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Relativistic Nonlinear Physics of the Lower Transversal Spaces and Consequences

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Abstract. In this paper we introduced the fundamental elements of a new relativistic physics for the lower transversal spaces. The classical Einstein's theory (physics) of the upper transversal spaces is linear via Lorentz's transformations. On the other hand, the physics of the lower transversal spaces is nonlinear via the nonlinear transformations. Also, for the upper transversal spaces, as and Einstein's physics, is essential that the "live" is finite , but in the lower transversal spaces the "live" is infinite. This is a result (fact) of the deepest connection between new nonlinear physics and the geometry of the lower transversal spaces. The relativistic physics and the new nonlinear physics are essential different, but the equation for Energy in the form $E = mc^2$ is in the same in both physics!

1. Introduction and history

It is well known that Einstein's theory of relativity has been developed in two fundamental papers, which appeared during the years 1905 (special theory of relativity) and 1916 (general theory of relativity). The special theory of relativity begins with the **principle of relativity** in form: *All physical processes have the same form for all inertial systems.*

The general theory of relativity represents an extension of Newton's theory of gravity to arbitrary systems of reference. It represents the deepest known connection between physics and mathematics in form: *Physical interactions can be reduced to geometrical properties.*

Because the velocity of light is constant, a change of space and time between inertial systems is given by Lorentz's transformations. This can be achieved by formulating these laws as geometrical laws for Minkowski's uncurved four-dimensional space-time of suitable manifold. In general this is connection with

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the metrical Fréchet's spaces in 1905 and further general with the upper transversal spaces of Tasković in 1998 and 2005.

The possibility of defining such notions as *limit* and *continuity* in an arbitrary set is an idea which undoubtedly was first put forward by M. Fréchet in 1904, and developed by him in his famous thesis in 1906.

The simplest and most fruitful method which be proposed for such definitions was the introduction of the notion of distance.

In connection with this, first, in Tasković [4] we introduced the concept of transversal (upper and lower) spaces as a natural extension of Fréchet's Kurepa's and Menger's spaces.

Let X be a nonempty set. The function $\rho : X \times X \to \mathbb{R}_+^0 := [0, +\infty)$ is called an **upper transverse** on X (or *upper transversal*) iff: $\rho[x, y] = \rho[y, x]$, $\rho[x, y] = 0$ if and only if $x = y$, and if there is function $\psi : (\mathbb{R}_+^0)^2 \to \mathbb{R}_+^0$ such that

$$\text{(As)} \qquad \rho[x, y] \leq \max\Big\{\rho[x, z], \rho[z, y], \psi\Big(\rho[x, z], \rho[z, y]\Big)\Big\}$$

for all $x, y, z \in X$. An **upper transversal space** is a set X together with a given upper transverse on X. The function ψ in (As) is called *upper bisection function.*

For the upper transversal spaces (X, ρ), as and for Einstein's physic, are essential the mappings $T : X \to X$ which are *bounded variation*, i.e., if

$$\sum_{n=0}^{\infty} \rho\Big(T^n x, T^{n+1} x\Big) < +\infty$$

for arbitrary $x \in X$, where $T^n(x)$ for $n \in \mathbf{N} \cup \{0\}$ is an iteration sequence of mapping T, in further.The upper transversal spaces are spaces are spaces with the *leif finite*, where **spring of spaces** in the point $x = 0$.

On the other hand, the function $\rho : X \times X \to [0, +\infty] := \mathbb{R}_+^0 \cup \{+\infty\}$ is called a **lower transverse** on X (or *lower transversal*) iff: $\rho[x, y] = \rho[y, x]$, $\rho[x, y] = +\infty$ if and only if $x = y$, and if there is a lower bisection function $d : [0, +\infty]^2 \to [0, +\infty]$ such that

$$\text{(Am)} \qquad \rho[x, y] \geq \min\Big\{\rho[x, z], \rho[z, y], d\Big(\rho[x, z], \rho[z, y]\Big)\Big\}$$

for all $x, y, z \in X$. A **lower transversal space** is a set X together with a given lower transverse on X. The function d in (Am) is called *lower bisection function.*

Let (X, ρ) be a lower transversal space and $T : X \to X$. We shall introduce the concept of DS-convergence in a space X; i.e., a lower transversal space X satisfies the condition of **DS-convergence** (or X is *DS-complete*) iff: $\{x_n\}_{n \in \mathbb{N}}$ is an arbitrary sequence in X and $\sum_{i=1}^{\infty} \rho[x_i, x_{i+1}] = +\infty$ implies that $\{x_n\}_{n \in \mathbb{N}}$ has a convergent subsequence in X.

In connection with this, a lower transversal space X satisfies the condition of **orbitally DS-convergence** (or X is *orbitally DS-complete*) iff: $\{T^n x\}_{n \in \mathbb{N} \cup \{0\}}$ for $x \in X$ is an arbitrary iteration sequence in X and

$$\sum_{n=0}^{\infty} \rho[T^n x, T^{n+1} x] = +\infty \qquad (\text{for } x \in X)$$

implies that $\{T^n x\}_{n \in \mathbb{N} \cup \{0\}}$ has a convergent subsequence in X.

We notice that in [5] Tasković proved the following statement for a class of expansion mappings. Namely, if (X, ρ) is an orbitally DS-complete lower transversal space, if T :

$X \to X$, and if there exists a number $q > 1$ such that

$$\rho\Big(T(x), T(y)\Big) \geq q\rho(x, y) \tag{1}$$

for each $x, y \in X$, then T has a unique fixed point in the lower transversal space X.

Let (X, ρ_X) and (Y, ρ_Y) be two lower transversal spaces and let $T : X \to Y$. We notice, from Tasković [5], that T be **lower transversal continuous** (or *lower continuous*) at $x_0 \in X$ iff for every $\varepsilon > 0$ there exists a $\delta > 0$ such that the relation

$$\rho_X[x, x_0] > \delta \quad \text{implies} \quad \rho_Y[T(x), T(x_0)] > \varepsilon.$$

A typical first example of a lower transversal continuous mapping is the mapping $T : X \to X$ with property (1). Also, the lower transverse ρ need not be lower transversal continuous; but, for an arbitrary metric function $r(x, y)$ the lower transverse of the form $\rho[x, y] := 1/r(x, y)$ is a lower transversal continuous function. For further facts on the lower transversal continuous mappings see: Tasković [5].

In this sense, for any nonempty set S in the lower transversal space X the *diameter* of S is defined by

$$\operatorname{diam}(S) := \inf\Big\{\rho[x, y] : x, y \in S\Big\};$$

it is a positive real number or $+\infty$, and $A \subset B$ implies $\operatorname{diam}(B) \leq \operatorname{diam}(A)$. The relation $\operatorname{diam}(S) = 0$ holds if and only if S is a one point set. Also, for a point $x_0 \in X$ we have

$$\rho(x_0, S) := \sup\Big\{\rho[x_0, s] : s \in S\Big\}.$$

Elements of a lower transversal space will usually be called points. Given a lower transversal space (X, ρ), with the bisection function d and a point $z \in X$, the **open ball** of *center* z and *radius* $r > 0$ is the set

$$d(B(z, r)) := \Big\{x \in X : \rho[z, x] > r\Big\}.$$

In this sense, we have the following form of convergence on the lower transversal spaces. The *convergence* $x_n \to x$ as $n \to \infty$ in the lower transversal space (X, ρ) means that

$$\rho[x_n, x] \to +\infty \quad \text{as} \quad n \to \infty,$$

or equivalently, for every $\varepsilon > 0$ there exists an integer n_0 such that the relation $n \geq n_0$ implies $\rho[x_n, x] > \varepsilon$.

The sequence $\{x_n\}_{n \in \mathbb{N}}$ in the lower transversal space (X, ρ) is called **transversal sequence** (or *lower Cauchy sequence*) iff for every $\varepsilon > 0$ there is an $n_0 = n_0(\varepsilon)$ such that

$$\rho[x_n, x_m] > \varepsilon \qquad \text{for all} \qquad n, m \geq n_0.$$

Let (X, ρ) be a lower transversal space and $T : X \to X$. We notice, from Tasković [5], that a sequence of iterates $\{T^n(x)\}_{n \in \mathbb{N}}$ in X is said to be transversal sequence if and only if

$$\lim_{n \to \infty}\Big(\operatorname{diam}\Big\{T^k(x) : k \geq n\Big\}\Big) = +\infty.$$

In this sense, a lower transversal space is called **lower complete** iff every transversal sequence converges.

Also, a space (X, ρ) is said to be **lower orbitally complete** (or *lower T-orbitally complete*) iff every transversal sequence which is contained in the *orbit* $\mathcal{O}(x) := \{x, Tx, T^2x, \dots\}$ for some $x \in X$ converges in X.

For the lower transversal spaces (X, ρ) are essential the mappings $T : X \to X$ which are *unbounded variation*, i.e., if

$$\sum_{n=0}^{\infty} \rho\Big(T^n x, T^{n+1} x\Big) = +\infty$$

for arbitrary $x \in X$. The lower transversal spaces are spaces with *life infinite*, where **spring of spaces** in the point $x = +\infty$.

In this paper I give a physics and a geometry of lower transversal spaces beginning with some special transformations. The physics of lower transversal spaces is nonlinear.

Some annotations. From the property of the open bolls in the lower transversal spaces we can explain the problem widening of galaxies in universe.

We notice that are the lower transversal spaces a good way and frame for interpretation of the widening of galaxies. In this sense, let the open bolls of the form $d(B(z, r_k))$ for $k \in \mathbb{N} \cup \{0\}$ are galaxies with the earth as a center z for all galaxies. If the universe X is a lower transversal space, and if $r_k \to r_\infty = \infty$ when $k \to \infty$ as on Fig.1, then we obtain a reversed process for the galaxies, i.e., we have

$$(\downarrow) \qquad X \supset d(B(z, r_0)) \supset d(B(z, r_1)) \supset \cdots \supset d(B(z, r_\infty)) = \partial X,$$

where ∂X denoted a boundary of the universe X, which can be and a spring point $+\infty$, i.e., $\partial X = +\infty$. In this sense, $(\downarrow)$ means that we have a widening of galaxies (in the preceding context), but it is widening permanent in the frame of an universe to the completion (addition) of the universe; and then once more the same action.

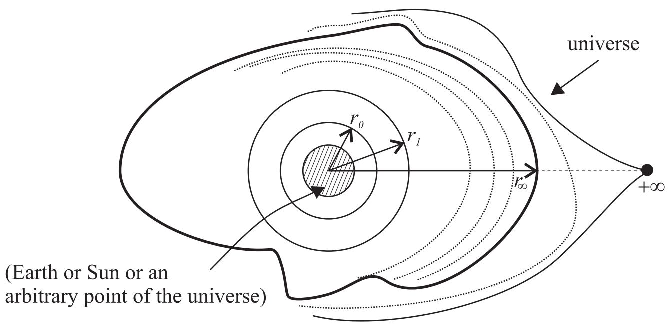

Figure 1.

2. NONLINEAR TRANSFORMATIONS

Consider two "lower" coordinate systems σ and σ' with corresponding space coordinates $\xi = (x, y, z)$ and $\xi' = (x', y', z')$, where axes x, y, z, x', y' and z' are hyperbolas. Assume also that σ and σ' are two **lower inertial systems** of the form as on Fig.2 with corresponding system times t and t', and with the fact that the point $(0, 0, 0, 0)$ corresponds with the point $(\infty, \infty, \infty, \infty)$ in the same lower inertial systems, and reverse.

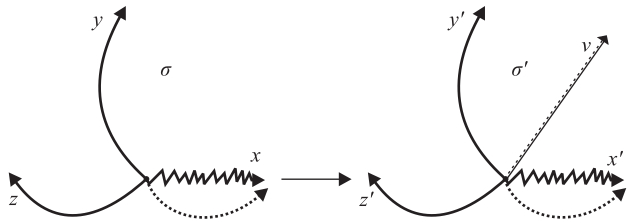

Figure 2.

In this sense, a "lower" *coordinate system* σ is a *lower inertial system* precisely if there exists a system time t for it such that each mass point, which is for enough away from other masses and shielded against fields, e.g., light pressure, remains at rest or moves rectilinearly with constant velocity.

At the beginning of this part I formulated the following three postulates in the form:

(A) All lower inertial systems are physically equivalent, i.e., physical processes are the same in all lower inertial systems when initial boundary conditions are the same.

(B) (*Constant velocity of light*). In every lower inertial system, light travels with the same constant velocity c in every direction.

(C) (*Principle of translation*). There exists a lower inertial system. If σ is a lower inertial system, then also each lower coordinate system σ', which is obtained from σ by a constant translatory motion, is a lower inertial system.

Recall that we mean by a translatory motion that σ' is not rotated compared with σ. By a constant translatory motion of σ' we mean a constant motion of σ' with tespect to σ with constant velocity vector v as on Fig. 2.

(D) (*Principle of time reciprocalness*). In every two lower inertial systems S and S' the times $t \in S$ and $t' \in S'$ are as an Fig. 3 what means that if in the system S time stream as t, then in the system S' time stream as $1/t'$; and reverse, if in the system S' time stream as t', then in the system S time stream as $1/t$.

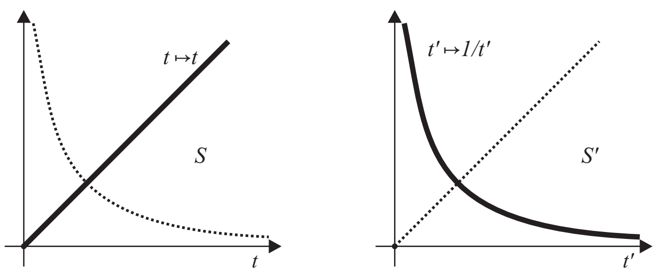

Figure 3.

In connection with the preceding we notice that σ' is obtained from system σ by a constant translatory motion with velocity v. Using a fixed rotation of σ and σ' and a translation of the coordinates ξ' and t', one can always get the following more simple situation: At time $t = 0$, the two lower inertial systems σ and σ' *lower coincide*, and we have $t' = +\infty$. Moreover, $v = Ve$, i.e., the translation is performed for $V > 0$ along x-axis, and for $V < 0$ in the opposite direction as on Fig.4.

The lower coincidence of σ and σ' means that the origins are at time $t = 0$, $t' = +\infty$ and reverse $t = +\infty$, $t' = 0$; and the corresponding coordinate axes have the same direction.

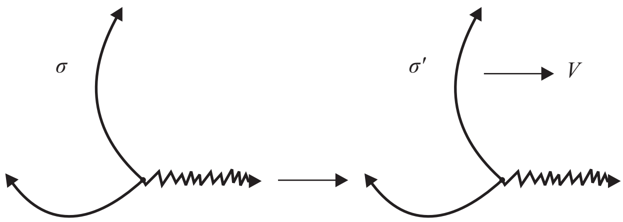

Figure 4.

With the choice of the coordinate axes shown in Fig.2, the plane $y = 0$ coincides with the plane $y' = \infty$ and the plane $z = 0$ with the plane $z' = \infty$. Therefore, y and y' can be related only by expressions of the kind $y = \varepsilon/y'$ where ε is a constant. Owing to the frames σ and σ' having equal rights, the reverse relation must hold, i.e., $y' = \varepsilon/y$ with the same value of the constant ε as in the first case. This means that ε can be an arbitrary constant, i.e., can be and $\varepsilon = 1$.

Similar reasoning yields $z = \varepsilon/z'$. Now let us turn to finding the transformations for x and t. It can be seen from above that the values y and z do not depend on x' and t'. Hence, the values x' and t' cannot depend on y and z, correspondingly, the values of x and t cannot depend on y' and z'. Thus, x and t can be nonlinear functions of only x' and t', from the preceding facts.

The origin of coordinates $(0, 0, 0, 0)$ of the frame σ has the coordinate $x = 0$ in the frame σ and $x' = \infty$, i.e., $x' = -vt'$ in the frame σ', where v is velocity. Consequently, the expression $(x' + vt')^{-1}$ must vanish simultaneously with coordinate x. For this to occur, the nonlinear transformation should have the form

$$x = \frac{\gamma}{x' + vt'}, \tag{1}$$

where γ is a constant. Similarly, the origin of coordinates $(\infty, \infty, \infty, \infty)$ of the frame σ' has the coordinate $x' = \infty$ in the frame σ' and $x = 0$, i.e., $x = vt$ in the frame σ. Hence,

$$x' = \frac{\gamma}{x - vt}; \tag{2}$$

it follows from the frames σ and σ' having equal rights that the constant of proportionality in both cases should be the same.

We shall use the principle of constancy of the speed of light to find the constant γ. Let us begin to count the time in both frames from the moment when their origins of coordinates coincide. Assume that at the moment $t = 0$ and $t' = 0$ a light signal is sent in the direction of the axes x and x' that causes a flesh of light to appear on a screen at a point with the coordinate x in the frame σ and with the coordinate x' in the frame σ'. This from (D) flash is described by the coordinate x and the moment t in the frame σ, and by the coordinate x' and the moment t' in the frame σ', and $x = c/t$ and $x' = ct'$; and $x' = c/t'$, $x = ct$. Using these values of x and x' in (1) and (2), we get

$$\frac{c}{t} = \frac{\gamma}{ct' + vt'} = \frac{\gamma}{t'}\frac{1}{c+v},$$
$$\frac{c}{t'} = \frac{\gamma}{ct - vt} = \frac{\gamma}{t}\frac{1}{c-v};$$

and thus multiplication of these two equation we obtain the following equality of the form

$$c^2 = \frac{\gamma^2}{c^2 - v^2},$$

i.e., $\gamma = c\sqrt{c^2 - v^2}$ and $\gamma = -c\sqrt{c^2 - v^2}$. In further we considered the case $\gamma > 0$. In this sense the symmetrical case $\gamma < 0$ can be considered as a technical totally analogy. Introduction of this value for $\gamma > 0$ into equation (1) gives

$$x = \frac{c\sqrt{c^2 - v^2}}{x' + vt'}. \tag{3}$$

To obtain an equation allowing us to find the value of t according to the known values of x' and t', let us delete the coordinate x from (1) and (2) and solve the resulting expression relative to t. We obtain, substituting for γ its value, the following form

$$t = \frac{-ct'\sqrt{c^2 - v^2}}{x'(x' + vt')} \tag{4}$$

The combination of equations $y = 1/y'$, $z = 1/z'$, (3) and (4) is called **nonlinear transformations** of lower transversal spaces.

If we solve this equations of nonlinear transformations relative to the primed quantities, we get the equations for transformation from the frame σ to σ' in the following form

$$x' = \frac{c\sqrt{c^2 - v^2}}{x - vt}, \quad y' = \frac{1}{y}, \quad z' = \frac{1}{z}, \quad t' = \frac{-ct\sqrt{c^2 - v^2}}{x(x - vt)}. \tag{5}$$

As it should be expected with a view to the equal rights of the frames σ and σ', equations (5) differ from their counterparts of nonlinear transformations only in the sign of v.

It is easy to understand that when $v < c$, i.e., $\gamma < c^2$, the nonlinear transformations become the same as the Galilean type ones for lower transversal spaces. The latter thus retain their importance for speeds that are small in comparison with the speed of light in a vacuum. See Figs. 5 and 6.

When $v > c$, equations of nonlinear transformations and (5) for x,t,x' and t' become imaginary. This agrees with the fact that motion at a speed exceeding that of light in a vacuum is impossible. For $v = c$ we can systems σ and σ' return in the origin positions.

In connection with the preceding, for $v < c$ the change from σ' to σ is given by the *special nonlinear transformations* in the form

$$x = \frac{c^2}{x' + vt'}, \quad y = \frac{1}{y'}, \quad z = \frac{1}{z'}, \quad t = \frac{-c^2 t'}{x'(x' + vt')}, \tag{6}$$

where c is the velocity of light. Equations (6) allow us to pass over from coordinates and time measured in the frame σ' to those measured in the frame σ.

If we solve equations (6) relative to the primed quantities, we get the equations for transformation from the frame σ to σ' in the form

$$x' = \frac{c^2}{x - vt}, \quad y' = \frac{1}{y}, \quad z' = \frac{1}{z}, \quad t' = \frac{-c^2 t}{x(x - vt)}. \tag{7}$$

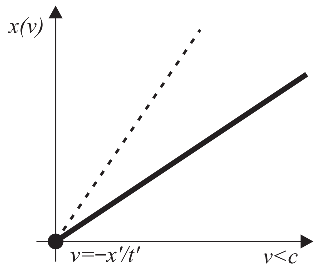

Figure 5. Einstein-Newton physics.

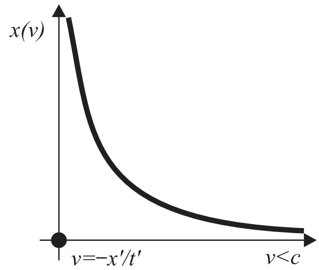

Figure 6. Nonlinear physics.

3. PHYSICS OF LOWER TRANSVERSAL SPACES

Length of bodies in different frames. Let us consider a rod arranged along the x'-axis and at rest relative to the reference frame σ'. Its length in this frame, from the facts for lower transversal spaces, is $l' = 1/|x'_2 - x'_1|$, where x'_1 and x'_2 are the coordinates of the rod ends that to not change with the time t'.

The rod travels with the velocity v_0 relative to the frame σ. To determine its length in this frame, we must note the coordinates of the rod ends x_1 and x_2 at the same moment $t_1 = t_2 = t$. Their "difference" l in lower transversal spaces will give the length of the rod measured in the frame σ. To find the relationship between $l := \triangle x$ and $l' := \triangle x'$ we must take the equation of nonlinear transformations that contains x, x' and t, i.e., the first of the equations (5). Thus we obtain x'_1 and x'_2, i.e.,

$$\triangle x' = \frac{1}{|x'_2 - x'_1|} = \frac{|(x_2 - vt)(x_1 - vt)|}{c\sqrt{c^2 - v^2}} \triangle x,$$

i.e., $l' = |(x_1 - vt)(x_2 - vt)|(c\sqrt{c^2 - v^2})^{-1} l$. Thus, the length l' measured in a frame relative to which it is moving is shorter than the length l measured in the frame relative to which the rod is at rest.

Simultaneity of events. Assume that two events occur simultaneously in the frame σ at points with the coordinates x_1 and x_2 and at the moment $t_1 = t_2 = t$. According to the last of equations (5), the moments t'_1 and t'_2 will correspond to these events in the frame σ'. Examination of these equations shows that if the events occur at different points of space ($x_1 \neq x_2$) in the frame σ, then they will not be simultaneous in the frame σ' ($t_1' \neq t_2'$). Then we obtain, consequently, in different frames σ', the difference $1/|t_2' - t_1'|$ will vary in magnitude and may differ in sign. It must be noted that what has been said above relates only to events between which there is no causal relationship. Causally related events will not be simultaneous in any reference frame, and in all frames event that is the cause will precede the effects.

Length of time. Let us suppose that two events occur at the same point of the frame σ'. The coordinate $x_1' = \alpha$ and the moment t_1' correspond to the first event in the frame, and the coordinate $x_2' = \alpha$ and the moment t_2' to the second one. According to the equation (4), the moments corresponding to these events in the frame σ will be t_1 and t_2 and thus, introducing the notations $\triangle t = 1/|t_2 - t_1|$ and $\triangle t' = 1/|t_2' - t_1'|$, we get the following equation in the form

$$\triangle t = \frac{|(x + vt_1')(x + vt_2')|}{c\sqrt{c^2 - v^2}} \triangle t' \tag{8}$$

that relates the lengths of time between two events measures in the frames σ and σ'.

We notice that $\triangle t'$ can be interpreted as the length of time measured on a clock that is stationary relative to the particle, or, in other words, measured on a clock that is moving together with the particle. The time measured on a clock moving together with a body is called the *proper time* of this body and is usually denoted by the τ, and thus $\triangle t' = \triangle \tau$. Now we can thus write (8) as follows

$$\triangle \tau = \frac{(c\sqrt{c^2 - v^2})\triangle t}{\Big|(x + vt_1')(x + vt_2')\Big|}. \tag{9}$$

where now this equation (9) relates the proper time of a body τ to the time t read on a clock of a reference frame relative to which the body is moving with the velocity v; this clock itself is moving relative to the body with the velocity $-v$.

Transformation of velocities. Let us consider the motion of a point particle. The position of the particle in the frame σ is determined at each moment t by the coordinates x, y, z. The expressions

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad \text{and} \quad v_z = \frac{dz}{dt}$$

are the projections of the vector of the particle's velocity relative (nonlinear) to the frame σ onto the axes x, y, z. The position of the particle in the frame σ' is characterized at each moment t' by the coordinates x', y', z'. The projections of the vector of the particle's velocity relative to the frame σ' onto the axes x', y', z' are determined by the expressions

$$v_x' = \frac{dx'}{dt'}, \quad v_y' = \frac{dy'}{dt'}, \quad \text{and } v_z' = \frac{dz'}{dt'};$$

and, thus, from (3) and (4) we have dx, dy, dz, and dt; and thus we get formulas for transformations of the velocities when passing over from one reference frame to another: v_x, v_y, and v_z.

It is simple to obtain expressions for velocities in the frame σ' through the velocities in the frame σ from (5) in the form: v'_x, v'_y, and v'_z.

Expression in nonlinear mechanics for the Momentum. It can be shown that the law of momentum conservation will be invariant with respect to the nonlinear transformations at any velocities if we substitute the proper time of a particle τ for the time t in classical expression $p = mv = m\,dr/dt$. Consequently, the nonlinear expression for the momentum has the form $p = m\,dr/d\tau$, where dr is the displacement of the particle in the reference frame in which the momentum p is determined, whereas the length of time $d\tau$ is determined on a clock travelling together with the particle.

We get an expression for the momentum trough the time t of the frame od reference relative to which the motion of bodies is being observed. By (9) we have $d\tau$, where v is the velocity of the body. This substitution in the proceeding equation of the momentum yields

$$p = \frac{m\Big|(x+vt'_1)(x+vt'_2)\Big|}{c\sqrt{c^2-v^2}}\frac{dr}{dt}. \tag{10}$$

We notice that as in Newtonian mechanics equals the product of the mass of a body and its velocity $p = m_{nl}v$. The mass of a body, however, is not a constant invariant quantity, but depends on the velocity according to the law

$$m_{nl} = \frac{m\Big|(x+vt'_1)(x+vt'_2)\Big|}{c\sqrt{c^2-v^2}} := \frac{m_0 c^k}{\sqrt{1-v^2/c^2}}, \tag{11}$$

where $m_0 := m_0(v) = c^{-k-2}m\Big|(x+vt'_1)(x+vt'_2)\Big|$, $k \in \mathbb{R}$ is an arbitrary fixed real number, and m is an invariant mass of body.

Thus, in this interpretation, m_0 is an invariant mass, where the noninvariant mass m_{nl} depending on the velocity. In further, m_{nl} we will call the **nonlinear mass**.

Nonlinear mechanics. As well, Newton's second law states that the time derivate of the momentum p of a particle (point particle) equals the resultant force F acting on the particle. Hence, the nonlinear expression of Newton's second law, from $p = m_{nl}v$ and (11) in the lower transversal spaces, has the form

$$\frac{d}{dt}\left(\frac{m_0 v c^k}{\sqrt{1-v^2/c^2}}\right) = F \tag{12}$$

for fixed arbitrary $k \in \mathbb{R}$. It should be borne in mind that the equation $ma = F$ cannot be used in the nonlinear case, the acceleration a and the force F, generally speaking, being noncollinear.

To find the nonlinear expression for the energy, let us proceed in the same way as a classical case. We shall multiply equation (12) by the displacement of a particle $ds = v\,dt$. The result is

$$\frac{d}{dt}\left(\frac{m_0 v c^k}{\sqrt{1-v^2/c^2}}\right) v\,dt = F\,ds,$$

where the right hand side of this equation gives the work dA done on the particle during the time dt.

It is well known that the work of the resultant of all the forces is spent on an increment of the kinetic energy of the perticle. Consequently, the left hand side of the equation should be interpreted as the increment of the kinetic energy E_k of the particle during time dt. Thus we obtain

$$dE_k = \frac{d}{dt}\left(\frac{m_0 v c^k}{\sqrt{1-v^2/c^2}}\right) v\,dt = v\,d\left(\frac{m_0 v c^k}{\sqrt{1-v^2/c^2}}\right), \tag{13}$$

and let us transform the obtained expression, bearing in mind that $v\,dv = d(v^2/2)$ we have

$$dE_k = dA = \frac{m_0 v c^k}{\left(1-\frac{v^2}{c^2}\right)^{3/2}} dv, \tag{14}$$

for the arbitrary fixed $k \in \mathbb{R}$.

On the other hand, from the (11), we obtain the following fact in the form

$$dm_{nl} = \frac{m_0 v c^k}{c^2\left(1-\frac{v^2}{c^2}\right)^{3/2}} dv; \tag{15}$$

and thus from (14) and (15) we obtain $dE_k = c^2 dm_{nl}$. This means that, in general case, for the mass m_{nl} and the energy E, we can brief that

$$E = m_{nl} c^2. \tag{16}$$

We notice that equality (16), on the other condition, follows from (13) by integration of this expression. In connection with this we find that from (11) and (16) we obtain that

$$E = \frac{m_{nl} c^{k+2}}{\sqrt{1-\frac{v^2}{c^2}}}, \tag{17}$$

for an arbitrary fixed $k \in \mathbb{R}$; and thus, when $v = 0$, equation (17) transforms into equation $E = m_{nl} c^{k+2}$ and into equation of the form (16) for $k = 0$. This energy is the internal energy of a particle not associated with its motion as a whole.

Further facts and problems. Does there exists *Ether* (lower ether) in the lower transversal spaces (in the preceding sense of the nonlinear physics)?! If there is a lower ether in the some lower transversal space, does he the same as and the Ether in Einstein's physics?

In connection with this, if Michelson's experiment to make in nonlinear physics does result the same as in the Einstein's physics?

Does in the nonlinear physics there is well known *photo-effect* as in the classical case which is to explain Einstein in 1905? If there exists *lower photo-effect* (i.e., photo-effect in nonlinear physics), does he can the same arguments of nonlinear physics to explain as in the classical case?

4. Asymptotic behaviour in springs of lower spaces

Lower general edges spaces. Let X be a nonempty set. The function $A : X \times X \to [a, b] \subset \mathbb{R}^0_+$ for $a < b$ is called a **lower general edges transverse** on X (or *lower general edges transversal*) iff: $A(x, y) = b$ if and only if $x = y$ for all $x, y \in X$.

A **lower general edges transversal space** (or *lower general edges space*) is a set X together with a given lower general edges transverse on X.

Otherwise, the function A is called a **semilower general edges transverse** on X (or *semilower general edges transversal*) iff: $A(x, y) = b$ implies $x = y$ for all $x, y \in X$. A **semilower general edges transversal space** $X := (X, A)$ is a set X together with a given semilower general edges transverse on X. For any nonempty set S in the lower general edges transversal space X the *diameter* of S is defined as

$$\operatorname{diam}(S) := \inf \big\{ A(x, y) \ : \ x, y \in S \big\};$$

it is a real number in $[a, b]$, $A \subset B$ implies $\operatorname{diam}(B) \leqslant \operatorname{diam}(A)$. The relation $\operatorname{diam}(S) = b$ holds if and only if S is a one point set.

Elements of a lower general edges transversal space will usually be called points. Given a lower general edges transversal space $X := (X, A)$ and a point $z \in X$, the **open ball** of *center* z and radius $r > 0$ is the set

$$B(z, r) := \big\{ x \in X \ : \ A(z, x) > b - r \big\}.$$

The *convergence* $x_n \to x$ as $n \to \infty$ in the lower general edges transversal space $X := (X, A)$ means that

$$A(x_n, x) \to b \ \text{ as } \ n \to \infty,$$

or equivalently, for every $\varepsilon > 0$ there exist an integer n_0 such that the relation $n \geqslant n_0$ implies $A(x_n, x) > b - \varepsilon$.

The sequence $\{x_n\}_{n \in \mathbb{N}}$ in the lower general edges transversal space $X := (X, A)$ is called **lower transversal sequence** (or *lower Cauchy sequence*) iff for every $\varepsilon > 0$ there is an $n_0 = n_0(\varepsilon)$ such that

$$A(x_n, x_m) > b - \varepsilon \ \text{ for all } \ n, m \geqslant n_0.$$

Let $X := (X, A)$ be a lower general edges transversal space and $T : X \to X$. We notice, from Tasković [5], that a sequence of iterates $\{T^n(x)\}_{n \in \mathbb{N}}$ in X is said to be lower transversal sequence if and only if

$$\lim_{n \to \infty} \Big(\operatorname{diam}\{T^k(x) \ : \ k \geqslant n\} \Big) = b.$$

In this sense, a lower general edges transversal space is called **lower complete** iff every lower transversal sequence converges.

Also, a space $X := (X, A)$ is said to be **lower orbitally complete** (or *lower T-orbitally complete*) iff every lower transversal sequence which is contained in $\mathcal{O}(x)$ for some $x \in X$ converges in X.

A function f mapping X into the reals is **T-orbitally upper semicontinuous** at $p \in X$ iff $\{x_n\}_{n\in\mathbb{N}}$ is a sequence in $\mathcal{O}(x)$ and $x_n \to p$ $(n \to \infty)$ implies that $f(p) \geqslant \lim.\sup f(x_n)$.

Let $X := (X, A)$ be a lower general edges transversal space. A mapping $T : X \to X$ is said to be **lower general edges contraction** if there exists an $0 \leqslant \lambda < 1$ such that

$$\text{(Le)} \qquad A\big(T(x), T(y)\big) \geqslant \lambda A(x, y) + b(1 - \lambda)$$

for all points $x, y \in X$. For further facts on the lower general edges contractions see: Tasković [5].

Let (X, A_X) and (Y, A_Y) be two lower general edges transversal spaces and let $T : X \to Y$. In order, we notice from Tasković [5], that T be **lower general edges continuous** at $x_0 \in X$ iff for every $\varepsilon > 0$ there exists a $\delta > 0$ such that for every $x \in X$ the following relation holds that

$$A_X(x_0, x) > b - \delta \ \text{ implies } \ A_Y\big(T(x_0), T(x)\big) > b - \varepsilon.$$

A typical first example of a lower general edges continuous mapping is the lower general edges contraction on the lower general edges transversal space $X := (X, A)$. For the further facts on the lower general edges continuous mappings see: Tasković [5].

Let X be a nonempty set, $T : X \to X$, and let $A : X \times X \to [a, b] \subset \mathbb{R}^0_+$ for $a < b$ be a given function. We shall introduce the concept of lower general edges TCS-convergence in a space X, i.e., a general edges transversal space $X := (X, A)$ satisfies the **condition of lower general edges TCS-convergence** iff $x \in X$ and if $A(T^n x, T^{n+1} x) \to b$ $(n \to \infty)$ implies that $\{T^n(x)\}_{n\in\mathbb{N}}$ has a convergent subsequence.

Theorem 1. *Let T be a mapping of semilower general edges space $X := (X, A)$ into itself, where X satisfies the condition of lower general edges TCS-convergence. Suppose that for all $x, y \in X$ there exist a sequence of nonnegative real functions $\{\alpha_n(x, y)\}_{n\in\mathbb{N}}$ such that $\alpha_n(x, y) \to b$ $(n \to \infty)$ and positive integer $m(x, y)$ such that*

$$\text{(D)} \qquad A(T^n(x), T^n(y)) \geqslant \alpha_n(x, y) \ \text{ for all } \ n \geqslant m(x, y),$$

where $A : X \times X \to [a, b] \subset \mathbb{R}^0_+$ for $a < b$. If $x \mapsto A(x, T(x))$ is a T-orbitally upper semicontinuous function or T is orbitally continuous, then T has a unique fixed point $\xi \in X$ and $T^n(x) \to \xi$ $(n \to \infty)$ for each $x \in X$.

Proof. For $y = T(x)$ from (D) we have that $A(T^n x, T^{n+1} x) \geqslant \alpha_n(x, Tx)$ for all $n \geqslant m(x, Tx))$, and thus we obtain that $A(T^n x, T^{n+1} x) \to b$ $(n \to \infty)$. This implies (from lower general edges TCS-convergence) that the sequence of iterates $\{T^n(x)\}_{n\in\mathbb{N}}$ has a convergent subsequence $\{T^{n(i)}(x)\}_{i\in\mathbb{N}}$ with the limit point $\xi \in X$. Since $x \mapsto A(x, T(x))$ is T orbitally upper semicontinuous, we get

$$A(\xi, T(\xi)) \geqslant \limsup_{i\to\infty} A(T^{n(i)}x, T^{n(i)+1}x) = \limsup_{n\to\infty} A(T^n x, T^{n+1}x) = b,$$

which implies that $A(\xi, T(\xi)) = b$, i.e., $\xi = T(\xi)$. On the other hand, if T is orbitally continuous the proof of previous fact is trivial. We complete the proof by showing that T can have at most one fixed point. Indeed, if we suppose that $\xi \neq \eta$ were two fixed points, then from (D) we have

$$A(\xi, \eta) = A(T^n(\xi), T^n(\eta)) \geqslant \alpha_n(\xi, \eta) \text{ for every } n \geqslant m(\xi, \eta);$$

taking limits as $n \to \infty$ we obtain a contradiction. Thus we obtain that $\xi = \eta$, i.e., T has a unique fixed point in X. The proof is complete. □

Applications of Theorem 1. In connection with the preceding facts we have the following two "asymptotic" statements for existence of a unique fixed point as applications of Theorem 1 on lower general edges transversal spaces.

Corollary 1. *Let $X := (X, \rho)$, with the continuous general edges transverse ρ, be a lower general edges complete lower general edges transversal space, $T : X \to X$ is a continuous function, and $\varphi_n : [a, b] \to [a, b]$ for $n \in \mathbb{N}$ sequence of continuous functions such that for every $n \in \mathbb{N}$ satisfying*

$$\rho\left[T^n(x), T^n(y)\right] \geqslant \varphi_n\left(\rho[x, y]\right) \quad \text{for all} \quad x, y \in X;$$

and assume also that there exists a function $\varphi : [a, b] \to [a, b]$ such that for any $t \in [a, b)$, $\varphi(t) > t$, $\varphi(t) = b$ iff $t = b$, and $\varphi_n \to \varphi$ $(n \to \infty)$ uniformly on the range of ρ. Then T has a unique fixed point in X.

Corollary 2. *Let $X := (X, \rho)$, with the continuous general edges transverse ρ, be a lower general edges complete lower general edges transversal space, $T : X \to X$ is a continuous function, and $\varphi_n : [a, b] \to [a, b]$ for $n \in \mathbb{N}$ sequence of continuous functions such that for every $n \in \mathbb{N}$ satisfying*

$$\rho\left[T^n(x), T^n(y)\right] \geqslant$$

$$\geqslant \min\left\{\varphi_n(\rho[x, y]), \varphi_n(\rho[x, Tx]), \varphi_n(\rho[y, Ty]), \varphi_n(\rho[x, Ty]), \varphi_n(\rho[y, Tx])\right\}$$

for all $x, y \in X$; and assume also that there exists function $\varphi : [a, b] \to [a, b]$ such that for any $t \in [a, b)$, $\varphi(t) > t$, $\varphi(t) = b$ iff $t = b$, and $\varphi_n \to \varphi$ $(n \to \infty)$ uniformly on the range of ρ. Then T has a unique fixed point in X.

Further applications of Theorem 1. In further we give the following examples of Theorem 1 as some examples of lower general edges transversal spaces.

Example 1. (*Metric spaces*). A fundamental first example of lower general edges transversal space is a metric space. Indeed, if (X, q) is a metric space, then for the lower general edges transverse $\rho : X \times X \to [a, b] \subset \mathbb{R}^0_+$ for $a < b$ defined by

$$\rho[x, y] = \frac{(a - b)q[x, y]}{1 + q[x, y]} + b$$

for all $x, y \in X$ we have that (X, ρ) is an example of a lower general edges transversal space. In general, every metric space is an example of a lower general edges transversal space.

Example 2. (*Lower probabilistic spaces*). A mapping $F : \mathbb{R} \to \mathbb{R}^0_+$ is called a **left distribution function** if it is nondecreasing, left-continuous with $\inf F = 0$ and $\sup F = 1$. We will denote by $\mathcal{L}$ the set of all left distribution functions. We shall denote the left distribution function $\mathcal{L}(p, q)$ by $F_{p,q}(x)$, whence $F_{p,q}(x)$ will denote the value of $F_{p,q}$ at $x \in \mathbb{R}$.

An example of lower general edges transversal space is a **lower probabilistic space** which is a nonempty set X together with the functions $F_{p,q}(x)$ with the following properties: $F_{p,q}(x) = F_{q,p}(x)$, $F_{p,q}(0) = 0$,

$$F_{p,q}(x) = 1 \text{ for } x > 0 \text{ if and only if } p = q, \tag{18}$$

and if there is a nondecreasing functions $\tau : [0, 1]^2 \to [0, 1]$ with the property $\tau(t, t) \geqslant t$ for all $t \in [0, 1]$ such that

$$F_{p,q}(x + y) \geqslant \tau(F_{p,r}(x), F_{r,q}(y)) \tag{Nm}$$

for all $p, q, r \in X$ and for all $x, y \geqslant 0$. Then, from (18), we immediately obtain that every lower probabilistic space, for $\rho[p, q] = F_{p,q}(x) : X \times X \to [0, 1]$ is a lower general edges transversal space.

Example 3. (*Lower parametric transversal spaces*). In connection with the preceding facts, the function $N : X \times X \times \mathbb{R} \to [a, b] \subset \mathbb{R}^0_+$ for $a < b$ is called a **lower parametric transverse** on X (or a *lower parametric transversal*) iff: for some $c \in \mathbb{R}^0_+$ is $N(u, v, t) = b$ for every $t > c$ if and only if $u = v$, and $\lim_{n\to\infty} N(u, v, x_n) = b$ for arbitrary nondecreasing sequence $\{x_n\}_{n\in\mathbb{N}}$ in $[c, +\infty)$ with $x_n \to +\infty$ $(n \to \infty)$.

A **transversal lower parametric space** is a set X together with a given lower parametric transverse $N : X \times X \times \mathbb{R} \to [a, b] \subset \mathbb{R}^0_+$ for $a < b$ in notation $X := (X, N)$. De facto, every transversal lower parametric space $X := (X, N)$, for $A = N$, is a lower general edges transversal space. For this spaces the following “asymptotic” statement holds.

Corollary 3. *Let $X := (X, N)$ be a transversal lower parametric space with the continuous lower parametric transverse, $T : X \to X$ is a continuous function, and X with the condition of lower general edges TCS-convergence. Suppose that there exists a function $\varphi : [c, +\infty) \to [c, +\infty)$ for some $c \in \mathbb{R}$ satisfying $\varphi(t) > t$ for every $t > c$ and*

$$\lim_{n\to\infty} \varphi^n(t) = +\infty \text{ for every } t > c, \tag{As}$$

and such that

$$N\Big(T^n(x), T^n(y), t\Big) \geqslant N\Big(x, y, \varphi^n(t)\Big) \text{ for every } n \in \mathbb{N}, \tag{A}$$

for every $t > c$, and for all $x, y \in X$. Then T has a unique fixed point in X.

Proof. (Application of Theorem 1).

We define a function $A : X \times X \to [a, b] \subset \mathbb{R}^0_+$ for $a < b$ by $A(u, v) := N(u, v, t)$ and define a sequence of functions $\{\alpha_n(u, v)\}_{n\in\mathbb{N}}$ by $\alpha_n(u, v) := N(u, v, \varphi^n(t))$.

Thus $\alpha_n(u, v) \to \lim_{n\to\infty} N(u, v, \varphi^n(t)) = b$ as $n \to \infty$ as in Theorem 1. Also, $x \mapsto A(x, Tx) := N(x, Tx, t)$ is T-orbitally upper semicontinuous, because T

and N are continuous. Since $A(u,v) := N(u,v,t) = b$ implies $u = v$ for every $t > c$, and since X satisfies the condition of lower general edges TCS-convergence, applying Theorem 1 we obtain that T has a unique fixed point in X. The proof is complete. □

We notice that the following "asymptotic" statement holds which is as a special case of the preceding Corollary 3.

Corollary 4. *Let $X := (X, N)$ be a transversal lower parametric space with the continuous lower parametric transverse, $T : X \to X$ is a continuous function, and X with the condition of lower general edges TCS-convergence. Suppose that there exists an increasing continuous function $\varphi : [c, +\infty) \to [c, +\infty)$ for some $c \in \mathbb{R}$ satisfying $\varphi(t) < t$ for every $t \in [c, +\infty)$ such that*

$$\text{(B')} \qquad N\Big(T^n(x), T^n(y), \varphi^n(t)\Big) \geqslant N(x, y, t)$$

for every $n \in \mathbb{N}$, for all $x, y \in X$ and for every $t > c$. Then T has a unique fixed point in X.

Proof. Since for the function $\varphi : [c, +\infty) \to [c, +\infty)$ for some $c \in \mathbb{R}$ there is the inverse function $\varphi^{-1} : [c, +\infty) \to [c, +\infty)$ with the property (As), thus from (B') we obtain a form of the inequality (A) in the form of the following inequality as

$$N\Big(T^n(x), T^n(y), t\Big) \geqslant N\Big(x, y, \varphi^{-n}(t)\Big) \ \text{ for } \ n \in \mathbb{N}$$

and for all $x, y \in X$. Thus, applying Corollary 3 we obtain this statement as a consequence. The proof is complete. □

An essential remark. *We notice that the lower parametric transversal spaces are, de facto, also the lower general edges transversal spaces.*

The lower spring transversal spaces. In connection with the preceding, we shall introduce the concept of lower spring transversal space. In this sense, the function $A : X \times X \to [0, +\infty] := \mathbb{R}^0_+ \cup \{+\infty\}$ is called a **lower spring transverse** on a nonempty set X (or *lower spring transversal*) iff: $A(x, y) = +\infty$ if and only if $x = y$ for all $x, y \in X$.

A **lower spring transversal space** $X := (X, A)$ is a nonempty set X together with a given lower spring transverse A on X.

Otherwise, the function A is called a **semilower spring transverse** on a nonempty set X iff: $A(x, y) = +\infty$ implies $x = y$ for all $x, y \in X$. A **semilower spring transversal space** $X := (X, A)$ is a nonempty set X together with a given semilower spring transverse A on X.

For any nonempty set S in the lower spring transversal space $X := (X, A)$ the **trs.diameter** of S is defined as

$$\operatorname{trs.diam}(S) := \inf \big\{A(x, y) : x, y \in S\big\};$$

where $Y \subset B$ implies $\operatorname{trs.diam}(B) \leqslant \operatorname{trs.diam}(Y)$. The relation $\operatorname{trs.diam}(S) = +\infty$ holds if and only if S is a one point set.

Elements of a lower spring transversal space will usually be called *points*. Given a lower spring transversal space $X := (X, A)$, and a point $z \in X$, the **open ball** of center z and radius $r > 0$ is the set

$$A(B(z, r)) = \big\{x \in X : A(z, x) > r\big\}.$$

On the other hand, from Tasković [5], the *lower spring convergence* $x_n \to x$ as $n \to \infty$ in the lower spring transversal space $X := (X, A)$ means that

$$A(x_n, x) \to +\infty \quad \text{as} \quad n \to \infty,$$

or equivalently, for every $\varepsilon > 0$ there exist an integer n_0 such that the relation $n \geqslant n_0$ implies $A(x_n, x) > \varepsilon$.

The sequence $\{x_n\}_{n\in\mathbb{N}}$ in the lower spring transversal space $X := (X, A)$ is called **lower spring transversal sequence** (or *lower spring Cauchy sequence*) iff for every $\varepsilon > 0$ there is an $n_0 = n_0(\varepsilon)$ such that

$$A(x_n, x_m) > \varepsilon \quad \text{for all} \quad n, m \geqslant n_0.$$

Let $X := (X, A)$ be a lower spring transversal space and $T : X \to X$. We notice, from Tasković [5], that a sequence of iterates $\{T^n(x)\}_{n\in\mathbb{N}}$ in X is said to be lower spring transversal sequence if and only if

$$\lim_{n\to\infty} \Big(\operatorname{trs.diam}\{T^k(x) : k \geqslant n\}\Big) = +\infty.$$

In this sense, a lower spring transversal space is called **lower spring complete** iff every lower spring transversal sequence lower spring converges.

Also, a space (X, ρ) is said to be **lower spring orbitally complete** (or *lower spring T-orbitally complete*) iff every lower spring transversal sequence which in contained in $\mathcal{O}(x) := \{x, Tx, T^2(x), \ldots\}$ for some $x \in X$ lower spring converges in X.

Annotation 1. We notice that in 1995 Tasković proved the following statement for a class of expansion mappings. Namely, if $X := (X, A)$ is a lower spring T-orbitally complete lower spring transversal space, if $T : X \to X$, and if there exists a number $q > 1$ such that

$$A\Big(T(x), T(y)\Big) \geqslant qA(x, y) \tag{19}$$

for each $x, y \in X$, then T has a unique fixed point in the lower spring transversal space X.

Annotation 2. Let $X := (X, A_X)$ and $Y := (Y, A_Y)$ be two lower spring transversal spaces and let $T : X \to Y$. We notice, from: Tasković [5], that T be **lower spring continuous** at $x_0 \in X$ iff for every $\varepsilon > 0$ there exists a $\delta > 0$ such that for every $x \in X$ the following relation holds as

$$A_X(x, x_0) > \delta \quad \text{implies} \quad A_Y(T(x), T(x_0)) > \varepsilon.$$

A typical first example of a lower spring continuous mapping is the mapping $T : X \to X$ with property (19). For further facts on the lower spring continuous mappings see: Tasković [5].

Asymptotic contractions on lower spring transversal spaces. Let X be a nonempty set, $T : X \to X$, and let $A : X \times X \to \mathbb{R}_+^0 \cup \{+\infty\}$ be a given function. We shall introduce the concept of lower spring TCS-convergence in a space X, i.e., a lower spring transversal space $X := (X, A)$ satisfies the **condition of lower spring TCS-convergence** iff $x \in X$ and if $A(T^n x, T^{n+1} x) \to +\infty$ $(n \to \infty)$ implies that $\{T^n(x)\}_{n\in\mathbb{N}}$ has a convergent subsequence.

Theorem 2. *Let T be a mapping of lower spring transversal space $X := (X, A)$ into itself, where X satisfies the condition of lower spring TCS-convergence. Suppose that for all $x, y \in X$ there exist a sequence of nonnegative real functions $\{\alpha_n(x, y)\}_{n\in\mathbb{N}}$ such that $\alpha_n(x, y) \to +\infty$ $(n \to \infty)$ and positive integer $m(x, y)$ such that*

$$\text{(D)} \qquad A(T^n(x), T^n(y)) \geqslant \alpha_n(x, y) \ \text{ for all } \ n \geqslant m(x, y),$$

where $A : X \times X \to \mathbb{R}_+^0 \cup \{+\infty\}$. If $x \mapsto A(x, T(x))$ is a T-orbitally upper semicontinuous function or T is orbitally continuous, then T has a unique fixed point $\xi \in X$ and $T^n(x) \to \xi$ $(n \to \infty)$ for each $x \in X$.

The proof of this statement is totally analogous with the preceding proof of Theorem 1 on the semilower general edges space. Thus the proof of this result we omit.

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