

Intuitionistic Fuzzy Strong Precompactness in Coker's Sense

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ABSTRACT. The concept of fuzzy strong precompactness in Coker's space has been introduced and studied. Several preservation properties and some characterizations concerning fuzzy strong precompactness have been obtained.

1. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh in his classic paper [8]. Using the concept of fuzzy sets Chang [2] introduced the fuzzy topological spaces. Since Atanassov [1] introduced the notion of intuitionistic fuzzy sets, Coker [3] defined the intuitionistic fuzzy topological spaces. This approach provided a wide field for investigation in the area of fuzzy topology and its applications. The authors introduced the class of intuitionistic fuzzy strongly preclosed sets [6]. This approach allows introducing of some new types of intuitionistic fuzzy topological spaces. The idea is to relate such defined concepts with existing ones.

Here, we introduce the concept of intuitionistic fuzzy strongly precompactness which is strictly stronger than the concept of intuitionistic fuzzy compactness. Also, we give an extension of this notion to arbitrary intuitionistic fuzzy set.

2. PRELIMINARIES

We introduce some basic notions and results that are used in the sequel.

Definition 2.1 ([1]). Let X be a nonempty fixed set and I the closed interval $[0, 1]$. An intuitionistic fuzzy set (IFS) A is an object of the following form

$$A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle \mid x \in X\}$$

where the mapping $\mu_A : X \rightarrow I$ and $\gamma_A : X \rightarrow I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) for each element $x \in X$ to the set A , respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

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Obviously, every fuzzy set A on a nonempty set X is an IFS of the following form

$$A = \{\langle x, \mu_A(x), 1 - \mu_A(x) \rangle \mid x \in X\}.$$

Definition 2.2 ([1]). Let A and B be IFS's of the form

$$A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle \mid x \in X\}$$

and

$$B = \{\langle x, \mu_B(x), \gamma_B(x) \rangle \mid x \in X\}.$$

Then

- (i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\gamma_A(x) \geq \gamma_B(x)$;
- (ii) $\overline{A} = \{\langle x, \gamma_A(x), \mu_A(x) \rangle \mid x \in X\}$;
- (iii) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \gamma_A(x) \vee \gamma_B(x) \rangle \mid x \in X\}$;
- (iv) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \gamma_A(x) \wedge \gamma_B(x) \rangle \mid x \in X\}$.

We will use the notation $A = \langle x, \mu_A, \gamma_A \rangle$ instead of $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle \mid x \in X\}$. A constant fuzzy set α taking value $\alpha \in [0, 1]$ will be denote by $\underline{\alpha}$. The IFS's 0_\sim and 1_\sim are defined by $0_\sim = \{\langle x, \underline{0}, \underline{1} \rangle \mid x \in X\}$ and $1_\sim = \{\langle x, \underline{1}, \underline{0} \rangle \mid x \in X\}$.

Let f be a mapping from an ordinary set X into an ordinary set Y . If

$$B = \{\langle y, \mu_B(y), \gamma_B(y) \rangle \mid y \in Y\}$$

is an IFS in Y , then the inverse image of B under f is IFS defined by

$$f^{-1}(B) = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x) \rangle \mid x \in X\}.$$

The image of IFS

$$A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle \mid x \in X\}$$

under f is IFS defined by

$$f(A) = \{\langle y, f(\mu_A)(y), f(\gamma_A)(y) \rangle \mid y \in Y\}$$

where

$$f(\mu_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x), & f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

and

$$f(\gamma_A)(y) = \begin{cases} \inf_{x \in f^{-1}(y)} \gamma_A(x), & f^{-1}(y) \neq \emptyset \\ 1, & \text{otherwise} \end{cases}$$

for each $y \in Y$.

Definition 2.3 ([3]). An intuitionistic fuzzy topology (IFT) in Coker's sense on a nonempty set X is a family τ of IFS's in X satisfying the following axioms:

- (T₁) $0_\sim, 1_\sim \in \tau$;
- (T₂) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;
- (T₂) $\cup G_i \in \tau$ for arbitrary family $\{G_i \mid i \in I\} \subseteq \tau$.

In this paper by (X, τ) or simply by X we will denote the Coker's intuitionistic fuzzy topological space (IFTS). Each IFS in τ is called intuitionistic fuzzy open set (IFOS) in X . The complement \bar{A} of an IFOS in X is called an intuitionistic fuzzy closed set (IFCS) in X .

Definition 2.4 ([3, 4]). Let $A = \langle x, \mu_A, \gamma_A \rangle$ be an IFS in IFTS X . Then

$\text{int } A = \cup\{G \mid G \text{ is an IFOS in } X \text{ and } G \subseteq A\}$ is called an intuitionistic fuzzy interior of A ;

$\text{cl } A = \cap\{G \mid G \text{ is an IFCS in } X \text{ and } G \supseteq A\}$ is called an intuitionistic fuzzy closure of A .

Definition 2.5 ([5]). An IFS A in an IFTS X is called an intuitionistic fuzzy preopen set (IFPOS) if $A \subseteq \text{int}(\text{cl } A)$.

The complement \bar{A} of an IFPOS in X is called an intuitionistic fuzzy preclosed set (IFPCS) in X .

Definition 2.6 ([6]). Let $A = \langle x, \mu_A, \gamma_A \rangle$ be an IFS in IFTS X . Then

$p\text{int } A = \cup\{G \mid G \text{ is an IFPOS in } X \text{ and } G \subseteq A\}$ is called an intuitionistic fuzzy preinterior of A ;

$p\text{cl } A = \cap\{G \mid G \text{ is an IFCS in } X \text{ and } G \supseteq A\}$ is called an intuitionistic fuzzy preclosure of A .

Definition 2.7 ([6]). An IFS A in an IFTS X is called an intuitionistic fuzzy strongly preopen set (IFSPOS) if $A \subseteq \text{int}(p\text{cl } A)$.

The complement \bar{A} of an IFSPOS in X is called an intuitionistic fuzzy preclosed set (IFSPCS) in X .

Definition 2.8 ([6, 7]). Let f be a mapping from an IFTS X into an IFTS Y . The mapping f is called

- (i) an intuitionistic fuzzy strong precontinuity if $f^{-1}(B)$ is an IFSPOS in X , for each IFOS B in Y .
- (ii) an intuitionistic fuzzy strong irresolute precontinuity if $f^{-1}(B)$ is an IFSPOS in X , for each IFSPOS B in Y .

Definition 2.9 ([3]). Let X be an IFTS. A family $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ of IFOS's in X satisfies the condition $\cup\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\} = 1_{\sim}$ is called a fuzzy open cover of X .

A finite subfamily of a fuzzy open cover $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ which is also a fuzzy open cover of X is called a finite subcover of $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$.

An IFTS X is called fuzzy compact if and only if every fuzzy open cover has a finite subcover.

Definition 2.10 ([3]). Let A be an IFS in an IFTS X . A family $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ of IFOS's in X satisfies the condition $A \subseteq \cup\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ is called a fuzzy open cover of A .

A finite subfamily of a fuzzy open cover $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ of A which is also a fuzzy open cover of A is called a finite subcover of $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$.

An IFS $A = \langle x, \mu_A, \gamma_A \rangle$ in an IFTS X is called fuzzy compact if and only if every fuzzy open cover of A has a finite subcover.

3. INTUITIONISTIC FUZZY STRONG PRECOMPACTNESS

Definition 3.1. Let X be an IFTS. A family $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ of IFSPOS's in X satisfies the condition $\cup\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\} = 1_{\sim}$ is called a fuzzy strongly preopen cover of X .

A finite subfamily of a fuzzy strongly preopen cover $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ which is also a fuzzy strongly preopen cover of X is called a finite subcover of $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$.

Definition 3.2. Let X be an IFTS. A family $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ of IFSPCS's in X has the finite intersection property if every finite subfamily $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i = 1, 2, \dots, n\}$ satisfies the condition

$$\bigcap_{k=1}^n \langle x, \mu_{G_i}, \gamma_{G_i} \rangle \neq 0_{\sim}.$$

Definition 3.3. An IFTS X is called fuzzy strongly precompact if and only if every fuzzy strongly preopen cover has a finite subcover.

We now derive an important criteria for intuitionistic fuzzy strong precompactness.

Theorem 3.1. *An IFTS X is fuzzy strongly precompact if and only if every family $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ of IFSPCS's with the finite intersection property has a nonempty intersection.*

Proof. Suppose X is fuzzy strongly precompact and let $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ be any family of IFSPCS's in X such that

$$\cap\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\} = 0_{\sim}.$$

Therefore $\wedge\{\mu_{G_i}(x) \mid i \in I\} = 0$ and $\vee\{\gamma_{G_i}(x) \mid i \in I\} = 1$. Then $\cup\{\langle x, \gamma_{G_i}, \mu_{G_i} \rangle \mid i \in I\} = 1_{\sim}$, so $\{\langle x, \gamma_{G_i}, \mu_{G_i} \rangle \mid i \in I\}$ is a fuzzy strongly preopen cover of X . Since X is fuzzy strongly precompact there is a finite subcover $\{\langle x, \gamma_{G_i}, \mu_{G_i} \rangle \mid i = 1, 2, \dots, n\}$. Then $\bigcup_{k=1}^n \langle x, \gamma_{G_i}, \mu_{G_i} \rangle = 1_{\sim}$. Hence $\vee\{\gamma_{G_i}(x) \mid i = 1, 2, \dots, n\} = 1$ and $\wedge\{\mu_{G_i}(x) \mid i = 1, 2, \dots, n\} = 0$. Finally

$$\bigcap_{k=1}^n \langle x, \mu_{G_i}, \gamma_{G_i} \rangle = 0_{\sim}.$$

We have proved that if X is fuzzy strongly precompact space, then given any family $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ of IFSPCS's whose intersection is empty, the intersection of some finite subfamily is empty.

Conversely, let X has the finite intersection property. It means that if the intersection of any family of IFSPCS's is empty, the intersection of each finite

subfamily is empty. Suppose $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ is any fuzzy strongly preopen cover of X . Then

$$\cup\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\} = 1_{\sim}.$$

Therefore $\vee\{\mu_{G_i}(x) \mid i \in I\} = 1$ and $\wedge\{\gamma_{G_i}(x) \mid i \in I\} = 0$. Hence $\cap\{\langle x, \gamma_{G_i}, \mu_{G_i} \rangle \mid i \in I\} = 0_{\sim}$, so $\{\langle x, \gamma_{G_i}, \mu_{G_i} \rangle \mid i \in I\}$ is a family of IFSPCS's whose intersection is empty. According to the assumption we can find finite subfamily $\{\langle x, \gamma_{G_i}, \mu_{G_i} \rangle \mid i = 1, 2, \dots, n\}$ such that

$$\bigcap_{k=1}^n \langle x, \gamma_{G_i}, \mu_{G_i} \rangle = 0_{\sim}.$$

Then $\bigcup_{k=1}^n \langle x, \mu_{G_i}, \gamma_{G_i} \rangle = 1_{\sim}$, so $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i = 1, 2, \dots, n\}$ is a finite subcover of $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$. Therefore X is fuzzy strongly precompact. \square

Remark 3.1. Since every IFOS is an IFSPOS, from the definition above we may conclude that every fuzzy strongly precompact compact IFTS is fuzzy compact.

Theorem 3.2. *Let $f : X \rightarrow Y$ be an intuitionistic fuzzy strongly irresolute precontinuous mapping from an IFTS X onto IFTS Y . If X is fuzzy strongly precompact, then Y is fuzzy strongly precompact, as well.*

Proof. Let $\{\langle y, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ be any fuzzy strongly preopen cover of Y . Then $\cup\{\langle y, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\} = 1_{\sim}$. From the relation $f^{-1}(\cup\{\langle y, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}) = 1_{\sim}$ follows that $\cup\{f^{-1}(\langle y, \mu_{G_i}, \gamma_{G_i} \rangle) \mid i \in I\} = 1_{\sim}$, so $\{f^{-1}(\langle y, \mu_{G_i}, \gamma_{G_i} \rangle) \mid i \in I\}$ is a fuzzy strongly preopen cover of X . Since X is fuzzy strongly precompact, there exists a finite subcover $\{f^{-1}(\langle y, \mu_{G_i}, \gamma_{G_i} \rangle) \mid i = 1, 2, \dots, n\}$. Therefore

$$\cup\{f^{-1}(\langle y, \mu_{G_i}, \gamma_{G_i} \rangle) \mid i = 1, 2, \dots, n\} = 1_{\sim}.$$

Hence

$$f(\cup\{f^{-1}(\langle y, \mu_{G_i}, \gamma_{G_i} \rangle) \mid i = 1, 2, \dots, n\}) = 1_{\sim},$$

so

$$\cup\{f(f^{-1}(\langle y, \mu_{G_i}, \gamma_{G_i} \rangle)) \mid i = 1, 2, \dots, n\} = 1_{\sim}.$$

From

$$\cup\{(\langle y, \mu_{G_i}, \gamma_{G_i} \rangle) \mid i = 1, 2, \dots, n\} = 1_{\sim}$$

follows that Y is fuzzy strongly precompact. \square

Theorem 3.3. *Let $f : X \rightarrow Y$ be an intuitionistic fuzzy strongly precontinuous mapping from an IFTS X onto IFTS Y . If X is fuzzy strongly precompact, then Y is fuzzy compact.*

Proof. It is similar to the proof of the Theorem 3.2. \square

Definition 3.4. Let A be an IFS in an IFTS X . A family $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ of IFSPOS's in X satisfies the condition $A \subseteq \cup\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ is called a fuzzy strongly preopen cover of A .

A finite subfamily of a fuzzy strongly preopen cover $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ of A which is also a fuzzy strongly preopen cover of A is called a finite subcover of $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$.

Definition 3.5. An IFS $A = \langle x, \mu_A, \gamma_A \rangle$ in an IFTS X is called fuzzy strongly precompact if and only if every fuzzy strongly preopen cover of A has a finite subcover.

Theorem 3.4. An IFS $A = \langle x, \mu_A, \gamma_A \rangle$ in an IFTS X is fuzzy strongly precompact if and only if for each family $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ of IFSPCS's with properties

$$\mu_A \leq \vee\{\mu_{G_i} \mid i \in I\} \quad \text{and} \quad 1 - \gamma_A \leq \vee\{1 - \gamma_{G_i} \mid i \in I\}$$

there exists a finite subfamily $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i = 1, 2, \dots, n\}$ such that

$$\mu_A \leq \vee\{\mu_{G_i} \mid i = 1, 2, \dots, n\} \quad \text{and} \quad 1 - \gamma_A \leq \vee\{1 - \gamma_{G_i} \mid i = 1, 2, \dots, n\}.$$

Proof. Suppose $A = \langle x, \mu_A, \gamma_A \rangle$ is a fuzzy strongly precompact set in IFTS X and $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ be any family of IFSPCS's in X satisfies the condition

$$\mu_A \leq \vee\{\mu_{G_i} \mid i \in I\} \quad \text{and} \quad 1 - \gamma_A \leq \vee\{1 - \gamma_{G_i} \mid i \in I\}.$$

Then $1 - \gamma_A \leq 1 - \wedge\{\gamma_{G_i} \mid i \in I\}$, so $\gamma_A \geq \wedge\{\gamma_{G_i} \mid i \in I\}$. Hence

$$A \subseteq \cup\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}.$$

According to the assumption there exists finite subfamily $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i = 1, 2, \dots, n\}$ such that

$$A \subseteq \cup\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i = 1, 2, \dots, n\}.$$

It follows that $\mu_A \leq \vee\{\mu_{G_i} \mid i = 1, 2, \dots, n\}$ and $\gamma_A \geq \wedge\{\mu_{G_i} \mid i = 1, 2, \dots, n\}$. Finally

$$\mu_A \leq \vee\{\mu_{G_i} \mid i = 1, 2, \dots, n\} \quad \text{and} \quad 1 - \gamma_A \leq \vee\{1 - \gamma_{G_i} \mid i = 1, 2, \dots, n\}.$$

Conversely, let $A = \langle x, \mu_A, \gamma_A \rangle$ be any IFS in IFTS X and let $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ be any family of IFSPCS's in X satisfies the condition

$$\mu_A \leq \vee\{\mu_{G_i} \mid i \in I\} \quad \text{and} \quad 1 - \gamma_A \leq \vee\{1 - \gamma_{G_i} \mid i \in I\}.$$

From $1 - \gamma_A \leq 1 - \wedge\{\gamma_{G_i} \mid i \in I\}$ follows that $\gamma_A \geq \wedge\{\gamma_{G_i} \mid i \in I\}$, so

$$A \subseteq \cup\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}.$$

Hence $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ is a fuzzy strongly preopen cover of IFS A . According to the assumption there exists finite subfamily $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i = 1, 2, \dots, n\}$ such that

$$\mu_A \leq \vee\{\mu_{G_i} \mid i = 1, 2, \dots, n\} \quad \text{and} \quad 1 - \gamma_A \leq \vee\{1 - \gamma_{G_i} \mid i = 1, 2, \dots, n\}.$$

From $\mu_A \leq \vee\{\mu_{G_i} \mid i = 1, 2, \dots, n\}$ and $\gamma_A \geq \wedge\{\mu_{G_i} \mid i = 1, 2, \dots, n\}$ we obtain that

$$A \subseteq \cup\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i = 1, 2, \dots, n\}.$$

Therefore A is fuzzy strongly precompact. \square

Remark 3.2. From the definition above it is not difficult to conclude that every fuzzy strongly precompact IFS in an IFTS is fuzzy compact.

Theorem 3.5. *Let $f : X \rightarrow Y$ be an intuitionistic fuzzy strongly irresolute precontinuous mapping from an IFTS X onto IFTS Y . If A is fuzzy strongly precompact, then $f(A)$ is fuzzy strongly precompact, as well.*

Proof. Let $\{\langle y, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ be any fuzzy strongly preopen cover of $f(A)$. Then $f(A) \subseteq \cup\{\langle y, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$. From the relation $A \subseteq f^{-1}(\cup\{\langle y, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\})$ follows that $A \subseteq \cup\{f^{-1}(\langle y, \mu_{G_i}, \gamma_{G_i} \rangle) \mid i \in I\}$, so $\{f^{-1}(\langle y, \mu_{G_i}, \gamma_{G_i} \rangle) \mid i \in I\}$ is a fuzzy strongly preopen cover of A . Since A is fuzzy strongly precompact, there exists a finite subcover $\{f^{-1}(\langle y, \mu_{G_i}, \gamma_{G_i} \rangle) \mid i = 1, 2, \dots, n\}$. Therefore $A \subseteq \cup\{f^{-1}(\langle y, \mu_{G_i}, \gamma_{G_i} \rangle) \mid i = 1, 2, \dots, n\}$. Hence

$$\begin{aligned} f(A) &\subseteq f(\cup\{f^{-1}(\langle y, \mu_{G_i}, \gamma_{G_i} \rangle) \mid i = 1, 2, \dots, n\}) = \\ &= \cup\{f(f^{-1}(\langle y, \mu_{G_i}, \gamma_{G_i} \rangle)) \mid i = 1, 2, \dots, n\} = \\ &= \cup\{(\langle y, \mu_{G_i}, \gamma_{G_i} \rangle) \mid i = 1, 2, \dots, n\}, \end{aligned}$$

so $f(A)$ is fuzzy strongly precompact. □

Theorem 3.6. *Let $f : X \rightarrow Y$ be an intuitionistic fuzzy strongly precontinuous mapping from an IFTS X onto IFTS Y . If A is fuzzy strongly precompact, then $f(A)$ is fuzzy compact.*

Definition 3.6. An IFTS X is called fuzzy strongly pre-Lindelof (fuzzy Lindelof) if and only if every fuzzy strongly preopen (fuzzy open) cover of X has a countable subcover.

Definition 3.7. An IFS $A = \langle x, \mu_A, \gamma_A \rangle$ in an IFTS X is called fuzzy strongly pre-Lindelof (fuzzy Lindelof) if and only if every fuzzy strongly preopen (fuzzy open) cover of A has a countable subcover.

Definition 3.8. An IFTS X is called countably fuzzy strongly precompact (countably fuzzy compact) if and only if every countable fuzzy strongly preopen (fuzzy open) cover of X has a finite subcover.

Definition 3.9. An IFS $A = \langle x, \mu_A, \gamma_A \rangle$ in an IFTS X is called countably fuzzy strongly precompact (countably fuzzy compact) if and only if every countable fuzzy strongly preopen (fuzzy open) cover of A has a finite subcover.

Remark 3.3. From the definitions above we may conclude that

- 1) every fuzzy strongly pre-Lindelof IFTS (IFS in IFTS) is fuzzy Lindelof;
- 2) every countably fuzzy strongly precompact IFTS (IFS in IFTS) is countably fuzzy compact;
- 3) every countably fuzzy strongly precompact IFTS (IFS in IFTS) is fuzzy strongly precompact.

It is not difficult to prove the following theorems.

Theorem 3.7. *If an IFTS X is both fuzzy strongly pre-Lindelof and countably fuzzy strongly precompact then it is fuzzy strongly precompact.*

Theorem 3.8. *If an IFS A in an IFTS X is both fuzzy strongly pre-Lindelof and countably fuzzy strongly precompact then A is fuzzy strongly precompact.*

Theorem 3.9. *Let X be a fuzzy strongly pre-Lindelof IFTS. Then X is countably fuzzy strongly precompact if and only if X is fuzzy strongly precompact.*

Proof. In the Remark 3.3 it is mentioned that if X is fuzzy strongly precompact then it is countably fuzzy strongly precompact.

Conversely, let $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$ be any fuzzy strongly preopen cover of X . Since X is fuzzy strongly pre-Lindelof, there exists countable subcover $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i = 1, 2, \dots\}$ of $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i \in I\}$. Therefore $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i = 1, 2, \dots\}$ is countably fuzzy strongly preopen cover of X , so there exists subcover $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i = 1, 2, \dots, n\}$ of $\{\langle x, \mu_{G_i}, \gamma_{G_i} \rangle \mid i = 1, 2, \dots\}$. Hence X is fuzzy strongly precompact. \square

Theorem 3.10. *Let an IFS A be fuzzy strongly pre-Lindelof in an IFTS. Then A is countably fuzzy strongly precompact if and only if A is fuzzy strongly precompact.*

Proof. The proof is similar to the proof of the previous theorem. \square

Theorem 3.11. *Let $f : X \rightarrow Y$ be an intuitionistic fuzzy strongly irresolute precontinuous mapping from an IFTS X onto IFTS Y . If X is fuzzy strongly pre-Lindelof (countably fuzzy strongly precompact), then Y is fuzzy strongly pre-Lindelof (countably fuzzy strongly precompact), as well.*

Proof. It is similar to the proof of the Theorem 3.2. \square

Theorem 3.12. *Let $f : X \rightarrow Y$ be an intuitionistic fuzzy strongly precontinuous mapping from an IFTS X onto IFTS Y . If X is fuzzy strongly pre-Lindelof (countably fuzzy strongly precompact), then Y is fuzzy Lindelof (countably fuzzy compact).*

Proof. It is similar to the proof of the Theorem 3.3. \square

Theorem 3.13. *Let $f : X \rightarrow Y$ be an intuitionistic fuzzy strongly irresolute precontinuous mapping from an IFTS X onto IFTS Y . If A is fuzzy strongly pre-Lindelof (countably fuzzy strongly precompact), then $f(A)$ is fuzzy strongly pre-Lindelof (countably fuzzy strongly precompact), as well.*

Proof. It is similar to the proof of the Theorem 3.5. \square

Theorem 3.14. *Let $f : X \rightarrow Y$ be an intuitionistic fuzzy strongly precontinuous mapping from an IFTS X onto IFTS Y . If A is fuzzy strongly pre-Lindelof (countably fuzzy strongly precompact), then $f(A)$ is fuzzy Lindelof (countably fuzzy compact).*

Proof. It is similar to the proof of the Theorem 3.6. \square

REFERENCES

- [1] K.T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems, **20** (1986), 87–96.
- [2] C.L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl., **24** (1986), 182–190.
- [3] D. Coker, *An introduction to intuitionistic fuzzy topological spaces*, Fuzzy Sets and Systems, **88** (1997), 81–89.
- [4] H. Gurcay and D. Coker, *On fuzzy continuity in intuitionistic fuzzy topological spaces*, J. Fuzzy Math., **5** (1997), 365–378.
- [5] J.K. Joen, Y.B. Jun and J.H. Park, *Intuitionistic fuzzy alpha-continuity and intuitionistic fuzzy precontinuity*, IJMMS, **19** (2005), 3091–3101.
- [6] B. Krsteska and S. Abbas, *Intuitionistic fuzzy strongly preopen sets and fuzzy strong precontinuity*, submitted in proceeding of ICTA 2006, Greece.
- [7] B. Krsteska and S. Abbas, *Intuitionistic fuzzy strongly irresolute precontinuous mappings in Coker's spaces*, to appear in Kragujevac Journal of Mathematics.
- [8] L.A. Zadeh, *Fuzzy sets*, Information and control, **8** (1965), 338–353.

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