2m} \in Q$ and for every $x_1^m \in Q^m$ the following equality holds:

(3) $$A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, x_1^m\right) = A\left(x_1^m, \alpha\left(b_1^{n-2m}\right), b_1^{n-2m}\right)$$

Remark. For $m = 1$ a notion of the central operation of the n-group was defined in [4]. Furthermore, for $(n, m) = (2, 1)$, $\alpha\left(a_1^{n-2m}\right) = \alpha(\emptyset) = c$, equality (3) is $A(c, x) = A(x, c)$, $\forall x \in Q$, that is a definition of the central element of a binary group.

Example 1.7. By definitions 1.4. and 1.6. and proposition 1.5. we conclude that the $\{1, n-m+1\}$-neutral operation of the (n,m)-group (Q,A) is the central operation.

2. Main Propositions

Proposition 2.1. *Let (Q,A) be an (n,m)-group, $n \geqslant 2m$ and α its central operation. Than for every $a_1^{n-2m}, b_1^{n-2m}, x_1^n \in Q$ the following equalities hold:*

a) $A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, A\left(x_1^n\right)\right) =$
$A\left(A\left(\alpha\left(b_1^{n-2m}\right), b_1^{n-2m}, x_1^m\right), x_{m+1}^n\right);$

b) $A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, A\left(x_1^n\right)\right) =$
$A\left(x_1^m, A\left(\alpha\left(b_1^{n-2m}\right), b_1^{n-2m}, x_{m+1}^{2m}\right), x_{2m+1}^n\right);$

c) $A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, A\left(x_1^n\right)\right) =$
$A\left(x_1^{n-m}, A\left(\alpha\left(b_1^{n-2m}\right), b_1^{n-2m}, x_{n-m+1}^n\right)\right).$

Proof. a):

$$A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, A\left(x_1^n\right)\right) \overset{(1)}{=}$$
$$A\left(A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, x_1^m\right), x_{m+1}^n\right) \overset{(3)}{=}$$
$$A\left(A\left(x_1^m, \alpha\left(b_1^{n-2m}\right), b_1^{n-2m},\right), x_{m+1}^n\right) \overset{(3)}{=}$$
$$A\left(A\left(\alpha\left(b_1^{n-2m}\right), b_1^{n-2m}, x_1^m\right), x_{m+1}^n\right).$$

Proof b):

$$A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, A\left(x_1^n\right)\right) \overset{(1)}{=}$$
$$A\left(A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, x_1^m\right), x_{m+1}^n\right) \overset{(3)}{=}$$
$$A\left(A\left(x_1^m, \alpha\left(b_1^{n-2m}\right), b_1^{n-2m},\right), x_{m+1}^n\right) \overset{(1)}{=}$$
$$A\left(x_1^m, A\left(\alpha\left(b_1^{n-2m}\right), b_1^{n-2m}, x_{m+1}^{2m}\right), x_{2m+1}^n\right).$$

Proof c):

$$A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, A\left(x_1^n\right)\right) \overset{(3)}{=}$$
$$A\left(A\left(x_1^n\right), \alpha\left(b_1^{n-2m}\right), b_1^{n-2m}\right) \overset{(1)}{=}$$
$$A\left(x_1^{n-m}, A\left(x_{n-m+1}^n, \alpha\left(b_1^{n-2m}\right), b_1^{n-2m}\right)\right) \overset{(3)}{=}$$
$$A\left(x_1^{n-m}, A\left(\alpha\left(b_1^{n-2m}\right), b_1^{n-2m}, x_{n-m+1}^n\right)\right).$$

□

Proposition 2.2. *Let (Q,A) be an (n,m)-group, $n \geqslant 2m$. Furthermore, let α be a mapping of the set Q^{n-2m} into the set Q^m and for every $a_1^{n-2m}, b_1^{n-2m}, x_1^m \in$*

Q the following equality holds:

(4) $$A\left(x_1^m, a_1^{n-2m}, \alpha\left(a_1^{n-2m}\right)\right) = A\left(b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right), x_1^m\right).$$

Than for every $a_1^{n-2m}, b_1^{n-2m}, x_1^n \in Q$ the following equalities hold:

a) $A\left(A\left(x_1^n\right), a_1^{n-2m}, \alpha\left(a_1^{n-2m}\right)\right) =$
$A\left(A\left(x_1^m, b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right)\right), x_{m+1}^n\right)$;

b) $A\left(A\left(x_1^n\right), a_1^{n-2m}, \alpha\left(a_1^{n-2m}\right)\right) =$
$A\left(x_1^{n-2m}, A\left(x_{n-2m+1}^{n-m}, b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right)\right), x_{n-m+1}^n\right)$;

c) $A\left(A\left(x_1^n\right), a_1^{n-2m}, \alpha\left(a_1^{n-2m}\right)\right) =$
$A\left(x_1^{n-m}, A\left(x_{n-m+1}^n, b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right)\right)\right)$.

Proof. a):

$$\begin{aligned}
&A\left(A\left(x_1^n\right), a_1^{n-2m}, \alpha\left(a_1^{n-2m}\right)\right) \overset{(4)}{=} \\
&A\left(b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right), A\left(x_1^n\right)\right) \overset{(1)}{=} \\
&A\left(A\left(b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right), x_1^m\right), x_{m+1}^n\right) \overset{(4)}{=} \\
&A\left(A\left(x_1^m, b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right)\right), x_{m+1}^n\right).
\end{aligned}$$

Proof b):

$$\begin{aligned}
&A\left(A\left(x_1^n\right), a_1^{n-2m}, \alpha\left(a_1^{n-2m}\right)\right) \overset{(1)}{=} \\
&A\left(x_1^{n-m}, A\left(x_{n-m+1}^n, a_1^{n-2m}, \alpha\left(a_1^{n-2m}\right)\right)\right) \overset{(4)}{=} \\
&A\left(x_1^{n-m}, A\left(b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right), x_{n-m+1}^n\right)\right) \overset{(1)}{=} \\
&A\left(x_1^{n-2m}, A\left(x_{n-2m+1}^{n-m}, b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right)\right), x_{n-m+1}^n\right).
\end{aligned}$$

Proof c):

$$\begin{aligned}
&A\left(A\left(x_1^n\right), a_1^{n-2m}, \alpha\left(a_1^{n-2m}\right)\right) \overset{(1)}{=} \\
&A\left(x_1^{n-m}, A\left(x_{n-m+1}^n, a_1^{n-2m}, \alpha\left(a_1^{n-2m}\right)\right)\right) \overset{(4)}{=} \\
&A\left(x_1^{n-m}, A\left(b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right), x_{n-m+1}^n\right)\right) \overset{(4)}{=} \\
&A\left(x_1^{n-m}, A\left(x_{n-m+1}^n, b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right)\right)\right).
\end{aligned}$$

□

Proposition 2.3. *Let (Q, A) be an (n, m)-group, $n \geqslant 2m$ and α its central operation. Than for every $a_1^{n-2m}, b_1^{n-2m}, x_1^n \in Q$ the following equalities hold:*

a) $A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, x_1^m\right) = A\left(x_1^m, b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right)\right)$;

b) $A\left(x_1^m, \alpha\left(a_1^{n-2m}\right), a_1^{n-2m}\right) = A\left(b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right), x_1^m\right)$.

Proof. a): Let $y_1^m \overset{def}{=} A\left(x_1^m, b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right)\right)$ for every $b_1^{n-2m}, x_1^m \in Q$. Than for arbitrary sequence $z_1^m \in Q$ the following sequence of equalities hold:

$$A\left(y_1^m, b_1^{n-2m}, z_1^m\right) = A\left(A\left(x_1^m, b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right)\right), b_1^{n-2m}, z_1^m\right) \overset{(1)}{\Leftrightarrow}$$

$$\Leftrightarrow A\left(y_1^m, b_1^{n-2m}, z_1^m\right) = A\left(x_1^m, b_1^{n-2m}, A\left(\alpha\left(b_1^{n-2m}\right), b_1^{n-2m}, z_1^m\right)\right) \overset{(2.1.c)}{\Leftrightarrow}$$

$$\Leftrightarrow A\left(y_1^m, b_1^{n-2m}, z_1^m\right) = A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, A\left(x_1^m, b_1^{n-2m}, z_1^m\right)\right) \overset{(1)}{\Leftrightarrow}$$

$$\Leftrightarrow A\left(y_1^m, b_1^{n-2m}, z_1^m\right) = A\left(A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, x_1^m\right), b_1^{n-2m}, z_1^m\right) \overset{(2)}{\Leftrightarrow}$$

$$\Leftrightarrow y_1^m = A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, x_1^m\right).$$

Proof b): Let $y_1^m \overset{def}{=} A\left(b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right), x_1^m\right)$ for every $b_1^{n-2m}, x_1^m \in Q$. Than for arbitrary sequence $z_1^m \in Q$ the following sequence of equalities hold:

$$A\left(b_1^{n-2m}, z_1^m, y_1^m\right) = A\left(b_1^{n-2m}, z_1^m, A\left(b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right), x_1^m\right)\right) \overset{(1)}{\Leftrightarrow}$$

$$A\left(b_1^{n-2m}, z_1^m, y_1^m\right) = A\left(b_1^{n-2m}, A\left(z_1^m, b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right)\right), x_1^m\right) \overset{(2.3.a)}{\Leftrightarrow}$$

$$A\left(b_1^{n-2m}, z_1^m, y_1^m\right) = A\left(b_1^{n-2m}, A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, z_1^m\right), x_1^m\right) \overset{(3)}{\Leftrightarrow}$$

$$A\left(b_1^{n-2m}, z_1^m, y_1^m\right) = A\left(b_1^{n-2m}, A\left(z_1^m, \alpha\left(a_1^{n-2m}\right), a_1^{n-2m}\right), x_1^m\right) \overset{(1)}{\Leftrightarrow}$$

$$A\left(b_1^{n-2m}, z_1^m, y_1^m\right) = A\left(b_1^{n-2m}, z_1^m, A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, x_1^m\right)\right) \overset{(3)}{\Leftrightarrow}$$

$$A\left(b_1^{n-2m}, z_1^m, y_1^m\right) = A\left(b_1^{n-2m}, z_1^m, A\left(x_1^m, \alpha\left(a_1^{n-2m}\right), a_1^{n-2m}\right)\right) \overset{(2)}{\Leftrightarrow}$$

$$y_1^m = A\left(x_1^m, \alpha\left(a_1^{n-2m}\right), a_1^{n-2m}\right).$$ □

Proposition 2.4. *Let (Q, A) be an (n, m)-group, $n \geqslant 2m$ and let α be a mapping of the set Q^{n-2m} into the set Q^m. Furthermore, let for every $a_1^{n-2m}, b_1^{n-2m} \in Q$ and for every $x_1^m \in Q^m$ the equality (4) holds. Than for every $a_1^{n-2m}, b_1^{n-2m} \in Q$ and for every $x_1^m \in Q^m$ the following equalities hold:*

a) $A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, x_1^m\right) = A\left(x_1^m, b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right)\right)$;
b) $A\left(x_1^m, \alpha\left(a_1^{n-2m}\right), a_1^{n-2m}\right) = A\left(b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right), x_1^m\right)$.

Proof. a): Let $y_1^m \overset{def}{=} A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, x_1^m\right)$ for every $a_1^{n-2m}, x_1^m \in Q$. Than for arbitrary sequence $z_1^m \in Q$ the following sequence of equalities hold:

$$A\left(z_1^m, a_1^{n-2m}, y_1^m\right) = A\left(z_1^m, a_1^{n-2m}, A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, x_1^m\right)\right) \overset{(1)}{\Leftrightarrow}$$

$$A\left(z_1^m, a_1^{n-2m}, y_1^m\right) = A\left(A\left(z_1^m, a_1^{n-2m}, \alpha\left(a_1^{n-2m}\right)\right), a_1^{n-2m}, x_1^m\right) \overset{(2.2.a)}{\Leftrightarrow}$$

$$A\left(z_1^m, a_1^{n-2m}, y_1^m\right) = A\left(A\left(z_1^m, a_1^{n-2m}, x_1^m\right), b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right)\right) \overset{(1)}{\Leftrightarrow}$$
$$A\left(z_1^m, a_1^{n-2m}, y_1^m\right) = A\left(z_1^m, a_1^{n-2m}, A\left(x_1^m, b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right)\right)\right) \overset{(2)}{\Leftrightarrow}$$
$$y_1^m = A\left(x_1^m, b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right)\right).$$

Proof b): Let $y_1^m \overset{def}{=} A\left(x_1^m, \alpha\left(a_1^{n-2m}\right), a_1^{n-2m}\right)$ for every $a_1^{n-2m}, x_1^m \in Q$. Than for arbitrary sequence $z_1^m \in Q$ the following sequence of equalities hold:

$$y_1^m = A\left(x_1^m, \alpha\left(a_1^{n-2m}\right), a_1^{n-2m}\right)$$
$$A\left(y_1^m, z_1^m, a_1^{n-2m}\right) = A\left(A\left(x_1^m, \alpha\left(a_1^{n-2m}\right), a_1^{n-2m}\right), z_1^m, a_1^{n-2m}\right) \overset{(1)}{\Leftrightarrow}$$
$$A\left(y_1^m, z_1^m, a_1^{n-2m}\right) = A\left(x_1^m, A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, z_1^m\right), a_1^{n-2m}\right) \overset{(2.4.a)}{\Leftrightarrow}$$
$$A\left(y_1^m, z_1^m, a_1^{n-2m}\right) = A\left(x_1^m, A\left(z_1^m, b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right)\right), a_1^{n-2m}\right) \overset{(4)}{\Leftrightarrow}$$
$$A\left(y_1^m, z_1^m, a_1^{n-2m}\right) = A\left(x_1^m, A\left(a_1^{n-2m}, \alpha\left(a_1^{n-2m}\right), z_1^m\right), a_1^{n-2m}\right) \overset{(1)}{\Leftrightarrow}$$
$$A\left(y_1^m, z_1^m, a_1^{n-2m}\right) = A\left(A\left(x_1^m, a_1^{n-2m}, \alpha\left(a_1^{n-2m}\right)\right), z_1^m, a_1^{n-2m}\right) \overset{(4)}{\Leftrightarrow}$$
$$A\left(y_1^m, z_1^m, a_1^{n-2m}\right) = A\left(A\left(b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right), x_1^m\right), z_1^m, a_1^{n-2m}\right) \overset{(2)}{\Leftrightarrow}. \quad \square$$

Theorem 2.5. *Let (Q, A) be an (n,m)-group, $n \geqslant 2m$ and let α be a mapping of the set Q^{n-2m} into the set Q^m. Than the following statements are equivalent:*

(i) *α is a central operations of the (n,m)-group (Q, A);*
(ii) *for every $x_1^m \in Q^m$ and $a_1^{n-2m}, b_1^{n-2m} \in Q$ the following equality holds*

$$A\left(x_1^m, a_1^{n-2m}, \alpha\left(a_1^{n-2m}\right)\right) = A\left(b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right), x_1^m\right).$$

Proof. *(i)* $\Rightarrow$ *(ii)*

$$A\left(x_1^m, a_1^{n-2m}, \alpha\left(a_1^{n-2m}\right)\right) \overset{(2.3.a)}{=} A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, x_1^m\right) \overset{(3)}{=}$$
$$A\left(x_1^m, \alpha\left(a_1^{n-2m}\right), a_1^{n-2m}\right) \overset{(2.3.b)}{=} A\left(b_1^{n-2m}, \alpha\left(b_1^{n-2m}\right), x_1^m\right).$$

(ii) $\Rightarrow$ *(i)*

$$A\left(\alpha\left(a_1^{n-2m}\right), a_1^{n-2m}, x_1^m\right) \overset{(2.4.a)}{=} A\left(x_1^m, a_1^{n-2m}, \alpha\left(a_1^{n-2m}\right)\right) \overset{(4)}{=}$$
$$A\left(a_1^{n-2m}, \alpha\left(a_1^{n-2m}\right), x_1^m\right) \overset{(2.4.b)}{=} A\left(x_1^m, \alpha\left(b_1^{n-2m}\right), b_1^{n-2m}\right). \quad \square$$

References

[1] G. Čupona, *Vector valued semigroups*, Semigroup forum **26**(1983), 65-74.

[2] R. Galić and A. Katić, *On neutral operations of* (n, m)*-groups*, Math. Moravica Vol. **9**(2005), 1-3.

[3] J. Ušan, *Neutral operations of* (n, m)*-groupoids*, (Russian), Rev. of Research, Fac. of. Sci. Univ. of Novi Sad, Math. Ser. **19**(1989) No. 2, 125-137.

[4] J. Ušan, *Description of super associative algebras with n-quasigroup operations*, Math. Moravica Vol. **5**(2001), 129-157.

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