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ORBITAL CONTINUITY AND COMMON FIXED POINTS IN MENGER PM-SPACES

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Abstract. In this paper, we prove that if a pair of semi R-commuting self-mappings defined on Menger PM-spaces with a nonlinear contractive condition posses a unique common fixed point, then these mappings are orbitally continuous. Also, we investigate whether this assertion and it converse holds if we replace semi R-commutativity with some other concept of commutativity in the weaker sense.

1. Introduction

The notion of orbital continuity was defined by Ćirić [4]. Shastri et al. [25] introduced the notion of orbital continuity for a pair of self-mappings.

Definition 1.1 ([25]). Let f and g be two self-mappings of a metric space (X, d) and let $\{x_n\}_{n\in\mathbb{N}\cup\{0\}}$ be a sequence in X such that $gx_n = fx_{n+1}$, $n = 0, 1, 2, \ldots$ Then the set $O(x_0, g, f) = \{gx_n \mid n = 0, 1, 2, \ldots\}$ is called the (g, f)-orbit at x_0 and f (or g) is called (g, f)-orbitally continuous if $gx_n \to u$ implies $x_n \to fu$, as $n \to +\infty$ (or $gx_n \to u$ implies $ggx_n \to gu$, as $n \to +\infty$).

Following the results obtained by Machuca [13] and Goebel [6], Jungck [9] generalized Banach contraction principle [1] by proving common fixed theorem for a pair of commutative self-mappings. Since then many common fixed point theorems have been obtained using various generalizations of commutativity (see e.g. [7, 10–12, 17–19, 21, 24]). Overview of weaker forms of commuting mappings and their systematic comparison can be found in [26].

Key words and phrases. Orbital continuity, probabilistic metric spaces, common fixed point, nonlinear contractive condition.

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Pant [19] introduced the notion of semi R-commutativity. Ješić et al. [8] extended this notion to Menger PM-spaces and obtained common fixed point theorem for two semi R-commuting self-mappings (Theorem 3.3. in [8]).

Patak et al. [20] defined the notion of R-weak commutativity of type A_f and of type A_g. Using the probabilistic version of this notion, Nikolić et al. [16] proved that orbital continuity for two self-mappings is a necessary and sufficient condition for the existence of a unique common fixed point for these mappings if they are R-weakly commuting of type A_f (or of type A_g) with nonlinear contractive condition in the sense of Boyd and Wong [2], for Menger PM-spaces.

In this paper we prove that converse of a slight modification of Theorem 3.3. in [8] (see Theorem 3.1. given below) holds under additional condition. Also, we investigate whether this theorem and it converse remain true if we replace a pair of semi R-commuting mappings with mappings that satisfy some other weaker form of commutativity. Nikolić et al. [16] gave a positive answer in this sense for a pair of R-weakly commutative mappings of type A_f (or of type A_g).

2. Preliminaries

In 1906, Fréchet introduced the concept of distance on an arbitrary set, described the properties of the distance function and thus founded the axiomatics of metric spaces. This abstractly introduced mathematical object found great applications in the study of not only mathematical objects in which the concept of distance appears. However, in many cases where metric spaces are used, assigning a unique real non-negative number to each pair of elements of a set is not sufficient to describe the observed phenomenon or problem. Namely, in many situations the concept of distance is more suitable to be viewed probabilistically, than as a quantity determined by a real number. In this way, in 1942, Menger [14] gave the definition of the statistical metric space using the notion of distribution function (in 1964, in the name of this spaces the adjective "statistical" was changed to "probabilistic"). Continuing the study of probabilistic metric spaces Schweizer and Sklar [22, 23] introduced some topological notions for this space and gave some properties devoted to the axiomatics of probabilistic metric spaces (in particular for triangle inequality).

Some function $F : \mathbb{R} \to [0, 1]$ is a distribution function if F is a left-continuous and non-decreasing mapping, which satisfies $F(0) = 0$ and $\sup_{x\in\mathbb{R}} F(x) = 1$. With ε_0 we will denote the distribution function given by

$$\varepsilon_0(t) = \begin{cases} 0, & t \leq 0, \\ 1, & t > 0. \end{cases}$$

Definition 2.1 ([23])**.** A mapping $T : [0, 1]^2 \mapsto [0, 1]$ is continuous t-norm if T satisfies the following conditions:

a) T is commutative and associative;
b) T is continuous;
c) $T(a, 1) = a$, for any $a \in [0, 1]$;

d) $T(a_1, b_1) \leq T(a_2, b_2)$ whenever $a_1 \leq a_2$ and $b_1 \leq b_2$, and $a_1, b_1, a_2, b_2 \in [0,1]$.

Definition 2.2. A Menger probabilistic metric space (briefly, Menger PM-space) is a triple $(X, \mathcal{F}, T)$ where X is a nonempty set, T is a continuous t-norm, and $\mathcal{F}$ is a mapping from $X \times X$ into the set of all distribution functions ($\mathcal{F}(x, y) = F_{x,y}$ for any $(x, y) \in X \times X$) if and only if the following conditions hold:

(PM1) $F_{x,y}(t) = \varepsilon_0(t)$ if and only if $x = y$;
(PM2) $F_{x,y}(t) = F_{y,x}(t)$;
(PM3) $F_{x,z}(t+s) \geq T\Big(F_{x,y}(t), F_{y,z}(s)\Big)$, for all $x, y, z \in X$ and all $s, t \geq 0$.

In 1960, Schweizer and Sklar [22] defined (ε, λ)-topology in a Menger PM-space $(X, \mathcal{F}, T)$ and proved that this topology is a Hausdorff topology. Since 1960 many other topics related to PM-spaces have been studied by various authors, such as convergence of sequences, continuity of mappings, completion, etc. We will only state the following definition.

Definition 2.3. Let $(X, \mathcal{F}, T)$ be a Menger PM-space.

(1) A sequence $\{x_n\}_{n\in\mathbb{N}}$ in X is said to be convergent to x in X if, for any $\varepsilon > 0$ and $\lambda \in (0,1)$ there exists positive integer N such that $F_{x_n,x}(\varepsilon) > 1 - \lambda$ whenever $n \geq N$.
(2) A sequence $\{x_n\}_{n\in\mathbb{N}}$ in X is called Cauchy sequence if, for any $\varepsilon > 0$ and $\lambda \in (0,1)$ there exists positive integer N such that $F_{x_n,x_m}(\varepsilon) > 1 - \lambda$ whenever $n, m \geq N$.
(3) A Menger PM-space is said to be complete if any Cauchy sequence in X is convergent to a point in X.

Also, the following two lemmas are stated and proved by Schweizer and Sklar [22].

Lemma 2.1 ([22])**.** *Let $\{x_n\}_{n\in\mathbb{N}}$ be a sequence such that $\lim_{n\to+\infty} x_n = x$. Then $F_{x,x_n}(t) \to F_{x,x}(t) = \varepsilon_0(t)$, for any $t > 0$, as $n \to +\infty$ and conversely.*

Lemma 2.2 ([22])**.** *Let $(X, \mathcal{F}, T)$ be a Menger PM-space and T is continuous. Then the function $\mathcal{F}$ is lower semi-continuous for any fixed $t > 0$, i.e., for any fixed $t > 0$ and any two convergent sequences $\{x_n\}_{n\in\mathbb{N}}$, $\{y_n\}_{n\in\mathbb{N}}$ such that $\lim_{n\to+\infty} x_n = x$ and $\lim_{n\to+\infty} y_n = y$, it follows that*

$$\liminf_{n\to+\infty} F_{x_n,y_n}(t) = F_{x,y}(t).$$

Lemma 2.3 ([23])**.** *Let y be a fixed point and suppose that $\{x_n\}_{n\in\mathbb{N}}$ is a convergent sequence such that $\lim_{n\to+\infty} x_n = x$. Then*

$$\liminf_{n\to+\infty} F_{x_n,y}(t) = F_{x,y}(t).$$

Remark 2.1. Lemma 2.3 is a corollary of Lemma 2.2.

3. Main Results

Fang et al. [5] defined the notion of algebraic sum for two distribution functions.

Definition 3.1 ([5])**.** The algebraic sum of distribution functions F and G, in denotation $F \oplus G$, is defined by:

$$(F \oplus G)(t) = \sup_{s_1+s_2=t} \min\{F(s_1), G(s_2)\}, \tag{3.1}$$

for any $t \in \mathbb{R}$.

From the previous definition, it is obvious that the following inequality

$$(F \oplus G)(t) \geq \min\{F(s_1), G(s_2)\} \tag{3.2}$$

holds for any $t > 0$, and arbitrary and fixed $s_1, s_2 > 0$, such that $s_1 + s_2 = t$.

Ješić et al. [8] extended definition of semi R-commutativity to Menger PM-spaces.

Definition 3.2 ([8])**.** Let $(X, \mathcal{F}, T)$ be a Menger PM-space and let f and g be two self-mappings of X. The mappings f and g will be called semi R-commuting if there exists $R > 0$ such that:

i) $F_{ffx,gfx}(Rt) \geq F_{fx,gx}(t)$ or
ii) $F_{fgx,gfx}(Rt) \geq F_{fx,gx}(t)$ or
iii) $F_{fgx,ggx}(Rt) \geq F_{fx,gx}(t)$ or
iv) $F_{ffx,ggx}(Rt) \geq F_{fx,gx}(t)$

is true for any $t > 0$, and for any $x \in X$ such that $fx, gx \in f(X) \cap g(X)$.

Using this notion Ješić et al. [8] proved the next theorem.

Theorem 3.1 ([8])**.** *Let $(X, \mathcal{F}, T)$ be a complete Menger PM-space, and let f and g be semi R-commuting mappings, $g(X)$ is a probabilistic bounded set and $g(X) \subseteq f(X)$ satisfying the condition*

$$F_{gx,gy}\Big(\varphi(t)\Big) \geq \min\Big\{F_{fx,fy}(2t), F_{fx,gx}(t), F_{fy,gy}(t), \Big(F_{fx,gy} \oplus F_{gx,fy}\Big)(\alpha t)\Big\}, \tag{3.3}$$

for all $x, y \in X$, any $t > 0$ and any $\alpha > 3$, and for some continuous function $\varphi : (0, +\infty) \to (0, +\infty)$ which satisfies condition $\varphi(t) < t$, for any $t > 0$. If (g, f)-orbitally continuous self-mappings on X, then f and g have a unique common fixed point.

Remark 3.1. In Theorem 3.3. from [8] the assumption for function φ is more general than assumption for function φ from assertion of Theorem 3.1 (see condition (1.1), page 2 in [8]).

For the proof of the main result, we need the following lemmas.

Lemma 3.1 ([16])**.** *Suppose that the function $\varphi : (0, +\infty) \to (0, +\infty)$ is continuous and satisfies condition $\varphi(t) < t$, for any $t > 0$ and let $(X, \mathcal{F}, T)$ be a Menger PM-space. Then the following assertion holds: if for $x, y \in X$ we have $F_{x,y}\Big(\varphi(t)\Big) \geq F_{x,y}(t)$ for any $t > 0$, then $x = y$.*

Lemma 3.2 ([16]). *Let $(X, \mathcal{F}, T)$ be a Menger PM-space. If for two convergent sequences $\{x_n\}_{n\in\mathbb{N}}$, $\{y_n\}_{n\in\mathbb{N}}$ holds that $\lim_{n\to+\infty} x_n = \lim_{n\to+\infty} y_n = p$, then $F_{x_n,y_n}(t) \to 1$, as $n \to +\infty$, for any $t > 0$.*

In the following theorem, we will prove that the converse of Theorem 3.1 holds.

Theorem 3.2. *Let the functions f and g satisfy all the assumptions of the Theorem 3.1 and let ggx_n converges for any sequence $\{x_n\}_{n\in\mathbb{N}}$ in X whenever gx_n converges. If f and g have a unique common fixed point, then mappings f and g are (g, f)-orbitally continuous.*

Proof. Let us suppose that z is a common fixed point for mappings f and g. Since, (g, f)-orbit of any point x_0 defined by $gx_n = fx_{n+1}$, $n = 0, 1, 2, \ldots$ converges to z, it follows that $\lim_{n\to+\infty} fx_n = \lim_{n\to+\infty} gx_n = z$. According to the definition of semi R-commutativity, we can observe next four cases.

Case 1. Firstly, we will suppose that mappings f and g satisfy condition *iii*) from Definition 3.2, i.e., we will suppose that there exists $R > 0$ such that inequality $F_{fgx,ggx}(Rt) \geq F_{fx,gx}(t)$ holds for any $t > 0$, and for any $x \in X$ such that $fx, gx \in f(X) \cap g(X)$. Then, it follows that there exists $R > 0$ such that $F_{fgx_n,ggx_n}(Rt) \geq F_{fx_n,gx_n}(t)$ holds, for any $t > 0$. If we apply Lemma 3.2 for such R and any $t > 0$, it follows

$$F_{fgx_n,ggx_n}(Rt) \to 1, \quad \text{as } n \to +\infty. \tag{3.4}$$

Now, if we put $x = gx_n$, $y = z$, and $gz = fz$ in contractive condition (3.3) and if we apply condition (PM3) from Definition 2.2, then we get that

$$\begin{aligned}
F_{ggx_n,gz}\Big(\varphi(t)\Big) &\geq \min\Big\{F_{fgx_n,gz}(2t), F_{fgx_n,ggx_n}(t), F_{gz,gz}(t), \Big(F_{fgx_n,gz} \oplus F_{ggx_n,fz}\Big)(\alpha t)\Big\} \\
&\geq \min\Big\{F_{fgx_n,gz}(2t), F_{fgx_n,ggx_n}(t), F_{gz,gz}(t), F_{fgx_n,gz}(2t), F_{gz,ggx_n}(t)\Big\} \\
&= \min\Big\{F_{fgx_n,gz}(2t), F_{fgx_n,ggx_n}(t), F_{gz,ggx_n}(t)\Big\} \\
&\geq \min\Big\{T\Big(F_{fgx_n,ggx_n}(t), F_{ggx_n,gz}(t)\Big), F_{fgx_n,ggx_n}(t), F_{gz,ggx_n}(t)\Big\}
\end{aligned} \tag{3.5}$$

holds, for all $x, y \in X$, any $t > 0$ and any $\alpha > 3$. Using assumption that ggx_n converges, having in mind condition (3.4) and conditions b), c) and d) from Definition 2.1, if take $\liminf$ as $n \to +\infty$ in inequality (3.5) and apply Lemma 2.3, we get

$$F_{\lim\limits_{n\to+\infty} ggx_n,gz}\Big(\varphi(t)\Big) \geq F_{\lim\limits_{n\to+\infty} ggx_n,gz}(t).$$

Finally, if we apply Lemma 3.1, then we get that $\lim_{n\to+\infty} ggx_n = gz$. Hence, g is (g, f)-orbitally continuous. Now, we will show that f is (g, f)-orbitally continuous. Indeed, using condition (PM3) from Definition 2.2 it follows that

$$F_{fgx_n,gz}(t) \geq T\left(F_{fgx_n,ggx_n}\left(\frac{t}{2}\right), F_{ggx_n,gz}\left(\frac{t}{2}\right)\right)$$

holds for any $t > 0$. Letting $n \to +\infty$ in previous inequality, from condition (3.4) and Lemma 2.1 we get that $F_{fgx_n,gz}(t) \to 1$, for any $t > 0$. Finally, applying Lemma 2.1 we get $\lim_{n\to+\infty} fgx_n = gz = fz$. Hence, f and g are orbitally continuous.

Case 2. Now, we will suppose that mappings f and g satisfy condition *i*) from Definition 3.2, i.e., we will suppose that there exists $R > 0$ such that inequality $F_{ffx,gfx}(Rt) \geq F_{fx,gx}(t)$ holds for any $t > 0$, and for any $x \in X$ such that $fx, gx \in f(X) \cap g(X)$. In this case, for such $R > 0$, it follows that

$$F_{ffx_n,gfx_n}(Rt) \to 1, \quad \text{as } n \to +\infty \tag{3.6}$$

holds for any $t > 0$. Similarly, as in Case 1, if we put $x = fx_n$, $y = z$, and $gz = fz$ in contractive condition (3.3) and if we apply condition (PM3) from Definition 2.2, then we obtain

$$F_{gfx_n,gz}\Big(\varphi(t)\Big) \geq \min\Big\{T\Big(F_{ffx_n,gfx_n}(t), F_{gfx_n,gz}(t)\Big), F_{ffx_n,gfx_n}(t), F_{gz,gfx_n}(t)\Big\}, \tag{3.7}$$

for all $x, y \in X$, and any $t > 0$. Having in mind condition (3.6) and conditions b), c) and d) from Definition 2.1, if taking $\liminf$ as $n \to +\infty$ in inequality (3.7) we get

$$\liminf_{n\to+\infty} F_{gfx_n,gz}\Big(\varphi(t)\Big) \geq \liminf_{n\to+\infty} F_{gfx_n,gz}(t). \tag{3.8}$$

From assumption that ggx_n converges, then from $gfx_n = ggx_{n-1}$ we get that gfx_n converges. Now, having in mind Lemma 2.3, and applying Lemma 3.1 for condition (3.8) we get that $\lim_{n\to+\infty} ggx_n = gz$. The remaining part of the proof is analogous as in the previous case.

Case 3. We will suppose that mappings f and g satisfy condition *ii*) from Definition 3.2, i.e., we will suppose that there exists $R > 0$ such that inequality $F_{fgx,gfx}(Rt) \geq F_{fx,gx}(t)$ holds for any $t > 0$, and for any $x \in X$ such that $fx, gx \in f(X) \cap g(X)$. In this case, for such $R > 0$, it follows that

$$F_{fgx_n,gfx_n}(Rt) \to 1, \quad \text{as } n \to +\infty,$$

i.e.,

$$F_{fgx_n,ggx_{n-1}}(Rt) \to 1, \quad \text{as } n \to +\infty, \tag{3.9}$$

for any $t > 0$. Applying condition (PM3) from Definition 2.2 it follows that

$$F_{fgx_n,ggx_n}(t) \geq T\left(F_{fgx_n,ggx_{n-1}}\left(\frac{t}{2}\right), F_{ggx_{n-1},ggx_n}\left(\frac{t}{2}\right)\right)$$

holds, for any $t > 0$. Letting $n \to +\infty$ in previous inequality, using assumption that ggx_n converges, condition (3.9) and Lemma 3.2, and having in mind conditions b) and c) from Definition 2.1 we get that $F_{fgx_n,ggx_n}(t) \to 1$, for any $t > 0$, i.e., we obtain condition (3.4). Therefore, the proof of this case reduces to the proof of Case 1.

Case 4. Finally, in this case we will suppose that mappings f and g satisfy condition *iv*) from definition of semi R-commutativity for Menger PM-spaces, i.e., we will suppose that there exists $R > 0$ such that inequality $F_{ffx,ggx}(Rt) \geq F_{fx,gx}(t)$ holds for any $t > 0$, and for any $x \in X$ such that $fx, gx \in f(X) \cap g(X)$. In this case, for such $R > 0$, we get that

$$F_{ffx_n,ggx_n}(Rt) \to 1, \quad \text{as } n \to +\infty,$$

i.e.,

$$F_{fgx_{n-1},ggx_n}(Rt) \to 1, \quad \text{as } n \to +\infty, \tag{3.10}$$

for any $t > 0$. Now, similarly as in previous case, condition

$$F_{fgx_{n-1},ggx_{n-1}}(t) \geq T\left(F_{fgx_{n-1},ggx_n}\left(\frac{t}{2}\right), F_{ggx_n,ggx_{n-1}}\left(\frac{t}{2}\right)\right)$$

is satisfied, for every $t > 0$. Letting $n \to +\infty$ in previous inequality, using assumption that ggx_n converges, condition (3.10) and Lemma 3.2, and having in mind conditions b) and c) from Definition 2.1 we get that $F_{fgx_{n-1},ggx_{n-1}}(t) \to 1$, i.e., we get $F_{fgx_n,ggx_n}(t) \to 1$, for any $t > 0$. The rest of the proof is the same as in the previous cases.

Now, the proof is completed. □

Now, we list some definitions of weaker forms of commuting mappings introduced for Menger PM-spaces by various authors.

Definition 3.3 ([7])**.** Let $(X, \mathcal{F}, T)$ be a Menger PM-space and let f and g be self-mappings of X. The mappings f and g will be called R-weakly commuting if there exists some positive real number R such that

$$F_{fgx,gfx}(Rt) \geq F_{fx,gx}(t),$$

for any $t > 0$ and any $x \in X$.

Definition 3.4 ([15])**.** Let $(X, \mathcal{F}, T)$ be a Menger PM-space and let f and g be self-mappings of X. The mappings f and g will be called compatible if

$$\lim_{n\to+\infty} F_{fgx_n,gfx_n}(t) = 1,$$

for any $t > 0$, whenever $\{x_n\}_{n\in\mathbb{N}}$ is a sequence in X such that $\lim_{n\to+\infty} fx_n = \lim_{n\to+\infty} gx_n = u$, for some u in X.

Definition 3.5 ([3])**.** Let $(X, \mathcal{F}, T)$ be a Menger PM-space and let f and g be self-mappings of X. The mappings f and g will be called compatible of type (A) if

$$\lim_{n\to+\infty} F_{fgx_n,ggx_n}(t) = 1 \quad \text{and} \quad \lim_{n\to+\infty} F_{gfx_n,ffx_n}(t) = 1,$$

for any $t > 0$, whenever $\{x_n\}_{n\in\mathbb{N}}$ is a sequence in X such that $\lim_{n\to+\infty} fx_n = \lim_{n\to+\infty} gx_n = u$, for some u in X.

Theorem 3.1 and Theorem 3.2 remain true if we replace assumption that a pair of mappings is semi R-commutative with assumption that these self-mappings are R-weakly commuting or compatible or compatible of type (A). These theorems can also be proved under the assumption that a pair of self-mappings satisfies some other types of compatibility (for instance type (E) or type (P) (in their probabilistic versions)).

Also, positive answer for Theorem 3.1 and Theorem 3.2 in this sense was obtained by Nikolić et al. [16] for a pair of R-weakly commutative mappings of type A_f (or type A_g).

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