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ON NEUTRAL OPERATIONS OF $(n,m)-$GROUPS

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Abstract. In this paper proposition on $\{1, n-m+1\}-$neutral operations of $(n,m)-$groups is proved.

1. Preliminaries

Definition 1.1 ([1])**.** Let $(Q;A)$ be an $(n,m)-$groupoid $[A : Q^n \to Q^m]$ and let $n \geq m+1$ $[n,m \in N]$. Then:

(*a*) we say that $(Q;A)$ is an $(n,m)-$ **semigroup** iff for every $i,j \in \{1,\dots,n-m+1\}$, $i<j$, the following law holds
$$A(x_1^{i-1}, A(x_i^{i+n-1}), x_{i+n}^{2n-m}) = A(x_1^{j-1}, A(x_j^{j+n-1}), x_{j+n}^{2n-m})$$
$[:< i,j > -$associative law$]$; and

(*b*) we say that $(Q;A)$ is an $(n,m)-$**group** iff $(Q;A)$ is an $(n,m)-$semigroup and for every $a_1^n \in Q$ there is exactly one sequence x_1^m over Q and exactly one sequence y_1^m over Q such that the following equalities hold
$$A(a_1^{n-m}, x_1^m) = a_{n-m+1}^n,$$
$$A(y_1^m, a_1^{n-m}) = a_{n-m+1}^n.$$

Remark 1.1. A notion of an $(n,m)-$group was introduced by Ǵ. Čupona in [1] as a generalization of a group ($n-$group, cf. [5]). The paper [2] is mainly a survey on the known results for vector valued groupoids, semigroups and groups (up to 1988).

Definition 1.2 ([3])**.** Let $(Q;A)$ be an $(n,m)-$groupoid and $n \geq 2m$. Let also $\mathbf{e}$ be a mapping of the set Q^{n-2m} into the set Q^m. Then, we say that $\mathbf{e}$ is a $\{1, n-m+1\}-$**neutral operation** of the $(n,m)-$groupoid $(Q;A)$ iff for all $x_1^m \in Q^m$ and for every sequence a_1^{n-2m} over Q the following equalities hold
$$A(x_1^m, a_1^{n-2m}, \mathbf{e}(a_1^{n-2m})) = x_1^m,$$
and
$$A(\mathbf{e}(a_1^{n-2m}), a_1^{n-2m}, x_1^m) = x_1^m.$$

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For $m = 1$, $\mathbf{e}$ is an $\{1, n\}-$neutral operation of the $n-$groupoid $(Q; A)$. Cf. [5]. See, also [4].

Proposition 1.1 ([3]). *Every $(n, m)-$groupoid $(n \geq 2m)$ has at most one $\{1, n-m+1\}-$neutral operation.*

See, also [4].

Proposition 1.2 ([3]). *Every $(n, m)-$group $(n \geq 2m)$ has an $\{1, n-m+1\}-$neutral operation.*

See, also [4].

2. Results

Theorem 2.1. *Let $(Q; A)$ be an $(n, m)-$group, $\mathbf{e}$ its $\{1, n-m+1\}-$neutral operation (cf. 1.5) and $n > 2m$. Then, for every a_1^{n-2m}, $x_1^m \in Q$ and for all $i \in \{1, \ldots, n-2m+1\}$, the following equalities hold*

$$A(x_1^m, a_i^{n-2m}, \mathbf{e}(a_1^{n-2m}), a_1^{i-1}) = x_1^m \tag{1}$$

and

$$A(a_i^{n-2m}, \mathbf{e}(a_1^{n-2m}), a_1^{i-1}, x_1^m) = x_1^m. \tag{2}$$

Proof. Let

$$F(x_1^m, b_1^{n-2m}) \overset{def}{=} A(x_1^m, b_i^{n-2m}, \mathbf{e}(b_1^{n-2m}), b_1^{i-1}) \tag{0}$$

for all $x_1^m, b_1^{n-2m} \in Q$.

Whence, we obtain

$$A(F(x_1^m, b_1^{n-2m}), b_i^{n-2m}, \mathbf{e}(b_1^{n-2m}), b_1^{i-1}) =$$
$$= A(A(x_1^m, b_i^{n-2m}, \mathbf{e}(b_1^{n-2m}), b_1^{i-1}), b_i^{n-2m}, \mathbf{e}(b_1^{n-2m}), b_1^{i-1})$$

for all $x_1^m, b_1^{n-2m} \in Q$. Hence, by definition 1.1 and by definition 1.3, we have

$$A(F(x_1^m, b_1^{n-2m}), b_i^{n-2m}, \mathbf{e}(b_1^{n-2m}), b_1^{i-1}) =$$
$$= A(x_1^m, b_i^{n-2m}, A(\mathbf{e}(b_1^{n-2m}), b_1^{i-1}, b_i^{n-2m}, \mathbf{e}(b_1^{n-2m})), b_1^{i-1}),$$

i.e.

$$A(F(x_1^m, b_1^{n-2m}), b_i^{n-2m}, \mathbf{e}(b_1^{n-2m}), b_1^{i-1}) =$$
$$= A(x_1^m, b_i^{n-2m}, \mathbf{e}(b_1^{n-2m}), b_1^{i-1})$$

for every $x_1^m, b_1^{n-2m} \in Q$.

In addition, hence, by definition 1.1 (cancelation), we obtain

$$F(x_1^m, b_1^{n-2m}) = x_1^m$$

for all $x_1^m, b_1^{n-2m} \in Q$, whence we have (1).

Similarly, we obtain, also, (2). □

Remark 2.1. For $m = 1$ $(Q; A)$ is an $n-$group. See, also proposition 1.1–IV in [5].

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