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TOTAL ABSOLUTE DIFFERENCE EDGE IRREGULARITY STRENGTH OF GRAPHS

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ABSTRACT. We introduce a new graph characteristic, the total absolute difference edge irregularity strength. We obtain the estimation on the total absolute difference edge irregularity strength and determine the precise values for some families of graphs.

1. INTRODUCTION

Throughout this paper, G is a simple graph, V and E are the sets of vertices and edges of G, with cardinalities $|V|$ and $|E|$ respectively. A labeling of a graph is a map that carries graph elements to the numbers.

A labeling is called a vertex labeling, an edge labeling or a total labeling, if the domain of the map is the vertex set, the edge set, or the union of vertex and edge sets respectively. Baca et al. in [2] started to investigate the total edge irregularity strength of a graph, an invariant analogous to the irregularity strength for total labeling. For a graph $G = (V(G), E(G))$, the weight of an edge $e = e_1e_2$ under a total labeling ξ is $wt_\xi(e) = \xi(e_1) + \xi(e) + \xi(e_2)$. For a graph G we define a labeling $\xi : V(G) \cup E(G) \to \{1, 2, \ldots, k\}$ to be an edge irregular total k-labeling of a graph G if for every two different edges xy and $x'y'$ of G one has $wt_\xi(xy) \neq wt_\xi(x'y')$. The total edge irregular strength, $tes(G)$, is defined as the minimum k for which G has an edge irregular total k-labeling. In [2], we can find that

$$tes(G) \geq \max\left\{\left\lceil \frac{|E(G)|+2}{3} \right\rceil, \left\lceil \frac{\Delta(G)+1}{2} \right\rceil\right\},$$

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where $\Delta(G)$ is the maximum degree of G, and also there are determined the exact values of the total edge irregularity strength for paths, cycles, stars, wheels and friendship graphs. Recently Ivanco and Jendrol [3] proved that for any tree T

$$tes(T) = \max\left\{\left\lceil\frac{|E(G)|+2}{3}\right\rceil, \left\lceil\frac{\Delta(G)+1}{2}\right\rceil\right\}.$$

Moreover, they posed a conjecture that for an arbitrary graph G different from K_5 and the maximum degree $\Delta(G)$

$$tes(G) = \max\left\{\left\lceil\frac{|E(G)|+2}{3}\right\rceil, \left\lceil\frac{\Delta(G)+1}{2}\right\rceil\right\}.$$

The Ivanco and Jendrol's conjecture has been verified for complete graphs and complete bipartite graphs in [4], for categorical product of cycle and path in [1] and [6], for corona product of paths with some graphs in [5].

A graceful labeling of a graph $G = (V, E)$ with $|V|$ vertices and $|E|$ edges is a one-to-one mapping Ψ of the vertex set $V(G)$ into the set $\{0, 1, 2, \ldots, |E|\}$ with the following property: If we define, for any edge $e = uv \in E(G)$, the value $\Psi'(e) = |\Psi(u) - \Psi(v)|$ then Ψ' is a one-to-one mapping of the set $E(G)$ onto the set $\{1, 2, \ldots, |E|\}$. Motivated by the total edge irregularity strength of a graph and motivated by the graceful labeling, we introduce and investigate the total absolute difference edge irregularity strength of graphs to reduce the edge weights.

A total labeling ξ is defined to be an edge irregular total absolute difference k-labeling of the graph G if for every two different edges e and f of G there is $wt(e) \neq wt(f)$ where weight of an edge $e = xy$ is defined as $wt(e) = |\xi(e) - \xi(x) - \xi(y)|$. The minimum k for which the graph G has an edge irregular total absolute difference labeling is called the total absolute difference edge irregularity strength of the graph G, $tades(G)$. The main aim of this paper is to obtain estimations on the parameter *tades* and determine the precise values of *tades* for some families of graphs.

2. Main Results

The following result shows that the absolute difference edge irregularity strength is defined for all graphs.

Theorem 2.1. *Let $G = (V, E)$ be a graph with vertex set V and a non-empty edge set E. Then $\left\lceil\frac{|E|}{2}\right\rceil \leq tades(G) \leq |E| + 1$.*

Proof. To get the upper bound we label each vertex of G with label 1 and the edges of G consecutively with labels $2, 3, \ldots, |E|+1$. Then $wt(e)$ are consecutively $0, 1, \ldots, |E|-1$ and the weights for any two distinct edges e and f are distinct.

To get the lower bound, let ξ be an optimal labeling with respect to the tades(G). The weight of the heaviest edge implies that $|\xi(e) - \xi(x) - \xi(y)| \geq |E| - 1$.

That is, $\xi(e)-\xi(x)-\xi(y) \geq |E|-1$ if $\xi(e) > \xi(x)+\xi(y)$, $\xi(x)+\xi(y)-\xi(e) \geq |E|-1$. If $\xi(e) < \xi(x)+\xi(y)$, then

$$\xi(x)+\xi(y)-\xi(e) \geq |E|-1,$$

which implies

$$\xi(x)+\xi(y) \geq |E|-1+\xi(e) \geq |E|-1+1 = |E|.$$

That is,

$$\xi(x)+\xi(y) \geq |E|\,.$$

So at least one label is at least $\left\lceil \frac{|E|}{2} \right\rceil$.

If $\xi(e) > \xi(x)+\xi(y)$, then $\xi(e)-\xi(x)-\xi(y) \geq |E|-1$.

Suppose $\xi(e) < \left\lceil \frac{|E|}{2} \right\rceil$, then

$$-\xi(x)-\xi(y) \geq |E|-1-\xi(e) > |E|-1-\left\lceil \frac{|E|}{2} \right\rceil > \left\lfloor \frac{|E|}{2} \right\rfloor - 1.$$

That is,

$$\xi(x)+\xi(y) < 1-\left\lfloor \frac{|E|}{2} \right\rfloor = 0,$$

which is not possible. Hence,

$$\xi(e) \geq \left\lceil \frac{|E|}{2} \right\rceil.$$

That is,

$$\left\lceil \frac{|E|}{2} \right\rceil \leq tades(G) \leq |E|+1. \qquad \square$$

The lower bound in the Theorem 2.1 is tight as can be seen from the following theorem.

Theorem 2.2. *Let P_n be a path on $n \geq 4$ vertices. Then $tades(P_n) = \left\lceil \frac{n-1}{2} \right\rceil$.*

Proof. From the Theorem 2.1 we have $tades(P_n) \geq \left\lceil \frac{n-1}{2} \right\rceil$. So, it is enough to prove that

$$tades(P_n) \leq \left\lceil \frac{n-1}{2} \right\rceil.$$

Let P_n be the path $v_1e_1v_2e_2v_3, \dots, v_{n-1}e_{n-1}v_n$, $n \geq 4$. Now define a mapping $\xi : V \cup E \to \left\{1, 2, \dots, \left\lceil \frac{n-1}{2} \right\rceil\right\}$ by $\xi(v_1) = 1$ and $\xi(v_i) = \left\lceil \frac{i-1}{2} \right\rceil$ for $2 \leq i \leq n$, $\xi(e_1) = 2$ and $\xi(e_i) = 1$ for $2 \leq i \leq n-1$. Now,

$$\max\left\{\{\xi(v)|v \in V(P_n)\} \cup \{\xi(e)|e \in E(P_n)\}\right\} = \left\lceil \frac{n-1}{2} \right\rceil$$

and the edge weights are given by

$$wt(e_1) = |\xi(e_1)-\xi(v_1)-\xi(v_2)| = |2-1-1| = 0,$$

for $2 \leq i \leq n-1$,

$$wt(e_i) = |\xi(e_i) - \xi(v_i) - \xi(v_{i+1})| = i - 1.$$

Hence, the weights are distinct. Therefore, $tades(P_n) = \left\lceil \frac{n-1}{2} \right\rceil$. □

Observation 1. We observe that $tades(P_3) = 2$, $tades(P_2) = 1$.

The upper bound in the Theorem 2.1 is not sharp. If we utilize the maximum degree $\Delta = \Delta(G)$ of the graph G, we obtain the following result.

Theorem 2.3. *Let $G = (V, E)$ be a graph with maximum degree $\Delta = \Delta(G)$. Then $tades(G) \geq \left\lceil \frac{\Delta+1}{2} \right\rceil$.*

Proof. Let $G = (V, E)$ be a graph with maximum degree $\Delta = \Delta(G)$. Let x be a vertex of G with maximum degree Δ in G. Let $e_j = xu_j$ be the edges incident with the vertex x, $1 \leq j \leq \Delta$. Assume to the contrary that, $tades(G) < \left\lceil \frac{\Delta+1}{2} \right\rceil$. Suppose ξ is an optimal total labeling of G. Then $wt(e_j)$ is either $\xi(e_j) - \xi(x) - \xi(u_j)$ or $-\xi(e_j) + \xi(x) + \xi(u_j)$ for $1 \leq j \leq \Delta$. Among the Δ edges, let i denote the number of edges that have weight $-\xi(e_j) + \xi(x) + \xi(u_j)$. Then $0 \leq i \leq \Delta$. Suppose $i = 0$, then all the edges have weight $\xi(e_j) - \xi(x) - \xi(u_j)$. Then $wt(e_j) = \xi(e_j) - \xi(x) - \xi(u_j)$ for $1 \leq j \leq \Delta$. Since $\xi(x) \geq 1$ and $\xi(u_j) \geq 1$, we have $-\xi(x) \leq -1$ and $-\xi(u_j) \leq -1$. Therefore, $wt(e_j) \leq \left\lfloor \frac{\Delta}{2} \right\rfloor - 2$. Then we have at most $\left\lfloor \frac{\Delta}{2} \right\rfloor - 1$ distinct weights, but we need at least Δ weights. Therefore, $i = 0$ is not possible. Among the edges e_j, $1 \leq j \leq \Delta$, there is an edge $e_k = xu_k$ with weight $wt(e_k) = -\xi(e_k) + \xi(x) + \xi(u_k)$. That is, $\xi(x) + \xi(u_k) \leq \xi(e_k) \leq \left\lfloor \frac{\Delta}{2} \right\rfloor$. That is, $\xi(x) \leq \left\lfloor \frac{\Delta}{2} \right\rfloor - 1$. Then the possible values of $\xi(x)$ are $1, 2, \dots, \left\lfloor \frac{\Delta}{2} \right\rfloor - 1$. Now, fix the value $\left\lfloor \frac{\Delta}{2} \right\rfloor - h$ to $\xi(x)$ for some $1 \leq h \leq \left\lfloor \frac{\Delta}{2} \right\rfloor - 1$. Then

$$wt(e_j) = -\xi(e_j) + \xi(x) + \xi(u_j) = -\xi(e_j) + \left\lfloor \frac{\Delta}{2} \right\rfloor - h + \xi(u_j).$$

Suppose $-\xi(e_j) + \left\lfloor \frac{\Delta}{2} \right\rfloor - h + \xi(u_j) \geq \Delta - h$, then $\xi(u_j) \geq \left\lceil \frac{\Delta}{2} \right\rceil + \xi(e_j) \geq \left\lceil \frac{\Delta+1}{2} \right\rceil + 1$, which is a contradiction to our assumption. Therefore, $-\xi(e_j) + \left\lfloor \frac{\Delta}{2} \right\rfloor - h + \xi(u_j) < \Delta - h$. Then we have at most $\Delta - 1$ distinct weights, but we need at least Δ weights. Therefore, $i \neq 0$ is also not possible. Therefore, $tades(G) \geq \left\lceil \frac{\Delta+1}{2} \right\rceil$. □

The lower bound in the Theorem 2.3 is tight as can be seen from the following theorem.

Theorem 2.4. *Let $S_n = K_{1,n}$ be a star on $n+1$ vertices, $n > 2$. Then $tades(S_n) = \left\lceil \frac{n+1}{2} \right\rceil$.*

Proof. From the Theorem 2.3

$$tades(S_n) \geq \left\lceil \frac{n+1}{2} \right\rceil.$$

Let the vertices of S_n be $\{u, v_1, v_2, \dots, v_n\}$ where u is a vertex of maximum degree. Let $e_i = uv_i$, $1 \le i \le n$, be the edges of the star S_n. Now define the labeling $\xi : V \cup E \to \left\{1, 2, \dots, \left\lceil \frac{n+1}{2} \right\rceil\right\}$ by $\xi(u) = \left\lfloor \frac{n-1}{2} \right\rfloor$. Then

$$\xi(v_i) = \begin{cases} i, & \text{if } 1 \le i < \left\lceil \frac{n+1}{2} \right\rceil, \\ \lceil \frac{n+1}{2} \rceil, & \text{if } \lceil \frac{n+1}{2} \rceil \le i \le n, \end{cases}$$

$$\xi(uv_i) = \begin{cases} \lceil \frac{n}{2} \rceil, & \text{if } 1 \le i \le \lceil \frac{n+1}{2} \rceil, \\ 2\lceil \frac{n+1}{2} \rceil - i, & \text{if } \lceil \frac{n+1}{2} \rceil < i \le n, \ n \text{ is an odd integer}, \\ 2\lceil \frac{n+1}{2} \rceil - i - 1, & \text{if } \lceil \frac{n+1}{2} \rceil < i \le n, \ n \text{ is an even integer}. \end{cases}$$

Now,

$$\max\{\{\xi(v) \mid v \in V(S_n)\} \cup \{\xi(e) \mid e \in E(S_n)\}\} = \left\lceil \frac{n+1}{2} \right\rceil.$$

Also, $0 \le wt(uv_i) \le \left\lceil \frac{n+1}{2} \right\rceil - 1$ for $1 \le i \le \left\lceil \frac{n+1}{2} \right\rceil$

$$wt(uv_{\lceil \frac{n+1}{2} \rceil}) = \left\lceil \frac{n+1}{2} \right\rceil - 1,$$

$\left\lceil \frac{n+1}{2} \right\rceil \le wt(uv_i) \le n - 1$ for $\left\lceil \frac{n+1}{2} \right\rceil < i \le n$. Hence, the edge weights are distinct. Therefore, $tades(S_n) = \left\lceil \frac{n+1}{2} \right\rceil$.

□

Observation 2. We observe that $tades(S_1) = 1$, $tades(S_2) = 2$.

In the next theorem, we discuss a technique to determine tades for some families of graphs.

Theorem 2.5. *Let G be a graph and $\phi : V(G) \to \{0, 1\}$ be a mapping and let $E_i(\phi) = \{xy \in E(G) \,|\phi(x) + \phi(y) = i\}$ for $i \in \{0, 1, 2\}$. If $|E_0(\phi)| \le k - 1$, $|E_1(\phi)| \le k - 1$, $|E_2(\phi)| = k - 1$ and $|E| \le 2k$, then G has a total edge-irregular absolute difference k-labeling.*

Proof. Let G be a graph and $\phi : V(G) \to \{0, 1\}$ be a mapping and

$$wt_\phi(e) = \phi(u) + \phi(v),$$

where u and v are end vertices of e. Let $E_i(\phi) = \{xy \in E(G) \mid \phi(x) + \phi(y) = i\}$ for $i = 0, 1, 2$. Suppose that $|E_0(\phi)| \le k - 1, |E_1(\phi)| \le k - 1, |E_2(\phi)| = k - 1$. Let $E_0(\phi) = \{e_1, e_2, \dots, e_{r_0}\}$, $E_1(\phi) = \{e'_1, e'_2, \dots, e'_{r_1}\}$ and $E_2(\phi) = \{e''_1, e''_2, \dots, e''_{k-1}\}$. Then define a mapping ξ_1 from $V(G)$ into the set of positive integers by $\xi_1(x) = k^{\phi(x)}$ if $x \in V(G)$. Under the labeling ϕ, there are r_0 edges with weight 0, r_1 edges with weight 1 and $k - 1$ edges with weight 2. Then under the vertex labling ξ_1, there are r_0 edges with weight 2, r_1 edges with weight $k + 1$ and $k - 1$ edges with weight $2k$. Now define a labeling ξ from $V(G) \cup E(G)$ into the set of positive integers as follows: $\xi(x) = \xi_1(x)$ for all $x \in V(G)$,

$$\xi(e_i) = \begin{cases} i, & \text{if } i = 1, 2, \\ i + 1, & \text{if } 3 \le i \le r_0, \end{cases}$$

$\xi(e_i') = i$ if $1 \le i \le r_1$, $\xi(e_i'') = i$ if $1 \le i \le k-1$. Then

$$\begin{aligned}\{wt_\xi(e_i) \mid 1 \le i \le r_0\} &= \{0, 1, 2, \dots, r_0 - 1\},\\ \{wt_\xi(e_i') \mid 1 \le i \le r_1\} &= \{k, k-1, k-2, \dots, k - r_1 + 1\},\\ \{wt_\xi(e_i'') \mid 1 \le i \le k-1\} &= \{2k-1, 2k-2, \dots, k+1\}.\end{aligned}$$

Since $|E| \le 2k$, we have

$$r_0 + r_1 + k - 1 \le 2k.$$

That is $r_0 + r_1 \le k + 1$. That is, $r_0 - 1 \le k - r_1 < 1 + k - r_1$. Hence, the edge weights are distinct. Therefore, the graph G has a total edge-irregular absolute difference k-labeling. □

We determine the *tades* for the graphs C_n, S_n and F_n using the Theorems 2.1 and 2.5.

Theorem 2.6. *For $n \ge 3$, $tades(C_n) = \left\lceil \frac{n}{2} \right\rceil$.*

Proof. For $n = 3, 4, 5$, from the labeling given in the Figure 1, we get the required result.

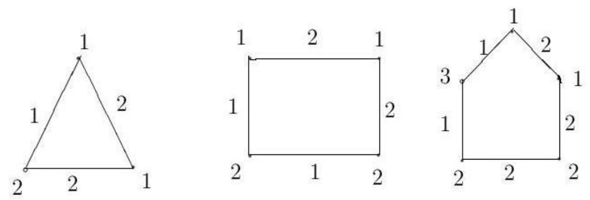

FIGURE 1. Tades for C_n, $n = 3, 4, 5$

For $n > 5$, the proof is as follows. From the Theorem 2.1, we have

$$tades(C_n) \ge \left\lceil \frac{n}{2} \right\rceil.$$

The vertex set of C_n is $\{u_i | 1 \le i \le n\}$ and the edge set of C_n is $\{u_i u_{i+1} | 1 \le i \le n-1\}$. Now, define the labeling $\phi : V(C_n) \to \{0, 1\}$ by

$$\phi(u_i) = \begin{cases} 1, & \text{if } 1 \le i \le \left\lceil \frac{n}{2} \right\rceil, \\ 0, & \text{if } \left\lceil \frac{n}{2} \right\rceil + 1 \le i \le n. \end{cases}$$

Then

$$E_0 = \left\{ u_{\lceil \frac{n}{2} \rceil + 1} u_{\lceil \frac{n}{2} \rceil + 2}, u_{\lceil \frac{n}{2} \rceil + 2} u_{\lceil \frac{n}{2} \rceil + 3}, \dots, u_{n-1} u_n \right\},$$

$$E_1 = \left\{ u_n u_1, u_{\lceil \frac{n}{2} \rceil} u_{\lceil \frac{n}{2} \rceil + 1} \right\}$$

and

$$E_2 = \left\{ u_1 u_2, u_2 u_3, \dots, u_{\lceil \frac{n}{2} \rceil - 1} u_{\lceil \frac{n}{2} \rceil} \right\}.$$

That is,

$$|E_0| = \left\lfloor \frac{n}{2} \right\rfloor - 1 \leq \left\lceil \frac{n}{2} \right\rceil - 1,$$
$$|E_1| = 2 \leq \left\lceil \frac{n}{2} \right\rceil - 1$$

and

$$|E_2| = \left\lceil \frac{n}{2} \right\rceil - 1.$$

Take $k = \left\lceil \frac{n}{2} \right\rceil$. Then, by Theorem 2.5, C_n has a total edge-irregular absolute difference k-labeling. Hence, $tades(C_n) = \left\lceil \frac{n}{2} \right\rceil$. □

Theorem 2.7. *Let $\Im_n$ denote the sun graph on $2n$ vertices. Then $tades(\Im_n) = n$ for $n > 3$.*

Proof. The vertex set of $\Im_n$ is $V(\Im_n) = \{u_i \mid 1 \leq i \leq n\} \cup \{u_i' \mid 1 \leq i \leq n\}$ and the edge set of $\Im_n$ is $E(\Im_n) = \{u_i u_{i+1} \mid 1 \leq i \leq n-1\} \cup \{u_i u_i' \mid 1 \leq i \leq n\}$. Then $|E| = 2n$. From the Theorem 2.1, we have $tades(\Im_n) \geq n$. Now, define the labeling $\phi : V(\Im_n) \to \{0, 1\}$ by

$$\phi(u_i) = \begin{cases} 1, & \text{if } 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil, \\ 0, & \text{if } \left\lceil \frac{n}{2} \right\rceil + 1 \leq i \leq n, \end{cases}$$

$$\phi(u_i') = \begin{cases} 1, & \text{if } 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor, \\ 0, & \text{if } \left\lfloor \frac{n}{2} \right\rfloor + 1 \leq i \leq n. \end{cases}$$

Then

$$E_0 = \begin{cases} \{u_i u_{i+1} \mid (\frac{n}{2}) + 1 \leq i \leq n-1\} \cup \{u_i u_i' \mid (\frac{n}{2}) + 1 \leq i \leq n\}, & \text{if } n \text{ is even}, \\ \{u_i u_{i+1} \mid \lceil \frac{n}{2} \rceil + 1 \leq i \leq n-1\} \cup \{u_i u_i' \mid \lfloor \frac{n}{2} \rfloor + 2 \leq i \leq n\}, & \text{if } n \text{ is odd}, \end{cases}$$

$$E_1 = \begin{cases} \{u_1 u_n, u_{\frac{n}{2}} u_{\frac{n}{2}+1}\}, & \text{if } n \text{ is even}, \\ \{u_1 u_n, u_{\lceil \frac{n}{2} \rceil} u_{\lceil \frac{n}{2} \rceil + 1}, u_{\lceil \frac{n}{2} \rceil} u'_{\lceil \frac{n}{2} \rceil}\}, & \text{if } n \text{ is odd}, \end{cases}$$

and

$$E_2 = \left\{ u_i u_{i+1} \middle| 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - 1 \right\} \cup \left\{ u_i u_i' \middle| 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \right\}.$$

Here

$$|E_0| = \begin{cases} n-1, & \text{if } n \text{ is even}, \\ n-2, & \text{if } n \text{ is odd}, \end{cases}$$

$$|E_1| = \begin{cases} 2, & \text{if } n \text{ is even}, \\ 3, & \text{if } n \text{ is odd}, \end{cases}$$

and

$$|E_2| = n - 1.$$

Then, by Theorem 2.5, $\Im_n$ has a total edge-irregular absolute difference k-labeling. Hence, $tades(\Im_n) = n$. □

Theorem 2.8. *Let $\boldsymbol{F}_n$ be the fan graph on $2n+1$ vertices, then $tades(\boldsymbol{F}_n) = \left\lceil \frac{3n}{2} \right\rceil$ for an odd integer n.*

Proof. Let $k = \left\lceil \frac{3n}{2} \right\rceil = \frac{3n+1}{2}$ for an odd integer n. The vertex set of $\boldsymbol{F}_n$ is

$$V(\boldsymbol{F}_n) = \{u, v_1, v_2, \dots, v_{2n}\}$$

and the edge set of $\boldsymbol{F}_n$ is

$$E(\boldsymbol{F}_n) = \{uv_i \mid 1 \le i \le 2n\} \cup \{v_{2i+1}v_{2i+2} \mid 0 \le i \le n-1\}.$$

From the Theorem 2.1, we have $tades(\boldsymbol{F}_n) \ge \lceil \frac{3n}{2} \rceil$. Now, define the labeling $\phi : V(\boldsymbol{F}_n) \to \{0, 1\}$ by $\phi(u) = 1$

$$\phi(v_i) = \begin{cases} 1, & \text{if } 1 \le i \le n, \\ 0, & \text{if } n+1 \le i \le 2n. \end{cases}$$

Then

$$\begin{aligned} E_0 =& \left\{ v_{2i+1}v_{2i+2} \,\middle|\, \frac{n+1}{2} \le i \le n-1 \right\}, \\ E_1 =& \{uv_i \mid n+1 \le i \le 2n\} \cup \{v_n v_{n+1}\} \end{aligned}$$

and

$$\begin{aligned} E_2 =& \{uv_i \mid 1 \le i \le n\} \cup \left\{ v_{2i+1}v_{2i+2} \,\middle|\, 0 \le i \le \frac{n-3}{2} \right\}, \\ |E_0| =& \frac{n-1}{2} \le k-1, \\ |E_1| =& n+1 \le k-1 \end{aligned}$$

and

$$|E_2| = \frac{3n+1}{2} - 1 = k - 1.$$

Then, by Theorem 2.5, $\boldsymbol{F}_n$ has a total edge-irregular absolute difference k-labeling. Hence, $tades(\boldsymbol{F}_n) = \left\lceil \frac{3n}{2} \right\rceil$. □

3. Open Problem and Conjectures

Problem. Determine $tades(\boldsymbol{F}_n)$ when n is even.

From our experience on this labeling, we propose the following conjectures.

Conjectures:

- for every tree T of maximum degree Δ on p vertices,

$$tades(T) = \max \left\{ \left\lceil \frac{p}{2} \right\rceil, \left\lceil \frac{\Delta+1}{2} \right\rceil \right\};$$

- for any graph G, $tes(G) \le tades(G)$.

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