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On Separation Axioms and Sequences

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Abstract. In 2003, Noiri has introduced the notion of β-θ-open sets. By using these sets, the aim of this paper is to investigate the relationships between separation axioms and sequences.

1. Introduction

Noiri [10] has introduced the notion of β-θ-open sets in topological spaces in 2003. Noiri has shown that β-θ-open sets are weaker form of β-regular sets and stronger form of β -open sets. Separation axioms and sequences are two main topics of general topology. In literature, many papers have been studied on these subjects [4, 5, 6, 9, 11, 12, etc.]. In this paper, we investigate the relationships between separation axioms and sequences by using β-θ-open sets.

2. Preliminaries

Throughout the present paper, spaces X and Y mean topological spaces. Let X be a topological space and A a subset of X. The closure of A and the interior of A are denoted by $cl(A)$ and $int(A)$, respectively.

A subset A is said to be β-open [1] or semi-preopen [3] (resp. α-open [8]) if $A \subset cl(int(cl(A)))$ (resp. $A \subset int(cl(int(A)))$). The complement of a β-open set is said to be β-closed [2] or semi-preclosed [3]. The intersection of all β-closed sets of X containing A is called the semi-preclosure [3] or β-closure [2] of A and is denoted by β-$cl(A)$. The union of all β-open sets of X contained in A is called the semi-preinterior or β-interior of A and is denoted by β-$int(A)$. A subset A is said to be β-regular if it is β-open and β-closed.

The family of all β-open (resp. β-regular) sets of X containing a point $x \in X$ is denoted by $\beta O(X, x)$ (resp. $\beta R(X, x)$). The family of all β-open (resp. β-closed, β-regular) sets in X is denoted by $\beta O(X)$ (resp. $\beta C(X)$, $\beta R(X)$).

A topological space X is called β-T_0 [7] if for any distinct pair of points in X, there exists a β-open set containing one of the points but not the other. A space X is said to be β-T_1 [7] if for each pair of distinct points x and y of X, there exist β-open sets U and V such that $x \in U$, $y \notin U$ and $x \notin V$, $y \in V$. A space X

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is said to be β-T_2 [7] if for each pair of distinct points x and y of X, there exist β-open sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.

Theorem 2.1. *([10]) Let A be a subset of a topological space X. Then*
(1) $A \in \beta O(X)$ *if and only if* $\beta\text{-}cl(A) \in \beta R(X)$,
(2) $A \in \beta C(X)$ *if and only if* $\beta\text{-}int(A) \in \beta R(X)$.

Definition 2.1. *([10]) Let X be a topological space. A point x of X is called a β-θ-cluster point of S if $\beta\text{-}cl(U) \cap S \neq \varnothing$ for every $U \in \beta O(X, x)$. The set of all β-θ-cluster points of S is called the β-θ-closure of S and is denoted by β-θ-$cl(S)$. A subset S is said to be β-θ-closed if $S = \beta\text{-}\theta\text{-}cl(S)$. The complement of a β-θ-closed set is said to be β-θ-open.*

Theorem 2.2. *([10]) For any subset A of a topological space X , the following hold:*

$$\begin{aligned}\beta\text{-}\theta\text{-}cl(A) &= \cap\{V : A \subset V \text{ and } V \text{is } \beta\text{-}\theta\text{-closed}\} \\ &= \cap\{V : A \subset V \text{ and } V \in \beta R(X)\}.\end{aligned}$$

Theorem 2.3. *([10]) Let A and B be any subsets of a topological space X. Then the following properties hold:*
(1) $x \in \beta\text{-}\theta\text{-}cl(A)$ *if and only if* $V \cap A \neq \varnothing$ *for each* $V \in \beta R(X, x)$,
(2) *if* $A \subset B$, *then* $\beta\text{-}\theta\text{-}cl(A) \subset \beta\text{-}\theta\text{-}cl(B)$,
(3) $\beta\text{-}\theta\text{-}cl(\beta\text{-}\theta\text{-}cl(A)) = \beta\text{-}\theta\text{-}cl(A)$,
(4) *if* A_α *is β-θ-closed in X for each* $\alpha \in \Delta$, *then* $\bigcap_{\alpha \in \Delta} A_\alpha$ *is β-θ-closed in X.*

The family of all β-θ-open (resp. β-θ-closed) sets of X containing a point $x \in X$ is denoted by $\beta\theta O(X, x)$ (resp. $\beta\theta C(X, x)$). The family of all β-θ-open (resp. β-θ-closed) sets in X is denoted by $\beta\theta O(X)$ (resp. $\beta\theta C(X)$).

3. SEPARATION AXIOMS AND SEQUENCES

In this section, we introduce and study β-θ-separation axioms, β-θ-convergences and some functions. Also, we investegate the relationships among β-θ-separation axioms, β-θ-convergences and some functions.

Definition 3.1. *A topological space X is called β-θ-T_0 if for any distinct pair of points in X, there exists a β-θ-open set containing one of the points but not the other.*

Definition 3.2. *A space X is said to be β-θ-T_1 if for each pair of distinct points x and y of X, there exist β-θ-open sets U and V such that $x \in U$, $y \notin U$ and $x \notin V$, $y \in V$.*

Definition 3.3. *A space X is said to be β-θ-T_2 if for each pair of distinct points x and y of X , there exist β-θ-open sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.*

Theorem 3.1. *For a topological space (X,τ), the following properties are equivalent:*

1. *X is β-θ-T_2,*
2. *for each pair of distinct points x and y of X , there exist β-regular sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$,*
3. *X is β-T_2,*
4. *for each pair of distinct points x and y of X , there exist β-θ-open and β-θ-closed sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.*

Proof. (1)⇔(2) It is obvious since A is β-θ-open in X if and only if for each $x \in A$ there exists $V \in \beta R(X,x)$ such that $x \in V \subset A$.

(2)⇔(3) It is obvious that $A \in \beta O(X)$ if and only if β-$cl(A) \in \beta R(X)$ and $A \in \beta C(X)$ if and only if β-$int(A) \in \beta R(X)$.

(2)⇔(4) It is obvious that $A \in \beta R(X)$ if and only if A is β-θ-open and β-θ-closed. □

Remark 3.1. *Every β-θ-T_0 space is β-T_0 and every β-θ-T_1 space is β-T_1.*

Example 3.1. *Let $X = \{a,b,c\}$ and $\tau = \{\varnothing, X, \{a\}\}$. Then (X,τ) is β-T_0 but not β-θ-T_0.*

Question *Does there exist a space which is β-T_1 and it is not β-θ-T_1?*

Definition 3.4. *A topological space X is said to be β-θ-R_1 if for $x, y \in X$ with β-θ-$cl(\{x\}) \neq \beta$-θ-$cl(\{y\})$, there exist disjoint β-θ-open sets U and V such that β-θ-$cl(\{x\}) \subset U$ and β-θ-$cl(\{y\}) \subset V$.*

Theorem 3.2. *A topological space X is β-θ-T_1 if and only if the singletons are β-θ-closed sets.*

Definition 3.5. *Let X be a topological space and $S \subset X$. The β-θ-kernel of S, denoted by β-θ-$ker(S)$, is defined to be the set β-θ-$ker(S) = \cap\{U \in \beta\theta O(X) : S \subset U\}$.*

Theorem 3.3. *A topological space X is β-θ-R_1 if and only if there exist disjoint β-θ-open sets U and V such that β-θ-$cl(\{x\}) \subset U$ and β-θ-$cl(\{y\}) \subset V$ whenever β-θ-$ker(\{x\}) \neq \beta$-θ-$ker(\{y\})$ for x, $y \in X$.*

Theorem 3.4. *A topological space X is β-θ-T_2 if and only if it is β-θ-R_1 and β-θ-T_0.*

Proof. (⇒): Let X be a β-θ-T_2 space. Then X is β-θ-T_1 and then β-θ-T_0. Since X is β-θ-T_2, by the Theorem 3.2, $\{x\} = \beta$-θ-$cl(\{x\}) \neq \beta$-θ-$cl(\{y\}) = \{y\}$ for x, $y \in X$, there exist disjoint β-θ-open sets U and V such that β-θ-$cl(\{x\}) \subset U$ and β-θ-$cl(\{y\}) \subset V$. Thus, X is a β-θ-R_1 space.

(⇐): Let X be β-θ-R_1 and β-θ-T_0. Let x, y be any two distinct points of X. Since X is β-θ-T_0, then there exists a β-θ-open set U such that $x \in U$ and $y \notin U$ or there exists a β-θ-open set V such that $y \in V$ and $x \notin V$. Let $x \in U$ and $y \notin U$. Then $y \notin \beta$-θ-$ker(\{x\})$ and then β-θ-$ker(\{x\}) \neq \beta$-θ-$ker(\{y\})$. Since X

is β-θ-R_1, by Theorem 3.3 there exist disjoint β-θ-open sets U and V such that $x \in \beta$-θ-$cl(\{x\}) \subset U$ and $y \in \beta$-θ-$cl(\{y\}) \subset V$. Thus, X is β-θ-T_2. □

Lemma 3.1. *([1]) Let A and Y be subsets of a space X. If $A \in \beta O(X)$ and Y is α-open in X, then $A \cap Y \in \beta O(Y)$.*

Theorem 3.5. *If X_0 be an α-open set and A be a β-θ-open set in X, then $X_0 \cap A \in \beta\theta O(X_0)$.*

Definition 3.6. *A sequence (x_n) is said to be β-θ-convergent to a point x of X, denoted by $(x_n) \overset{\beta\theta}{\to} x$, if (x_n) is eventually in every β-θ-open set containing x.*

Definition 3.7. *A space X is said to be β-θ-US if every β-θ-convergent sequence (x_n) in X β-θ-converges to a unique point.*

Definition 3.8. *A set F of a space X is said to be sequentially β-θ-closed if every sequence in F β-θ-converging in X β-θ-converges to a point in F.*

Definition 3.9. *A subset G of a space X is said to be sequentially β-θ-compact if every sequence in G has a subsequence which β-θ-converges to a point in G.*

Theorem 3.6. *Every β-θ-T_2 space is β-θ-US.*

Proof. Let X be a β-θ-T_2 space and (x_n) be a sequence in X. Suppose that (x_n) β-θ-converges to two distinct points x and y. That is, (x_n) is eventually in every β-θ-open set containing x and also in every β-θ-open set containing y. This is contradiction since X is a β-θ-T_2 space. Hence, the space X is β-θ-US. □

Theorem 3.7. *Every β-θ-US space is β-θ-T_1.*

Proof. Let X be a β-θ-US space. Let x and y be two distinct points of X. Consider the sequence (x_n) where $x_n = x$ for every n. Clearly, (x_n) β-θ-converges to x. Also, since $x \neq y$ and X is β-θ-US, (x_n) cannot β-θ-converge to y, i.e, there exists a β-θ-open set V containing y but not x. Similarly, if we consider the sequence (y_n) where $y_n = y$ for all n, and proceeding as above we get a β-θ-open set U containing x but not y. Thus, the space X is β-θ-T_1. □

Theorem 3.8. *A space X is β-θ-US if and only if the set $A = \{(x, x) : x \in X\}$ is a sequentially β-θ-closed subset of $X \times X$.*

Proof. Let X be β-θ-US. Let (x_n, x_n) be a sequence in A. Then (x_n) is a sequence in X. As X is β-θ-US, $(x_n) \overset{\beta\theta}{\to} x$ for a unique $x \in X$. i.e., (x_n) β-θ-converge to x and y. Thus, $x = y$. Hence, A is a sequentially β-θ-closed set.

Conversely, let A be sequentially β-θ-closed. Let a sequence (x_n) β-θ-converge to x and y. Hence, sequence (x_n, x_n) β-θ-converges to (x, y). Since A is sequentially β-θ-closed, $(x, y) \in A$ which means that $x = y$ implies space X is β-θ-US. □

Theorem 3.9. *A space X is β-θ-US if and only if every sequentially β-θ-compact set of X is sequentially β-θ-closed.*

Proof. $(\Rightarrow)$: Let X be a β-θ-US space. Let Y be a sequentially β-θ-compact subset of X. Let (x_n) be a sequence in Y. Suppose that (x_n) β-θ-converges to a point x in $X \backslash Y$. Let (x_{n_k}) be a subsequence of (x_n) which β-θ-converges to a point $y \in Y$ since Y is sequentially β-θ-compact. Also, a subsequence (x_{n_k}) of (x_n) β-θ-converges to $x \in X \backslash Y$. Since (x_{n_k}) is a sequence in the β-θ-US space X, $x = y$. Thus, Y is sequentially β-θ-closed set.

$(\Leftarrow)$: Suppose that (x_n) is a sequence in X β-θ-converging to distinct points x and y. Let $K_x = \{x_n : n \in N\} \cup \{x\}$. Then K_x is a sequentially β-θ-compact set and it is sequentially β-θ-closed. Since $y \notin K_x$, there exists $U \in \beta R(X, x)$ such that $U \cap K_x = \varnothing$. This contradicts that (x_n) is eventually in U. Therefore, $x = y$ and X is β-θ-US. □

Theorem 3.10. *Every α-open set of a β-θ-US space is β-θ-US.*

Proof. Let X be a β-θ-US space and $Y \subset X$ be an α-open set. Let (x_n) be a sequence in Y. Suppose that (x_n) β-θ-converges to x and y in Y. We shall prove that (x_n) β-θ-converges to x and y in X. Let U be any β-θ-open subset of X containing x and V be any β-θ-open set of X containing y. Then, $U \cap Y$ and $V \cap Y$ are β-θ-open sets in Y. Therefore, (x_n) is eventually in $U \cap Y$ and $V \cap Y$ and so in U and V. Since X is β-θ-US, this implies that $x = y$. Hence the subspace Y is β-θ-US. □

Theorem 3.11. *A space X is β-θ-T_2 if and only if it is both β-θ-R_1 and β-θ-US.*

Proof. Let X be a β-θ-T_2 space. Then X is β-θ-R_1 by Theorem 3.4 and β-θ-US by Theorem 3.6.

Conversely, let X be both β-θ-R_1 and β-θ-US space. By Theorem 3.7 we obtain that every β-θ-US space is β-θ-T_1 and X is both β-θ-T_1 and β-θ-R_1 and, it follows from Theorem 3.4 that the space X is β-θ-T_2. □

Next, we prove the product theorem for β-θ-US spaces.

Theorem 3.12. *If X_1 and X_2 are β-θ-US spaces, then $X_1 \times X_2$ is β-θ-US.*

Proof. Let $X = X_1 \times X_2$ where X_i is β-θ-US and $I = \{1, 2\}$. Let a sequence (x_n) in X β-θ-converge to $x = (x_i)_{i \in I}$ and $y = (y_i)_{i \in I}$. Then the sequence (x_{n_i}) β-θ-converges to x_i and y_i for all $i \in I$. Suppose that for $2 \in I$, (x_{n_2}) does not β-θ-converges to x_2. Then there exists β-θ-open set U_2 containing x_2 such that (x_{n_2}) is not eventually in U_2. Consider the set, $U = X_1 \times U_2$. Then U is a β-θ-open subset of X and $x \in U$. Also, (x_n) is not eventually in U which contradicts the fact that (x_n) β-θ-converges to x. Thus we get (x_{n_i}) β-θ-converges to x_i and y_i for all $i \in I$. Since X_i is β-θ-US for each $i \in I$, we obtain $x = y$. Hence, X is β-θ-US. □

Now we define the notion of sequentially β-θ-continuous functions in the following:

Definition 3.10. *A function $f : X \to Y$ is said to be*

(1) *sequentially β-θ-continuous at $x \in X$ if $f(x_n)$ β-θ-converges to $f(x)$ whenever (x_n) is a sequence β-θ-converging to x,*
(2) *sequentially β-θ-continuous if f is sequentially β-θ-continuous at all $x \in X$.*

Definition 3.11. *A function $f : X \to Y$ is said to be sequentially nearly β-θ-continuous if for each point $x \in X$ and each sequence (x_n) in X β-θ-converging to x, there exists a subsequence (x_{n_k}) of (x_n) such that $f(x_{n_k}) \overset{\beta\theta}{\to} f(x)$.*

Theorem 3.13. *Let $f : X \to Y$ and $g : X \to Y$ be two sequentially β-θ-continuous functions. If Y is β-θ-US, then the set $A = \{x : f(x) = g(x)\}$ is sequentially β-θ-closed.*

Proof. Let Y be β-θ-US and suppose that there exists a sequence (x_n) in A β-θ-converging to $x \in X$. Since f and g are sequentially β-θ-continuous functions, $f(x_n) \overset{\beta\theta}{\to} f(x)$ and $g(x_n) \overset{\beta\theta}{\to} g(x)$. Hence $f(x) = g(x)$ and $x \in A$. Therefore, A is sequentially β-θ-closed. □

Theorem 3.14. *Let $f : X \to Y$ be a sequentially β-θ-continuous function. If Y is β-θ-US, then the set $E = \{(x, y) \in X \times X : f(x) = f(y)\}$ is sequentially β-θ-closed in $X \times X$.*

Proof. Suppose that there exists a sequence (x_n, y_n) in E β-θ-converging to $(x, y) \in X \times X$. Since f is sequentially β-θ-continuous functions, $f(x_n) \overset{\beta\theta}{\to} f(x)$ and $f(y_n) \overset{\beta\theta}{\to} f(y)$. Hence $f(x) = f(y)$ and $(x, y) \in E$. Hence, E is sequentially β-θ-closed. □

Definition 3.12. *A function $f : X \to Y$ is said to be sequentially sub-β-θ-continuous if for each point $x \in X$ and each sequence (x_n) in X β-θ-converging to x, there exists a subsequence (x_{n_k}) of (x_n) and a point $y \in Y$ such that $f(x_{n_k}) \overset{\beta\theta}{\to} y$.*

Definition 3.13. *A function $f : X \to Y$ is said to be sequentially β-θ-compact preserving if the image $f(K)$ of every sequentially β-θ-compact set K of X is sequentially β-θ-compact in Y.*

Theorem 3.15. *Every function $f : X \to Y$ is sequentially sub-β-θ-continuous if Y is a sequentially β-θ-compact.*

Proof. Let (x_n) be a sequence in X β-θ-converging to a point x of X. Then $(f(x_n))$ is a sequence in Y and as Y is sequentially β-θ-compact, there exists a subsequence $(f(x_{n_k}))$ of $(f(x_n))$ β-θ-converging to a point $y \in Y$. Hence, $f : X \to Y$ is sequentially sub-β-θ-continuous. □

Theorem 3.16. *A function $f : X \to Y$ is sequentially β-θ-compact preserving if and only if $f \mid M : M \to f(M)$ is sequentially sub-β-θ-continuous for each sequentially β-θ-compact subset M of X.*

Proof. Suppose that $f : X \to Y$ is a sequentially β-θ-compact preserving function. Then $f(M)$ is a sequentially β-θ-compact set in Y for each sequentially β-θ-compact set M of X. Therefore, by Theorem 3.15, $f \mid M : M \to f(M)$ is a sequentially sub-β-θ-continuous function.

Conversely, let M be any sequentially β-θ-compact set of X. We shall show that $f(M)$ is a sequentially β-θ-compact set in Y. Let (y_n) be any sequence in $f(M)$. Then for each positive integer n, there exists a point $x_n \in M$ such that $f(x_n) = y_n$. Since (x_n) is a sequence in the sequentially β-θ-compact set M, there exists a subsequence (x_{n_k}) of (x_n) β-θ-converging to a point $x \in M$. Since $f \mid M : M \to f(M)$ is sequentially sub-β-θ-continuous, then there exists a subsequence (x_{n_k}) of (x_n) such that $f\left(x_{n_k}\right) \overset{\beta\theta}{\to} y$ and $y \in f(M)$. This implies that $f(M)$ is a sequentially β-θ-compact set in Y. Thus, $f : X \to Y$ is a sequentially β-θ-compact preserving function. □

The following theorem gives a sufficient condition for a sequentially sub-β-θ-continuous function to be sequentially β-θ-compact preserving.

Theorem 3.17. *If a function $f : X \to Y$ is sequentially sub-β-θ-continuous and $f(M)$ is a sequentially β-θ-closed set in Y for each sequentially β-θ-compact set M of X, then f is a sequentially β-θ-compact preserving function.*

Proof. We use the previous theorem. It suffices to prove that $f \mid M : M \to f(M)$ is sequentially sub-β-θ-continuous for each sequentially β-θ-compact subset M of X. Let (x_n) be any sequence in M β-θ-converging to a point $x \in M$. Then since f is sequentially sub-β-θ-continuous, there exist a subsequence (x_{n_k}) of (x_n) and a point $y \in Y$ such that $f(x_{n_k})$ β-θ-converges to y. Since $f(x_{n_k})$ is a sequence in the sequentially β-θ-closed set $f(M)$ of Y, we obtain $y \in f(M)$. This implies that $f \mid M : M \to f(M)$ is sequentially sub-β-θ-continuous. □

Theorem 3.18. *Every sequentially nearly β-θ-continuous function is sequentially β-θ-compact preserving.*

Proof. Suppose that $f : X \to Y$ is a sequentially nearly β-θ-continuous function and let M be any sequentially β-θ-compact subset of X. We shall show that $f(M)$ is a sequentially β-θ-compact set of Y. Let (y_n) be any sequence in $f(M)$. Then for each positive integer n, there exists a point $x_n \in M$ such that $f(x_n) = y_n$. Since (x_n) is a sequence in the sequentially β-θ-compact set M, there exists a subsequence (x_{n_k}) of (x_n) β-θ-converging to a point $x \in M$. Since f is sequentially nearly β-θ-continuous, then there exists a subsequence (x_j) of (x_{n_k}) such that $f\left(x_j\right) \overset{\beta\theta}{\to} f\left(x\right)$. Thus, there exists a subsequence (y_j) of (y_n) β-θ-converging to $f(x) \in f(M)$. This shows that $f(M)$ is a sequentially β-θ-compact set in Y. □

Theorem 3.19. *Every sequentially β-θ-compact preserving function is sequentially sub-β-θ-continuous.*

Proof. Suppose $f : X \to Y$ is a sequentially β-θ-compact preserving function. Let x be any point of X and (x_n) any sequence in X β-θ-coverging to x. We shall denote the set $\{x_n : n = 1, 2, 3, \ldots\}$ by N and $M = N \cup \{x\}$. Then M is sequentially β-θ-compact since $x_n \overset{\beta\theta}{\to} x$. Since f is sequentially β-θ-compact preserving, it follows that $f(M)$ is a sequentially β-θ-compact set of Y. Since $(f(x_n))$ is a sequence in $f(M)$, there exists a subsequence $(f(x_{n_k}))$ of $(f(x_n))$ β-θ-converging to a point $y \in f(M)$. This implies that f is sequentially sub-β-θ-continuous. □

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