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VERTICAL TIME: SOUND AND VISION

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ABSTRACT. The concept of poetic time, experienced through poetry, music and plastic arts, has firstly been individuated by Gaston Bachelard in the 1930s. The spectator's sensation of time rupture and ecstasy has been defined as time rearrangement into a vertical structure, linking to eternity. Although this particular perception of time has been described as triggered by spectator's experience of sublime art forms, this paper finds earlier evidence of the phenomenon in Giacomo Leopardi's poem *L'Infinito* (1819), as result of nature contemplation. Analysis of the sound that induces the vision of vertical time in Leopardi's poetry is performed by signal processing techniques. Relations between time, frequency and lag representations of the signal are established, finding correspondence of the signal power spectral density and time verticality in the lag domain.

1. INTRODUCTION

The concept of vertical time has been introduced by the French philosopher of science Gaston Bachelard (1884–1962). Bachelard's work, pioneering historical epistemology, could not but be permeated by reflection on time [15]. Impressing time thought with a primordial intuition, Bachelard individuates different temporalities that structure life.

Whether Bachelard identifies horizontal time, cadenzed by clocks, social, phenomenal and vital frameworks of duration, as prosodic time, the time of poetry is vertical [2]. Vertical time, accessed through poetry, is the surging up of the instant, possessing metaphysical scope. In [2], he writes, "*Every true poem can reveal the elements of* suspended time, meterless time - a time we shall call vertical in order to distinguish

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it from everyday time, which sweeps along horizontally with the streaming waters and the blowing winds".

This rupture of the horizontal axis that spreads into the dormant depth of temporality is described by Bachelard as an ecstatic moment, into which past and future converge. In [1], Bachelard refers the capability of vertical time to possesses eternity when he writes "This line running perpendicular to the temporal axis of life alone in fact gives consciousness of the present the means to flee and escape, to expand and deepen which have very often led to the present instant being linked to an eternity".

Although the concept of vertical time has been identified by Bachelard, the experience of time reverberation from the horizontal axis to a vertical one linking to eternity has been vividly described by the Italian poet Giacomo Leopardi (1798–1837) in his poem *L'Infinito* [8]. Whether Bachelard, and later other authors [11, 19], identify in poetry and art in general the trigger of vertical time experience, Leopardi finds it in the sound of nature.

In his work Zibaldone [9], Leopardi states "Pure, simple reason and mathematics have never been able, and will never be able, to discover anything poetic. Because everything that is poetic is felt rather than being known or understood, or perhaps we should say, is known or understood in being felt; nor indeed may it be known, discovered, or understood save by being felt. But pure reason and mathematics have no sensorium whatsoever". Therefore, Leopardi claims that poetry cannot be inspired by pure reason and mathematics, but does not rule out the contrary.

This paper explores the relation between sound and vertical time by an analysis based on signal processing.

2. The Infinite: Form and Content

L'Infinito was written in Recanati in 1819, and published in the volume Versi in 1826 [7]. The poem is a canzone libera, an Italian metric form with uneven stanzas and free rhyme. Subtitled by the author as Idyll I, it recalls the Hellenistic tradition of short pastoral poems [5]. The term idyll, deriving from the diminutive of the ancient Greek "seeing, image", stands for small image or scene. The author himself, by labeling the poem, deceives the reader, by imposing the expectation of a pastoral's search for the sensory world. Despite, the fifteen verses are rather a description of a sublime sensory void, an idyll without figures [3, 10].

It is here reported with literal translation alongside.

L'Infinito

The Infinite

1 Sempre caro mi fu quest'ermo colle,	Always dear was to me this solitary hill,
2 E questa siepe, che da tanta parte	And this hedge, which from so much part
3 Dell'ultimo orizzonte il guardo esclude.	Of the last horizon the look excludes.
4 Ma sedendo e mirando, interminati	But sitting and staring, endless
5 Spazi di là da quella, e sovrumani	Spaces beyond that, and superhuman
6 Silenzi, e profondissima quiete	Silences, and deepest quietness
7 Io nel pensier mi fingo; ove per poco	I pretend in thinking; where almost
8 Il cor non si spaura. E come il vento	My heart scares. And like the wind
9 Odo stormir tra queste piante, io quello	I hear rustling through these trees, that
10 Infinito silenzio a questa voce	Infinite silence to this voice
11 Vo comparando: e mi sovvien l'eterno,	I go comparing: and I recall eternity,
12 E le morte stagioni, e la presente	And the dead seasons, and the present
13 E viva, e il suon di lei. Così tra questa	And alive, and its sound. So between this
14 Immensità s'annega il pensier mio:	Immensity drowns my thought:
15 E il naufragar m'è dolce in questo mare.	And shipwreck is sweet to me in this sea.

In the three opening verses, the author carries thrift to extreme in the material scenery description; an anonymous hill and a hedge that hinders the view of the horizon are the representation of the finite, recalling the need for an interior escape from the limits of the matter. By abandoning the sensory world of the first three verses, the syntactic structure frees from the meter, producing ten verses of incomparable discursive and imagery continuity. While the introductory verses were characterized by a uniform rhythm, suggesting a regular meter, the following ten verses (4–13) are lacking of pauses, forming a single movement with irregular dilatations and accelerations.

The obstacle (hedge) imposed to the horizon is rejected in a subjective monologue, generating a series of images unbounded by the limit of human perception (endless/Spaces, superhuman/Silences, and deepest quiet), converging into an overwhelming sensory oblivion. The loss of physical references flashes on a perceptive limb of missing content (where almost/My heart scares). It is worth noticing that in Buzzati's *Il deserto dei Tartari* [4] the horizon is the most insidious limit for the spectator's eye, a whirl absorbing all the observer references into a voluntary self-annihilation.

The tension by the vicinity of a void abyss releases by an abrupt incursion of the tangible. The sound of wind rustling in the trees, transiently rending the bare canvas of perception, evokes remembrance of the eternity (And like the wind/ I hear rustling among these plants, that/Infinite silence to this voice/I go comparing: and I recall eternity). All the sensory void is fulfilled by an atopic, yet absolute experience of temporal awareness.

The moment of free flow and apparent disorganization settles in the rhythmicity of the closing verses. The sense of abandonment is framed by a consolatory acceptance



FIGURE 1. View from the Colle Dell'Infinito, Recanati, Italy (left) [17], view from Mountain Avala, Belgrade, Serbia

of the new condition (So between this/Immensity drowns my thought:/And shipwreck is sweet to me in this sea). Recurrence of the initial composed rhythm produces a sensation of closure and circularity (Always dear was to me this solitary hill/And shipwreck is sweet to me in this sea), introducing a parallelism between space and time.

3. A Signal Processing Insight

The poem *L'Infinito* initially introduces the reader into a poetry of space, however the rising intensity of the verses reveals the intention of the author to address an ephemeral experience of gazing into the abyss of time. Remembrance of eternity unambiguously recalls the concept of vertical time, individuated by Bachelard as the time of poetry and permeating arts in general [11, 19]. However, in Leopardi's poem it is intimately related to the contemplation of nature. In fact, the sensation of time verticality is straightforwardly linked to the sound of wind rustling in the trees, psithurism (from the Ancient Greek $\psi \iota \theta \dot{\nu} \rho \iota \sigma \mu \alpha$ – whisper). This explicit relation opens to the possibility of a signal processing approach in the study of vertical time.

3.1. Materials and methods. It is known that the *idyll* was composed at the top of Monte Tabor, a hill in Recanati, Italy, known today as Colle Dell'Infinito (the Hill of The Infinite), Figure 1 (left). The name strikingly coincides to the Mount Tabor whereon the Christ transfiguration has taken place, which is an infinitive experience as well. The traditional icon of such an event is temporally organized in terms of vertical structuring [12]. Taking into account vegetation features of the original site (mixture of evergreens and caducous trees), psithurism samples have been collected at Mountain Avala near Belgrade, Serbia, Figure 1 (right). Samples were recorded with sampling frequency $f_s = 44100$ Hz, and a bit depth of 16 bits per sample.

A signal is a function of time (a real-valued function of time, throughout this paper), that conveys information about a phenomenon. An electrical analog signal is the result of conversion of the physical waveform (in this case, fluctuation of air pressure) by means of a transducer [16]. The normalized instantaneous power of such

an electrical signal x(t) (based on the Ohm's law and assuming a unitary resistor) is

(3.1)
$$p(t) = x^2(t),$$

with x(t) being, equivalently, the transducer's output current or voltage.

The energy of a signal is defined as

(3.2)
$$E_x = \lim_{T \to +\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t) \, dt,$$

while the signal average power is

(3.3)
$$P_x = \lim_{T \to +\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t) dt.$$

A signal is classified as *energy signal*, if and only if, it has nonzero but finite energy $0 < E_x < +\infty$ (and thus zero average power). On the other hand, a signal is referred to as *power signal* if it has nonzero but finite power $0 < P_x < +\infty$ (and thus infinite energy). The two categories are mutually exclusive. While signals that are both deterministic and nonperiodic are classified as *energy signals*, periodic, and random signals are treated as *power signals* [16].

3.1.1. Energy spectral density. The total energy (from $-\infty$ to $+\infty$) of an energy signal, is maintained in the frequency domain, according to Parseval's theorem [16], as

(3.4)
$$E_x = \int_{-\infty}^{+\infty} x^2(t) dt = \int_{-\infty}^{+\infty} |\hat{x}(f)|^2 df,$$

with $\hat{x}(f)$ being the Fourier transform of the signal x(t):

(3.5)
$$\hat{x}(f) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{+\infty} x(t) e^{-i2\pi f t} dt$$

The squared magnitude spectrum, $|\hat{x}(f)|^2$, thus, describes the signal's energy per unit bandwidth, and will be referred to as *energy spectral density* (ESD).

The autocorrelation function of the energy signal,

(3.6)
$$R_x(\tau) = \int_{-\infty}^{+\infty} x(t)x(t-\tau) \, dt,$$

is related to the ESD by means of the Fourier transform, as

(3.7)
$$\mathfrak{F}\{R_x(\tau)\} = |\hat{x}(f)|^2.$$

3.1.2. Power spectral density. For power signals, such as sample signals x(t) of a wide-sense-stationary (WSS) random process X(t), which extend over infinite time, the Fourier transform might not exist. To deal with such signals in the frequency domain, the Fourier transform of the random process truncated by some window T, $\mathcal{F}{X_T(t)}$, should be considered. According to the Wiener-Khinchin theorem [6], the

autocorrelation function of the WSS random process has its Fourier pair in the *power* spectral density (PSD) G(f), as

(3.8)
$$G(f) = \mathcal{F}\{R_x(\tau)\} = \lim_{T \to +\infty} \frac{1}{T} \mathbb{E}\{|\hat{X}_T(f)|^2\},$$

with $E\{\cdot\}$ denoting the expected value.

If the WSS random process is also ergodic in the autocorrelation, in terms of one sample signal x(t) for $\tau = 0$ (taking into account Exp. (3.8)), the autocorrelation takes the form

(3.9)
$$R_x(0) = \lim_{T \to +\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t) dt = \int_{-\infty}^{+\infty} G(f) df.$$

Since the left-hand-term and the middle term in Eq. (3.9) represent the average power of the signal, the PSD in the right-hand-term describes the distribution of the signal's power in the frequency domain.

3.2. Signals in different domains - an interpretation of vertical time. The frequency representation of one sample of psithurism, being an *energy signal*, is reported in Figure 2 (a) by means of its ESD. Even though the energy distribution presents slightly dominant components in the lower frequency range, the ESD appears essentially flat within the interval of audible frequencies [14] (the ESD peak close to 5 kHz is the result of a cricket chirp, clearly audible in the recording and recognizable from the spectrogram). However, evenly distributed energy in the frequency domain cannot be considered a feature of the sound of wind in general; for comparison, the sound of wind in an open space produces an ESD with more pronounced characteristics of flicker noise [18], Figure 2 (b).

3.2.1. The impulse function model. Since we would acquiesce that, in the audible frequency range, psithurism is perceived as white noise, its characteristics will be analysed in different domains based on the theoretical white noise model [16]. White noise is modeled as a WSS, zero-mean, ergodic random process, with its PSD being constant for all frequencies:

(3.10)
$$G_N(f) = \frac{N_0}{2}, \quad W/Hz,$$

where the factor of 2 indicates the G(f) is a two-sided PSD.

The autocorrelation function of white noise, given as the inverse Fourier transform of the PSD G(f) (3.8), is denoted as:

(3.11)
$$R_N(\tau) = \mathcal{F}^{-1}\{G_N(f)\} = \frac{N_0}{2}\delta(\tau).$$

This relation is represented in Figure 3.

The Dirac delta function, defined by

(3.12)
$$\delta(\tau) = \begin{cases} +\infty, & \tau = 0, \\ 0, & \text{otherwise,} \end{cases}$$

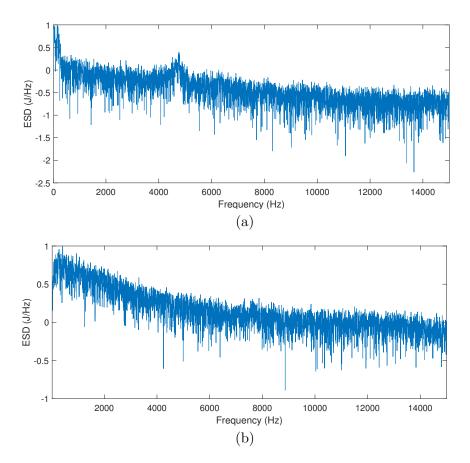


FIGURE 2. ESD of one sample of sound of wind in trees from Mountain Avala location (a), ESD of one sample of sound of wind in open space (b)

and

(3.13)
$$\int_{-\infty}^{+\infty} \delta(\tau) \, d\tau = 1$$

is an infinite impulse in the lag domain, supported only by $\tau = 0$ [13].

Expression (3.11), thus, realizes the implicit link between the perceived sound of psithurism and the experience of an infinite impulse, materialized in the domain of the time lag. This relation between sound and the vertical impulse, however, is conceded only for sound with spectral characteristics of white noise.

3.2.2. The sifting property of the impulse function. The delta function, thus, meets the static description of vertical time as a meterless impulse [2]. However, it is through its dynamics that vertical time materializes eternity. As described by Bachelard, vertical time runs perpendicularly to the temporal axis, linking the present instant to eternity [1].

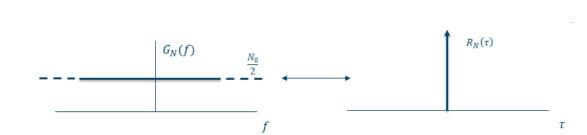


FIGURE 3. White noise in the frequency and lag domains

In order to enlighten the described behavior of vertical time across the horizontal axis, which according to Bachelard gives access to all time at once, a deeper insight into the delta function model and its properties will be required. As obvious from (3.12), $\delta(\tau)$ is not a function in the ordinary mathematical sense. It would be more appropriate to refer to $\delta(\tau)$ as to a functional quantity with a certain well-defined symbolic meaning [13]. For instance, one could define a sequence of functions $\delta(\tau, \epsilon)$ [13]

(3.14)
$$\delta(\tau, \epsilon) = \epsilon \operatorname{rect}(\epsilon \tau) \equiv \begin{cases} \frac{\epsilon}{2}, & |\tau| < \frac{1}{\epsilon}, \\ 0, & \text{otherwise,} \end{cases}$$

with

(3.15)
$$\int_{-\infty}^{+\infty} \delta(\tau, \epsilon) \, d\tau = 1$$

With increasing values of the parameter ϵ , the sequence of functions $\delta(\tau, \epsilon)$ differ from zero only on a decreasing interval about the origin, with

(3.16)
$$\lim_{\epsilon \to +\infty} \int_{-\infty}^{+\infty} \delta(\tau, \epsilon) \, d\tau = 1$$

still holding true. Thus any operation involving the delta function $\delta(\tau)$ should be performed with a function $\delta(\tau, \epsilon)$ and the limit $\epsilon \to +\infty$ introduced at the conclusion of the calculation.

Let us now consider a continuous and well-behaved function $f(\tau)$, and the value of the integral

(3.17)
$$\int_{-\infty}^{+\infty} f(\tau) \,\delta(\tau-a) \,d\tau = \lim_{\epsilon \to +\infty} \int_{-\infty}^{+\infty} f(\tau) \,\delta(\tau-a,\epsilon) \,d\tau.$$

The value of the integral on the right-hand side depends on the behavior of $f(\tau)$ in the vicinity of the $\tau = a$. Thus, taking an arbitrarily large ϵ allows the error originated from replacing $f(\tau)$ by f(a) to become negligibly small. In accordance

(3.18)
$$\lim_{\epsilon \to +\infty} \int_{-\infty}^{+\infty} f(\tau) \,\delta(\tau - a, \epsilon) \,d\tau = f(a) \lim_{\epsilon \to +\infty} \int_{-\infty}^{+\infty} \delta(\tau - a, \epsilon) \,d\tau,$$

and hence

(3.19)
$$\int_{-\infty}^{+\infty} f(\tau) \,\delta(\tau-a) \,d\tau = f(a).$$

Equation (3.19), referred to as the delta function *sifting property*, completes the explication of vertical time by means of the delta function. In fact, if the function $f(\tau) = \tau$ is considered in (3.19), as

(3.20)
$$\int_{-\infty}^{+\infty} \tau \,\delta(\tau - a) \,d\tau = a,$$

the delta function gives access to any time instant. By its *sifting property*, the delta function draws from the infinity of the horizontal axis. Furthermore, elimination of the privileged position of the present instant from the perspective of vertical time ("*This line running perpendicular to the temporal axis of life alone gives consciousness of the present the means to flee and escape...*" [1]), finds correspondence in the delta function model.

Disanchoring from the referent *now* by the vertical time perspective, reveals the unbearable depth of temporality, as described by Leopardi in the last verses of L'Infinito (So between this/ Immensity drowns my thought:/ And shipwreck is sweet to me in this sea).

4. CONCLUSION

Vertical time has been described by Bachelard as an impulse which runs perpendicular on the temporal axis, being in possession of eternity within an instant. This moment of metaphysical perspective, in the poem L'Infinito by Leopardi, is induced by the annihilation of the material world, except for the sound of wind in trees.

The direct link between sound and vision of vertical time in Leopardi's poetry allows to establish a correspondence between signals in different domains by means of spectral analysis. The moment of rupture of the temporal axis, generating an infinite impulse, has been identified as a delta function in the domain of time lag. The ability of vertical time to reach for any time form the past and future is reflected in the delta function *sifting property*. In Leopardi's poetry spatial and temporal dimensions are fluid and interchangeable, since the all-embracing view from the top of the solitary hill becomes a panoramic of the eternity from an infinite impulse standing above time.

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