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WHEN IS A BI-JORDAN HOMOMORPHISM BI-HOMOMORPHISM?

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Abstract. For Banach algebras $\mathcal{A}$ and $\mathcal{B}$, we show that if $\mathcal{U} = \mathcal{A}\times\mathcal{B}$ is commutative (weakly commutative), then each bi-Jordan homomorphism from $\mathcal{U}$ into a semisimple commutative Banach algebra $\mathcal{D}$ is a bi-homomorphism. We also prove the same result for 3-bi-Jordan homomorphism with the additional hypothesis that the Banach algebra $\mathcal{U}$ is unital.

1. Introduction

Let $\mathcal{A}$ and $\mathcal{B}$ be complex Banach algebras and $\varphi : \mathcal{A} \to \mathcal{B}$ be a linear map. Then φ is called an n-homomorphism if for all $a_1, a_2, ...a_n \in \mathcal{A}$,

$$\varphi(a_1a_2...a_n) = \varphi(a_1)\varphi(a_2)...\varphi(a_n).$$

The concept of an n-homomorphism was studied for complex algebras by Hejazian et al. in [5]. A 2-homomorphism is then just a homomorphism, in the usual sense. One may refer to [1] for certain properties of 3-homomorphisms.

The notion of n-Jordan homomorphisms was dealt with firstly by Herstein in [6]. A linear map φ between Banach algebras $\mathcal{A}$ and $\mathcal{B}$ is called an n-Jordan homomorphism if

$$\varphi(a^n) = \varphi(a)^n, \quad a \in \mathcal{A}.$$

A 2-Jordan homomorphism is called simply a Jordan homomorphism.

It is obvious that each n-homomorphism is an n-Jordan homomorphism, but in general the converse is false. The converse statement may be true under certain conditions. For example, it is shown in [2] that every n-Jordan homomorphism between two commutative Banach algebras is an n-homomorphism for $n \in \{2, 3, 4\}$,

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and this result extended to the case $n = 5$ in [3]. Lee in [7] generalized this result and proved it for all $n \in \mathbb{N}$. See also [4] for another proof of Lee's theorem.

Zelazko in [9] has given a characterization of Jordan homomorphism, that we mention in the following (see also [8]). We refer to [10] for another approach to the same result.

Theorem 1.1. *Suppose that $\mathcal{A}$ is a Banach algebra, which need not be commutative, and suppose that $\mathcal{B}$ is a semisimple commutative Banach algebra. Then each Jordan homomorphism $\varphi : \mathcal{A} \to \mathcal{B}$ is a homomorphism.*

Also it is shown in [11] that Theorem 1.1 is valid for 3-Jordan homomorphism with the extra condition that the Banach algebra $\mathcal{A}$ is unital. Some significant results concerning Jordan homomorphisms and their generalizations on Banach algebras obtained by the author in [12].

Throughout the paper, let $\mathcal{U} = \mathcal{A} \times \mathcal{B}$. Then $\mathcal{U}$ is a Banach algebra for the multiplication

$$(a, b)(x, y) = (ax, by), \quad (a, b), (x, y) \in \mathcal{U},$$

and with norm

$$\|(a, b)\| = \|a\| + \|b\|.$$

Let $\mathcal{D}$ be a complex Banach algebra. A bilinear map is a function $\varphi : \mathcal{U} \to \mathcal{D}$ such that for any $a \in \mathcal{A}$ the map $b \mapsto \varphi(a, b)$ is linear map from $\mathcal{B}$ to $\mathcal{D}$, and for any $b \in \mathcal{B}$ the map $a \mapsto \varphi(a, b)$ is linear map from $\mathcal{A}$ to $\mathcal{D}$.

A bilinear map φ is called an n-bi-homomorphism if for all $(a_i, b_i) \in \mathcal{U}$,

$$\varphi(a_1a_2...a_n, b_1b_2...b_n) = \varphi(a_1, b_1)\varphi(a_2, b_2)...\varphi(a_n, b_n),$$

and it is called an n-bi-Jordan homomorphism if

$$\varphi(a^n, b^n) = \varphi(a, b)^n, \quad (a, b) \in \mathcal{U}.$$

The concept of an n-bi-Jordan homomorphism introduced by the author in [13]. A (2-bi-Jordan) 2-bi-homomorphism is called simply a (bi-Jordan) bi-homomorphism.

It is obvious that each n-bi-homomorphism is n-bi-Jordan homomorphism, but in general the converse is not true.

Recently, the author proved [13] that every bi-Jordan homomorphism from unital commutative Banach algebra $\mathcal{U}$ into a semisimple commutative Banach algebra $\mathcal{D}$ is a bi-homomorphism.

In this paper, we extended this result for nonunital Banach algebra $\mathcal{U}$. We also prove the same result for 3-bi-Jordan homomorphism with the additional hypothesis that the Banach algebra $\mathcal{U}$ is unital.

2. Characterization of Bi-Jordan Homomorhisms

The following Theorem is the generalization of Theorem 4 of [13].

Theorem 2.1. *Every bi-Jordan homomorphism φ from commutative Banach algebra $\mathcal{U}$ into a semisimple commutative Banach algebra $\mathcal{D}$ is a bi-homomorphism.*

Proof. We first assume that $\mathcal{D} = \mathbb{C}$ and let $\varphi : \mathcal{U} \to \mathbb{C}$ be a bi-Jordan homomorphism. Then for all $(a, b) \in \mathcal{U}$,

$$\varphi(a^2, b^2) = \varphi(a, b)^2. \tag{2.1}$$

Replacing a by $a + x$ and b by $b + y$ in (2.1), gives

$$\varphi(a^2 + x^2 + 2ax, b^2 + y^2 + 2by) = \varphi(a + x, b + y)^2. \tag{2.2}$$

By Lemma 1 of [13], for all $(a, b), (x, y) \in \mathcal{U}$ we have

$$\varphi(a^2, by) = \varphi(a, b)\varphi(a, y) \quad \text{and} \quad \varphi(ax, b^2) = \varphi(a, b)\varphi(x, b). \tag{2.3}$$

It follows from (2.2) and (2.3) that

$$2\varphi(ax, by) = \varphi(a, b)\varphi(x, y) + \varphi(a, y)\varphi(x, b), \tag{2.4}$$

for all $(a, b), (x, y) \in \mathcal{U}$. Take $I = \varphi(a, b)\varphi(x, y)$, $J = \varphi(a, y)\varphi(x, b)$ and $t = I - J$. Then we get

$$t^2 = I^2 + J^2 - 2IJ, \quad 4\varphi(ax, by)^2 = I^2 + J^2 + 2IJ. \tag{2.5}$$

By (2.4) and (2.5), we deduce

$$\begin{aligned} 4\varphi(ax, by)^2 + t^2 &= 2(I^2 + J^2) \\ &= 2[\varphi(a, b)^2\varphi(x, y)^2 + \varphi(a, y)^2\varphi(x, b)^2] \\ &= 2[\varphi(a^2, b^2)\varphi(x^2, y^2) + \varphi(a^2, y^2)\varphi(x^2, b^2)] \\ &= 4\varphi(a^2x^2, b^2y^2) \\ &= 4\varphi(ax, by)^2. \end{aligned}$$

Hence, $t = 0$, which proves that $I = J$. Thus, by (2.4) we have

$$\varphi(ax, by) = \varphi(a, b)\varphi(x, y),$$

for all $(a, b), (x, y) \in \mathcal{U}$, so φ is a bi-homomorphism.

Now suppose that $\mathcal{D}$ is semisimple and commutative. Let $\mathfrak{M}(\mathcal{D})$ be the maximal ideal space of $\mathcal{D}$. We associate with each $f \in \mathfrak{M}(\mathcal{D})$ a function $\varphi_f : \mathcal{U} \to \mathbb{C}$ defined by

$$\varphi_f(a, b) := f(\varphi(a, b)), \quad (a, b) \in \mathcal{U}.$$

Pick $f \in \mathfrak{M}(\mathcal{D})$ arbitrary. It is easy to see that φ_f is a bi-Jordan homomorphism, so by the above argument it is a bi-homomorphism. Thus, by the definition of φ_f we have

$$f(\varphi(ax, by)) = f(\varphi(a, b))f(\varphi(x, y)) = f(\varphi(a, b)\varphi(x, y)).$$

Since $f \in \mathfrak{M}(\mathcal{D})$ was arbitrary and $\mathcal{D}$ is assumed to be semisimple,

$$\varphi(ax, by) = \varphi(a, b)\varphi(x, y),$$

for all $(a, b), (x, y) \in \mathcal{U}$. This complete the proof. □

A bilinear map $\varphi : \mathcal{U} \to \mathcal{D}$ is called co-bi-homomorphism if

$$\varphi(ax, by) = -\varphi(a, b)\varphi(x, y),$$

for all $(a, b), (x, y) \in \mathcal{U}$, and it is called co-bi-Jordan homomorphism if

$$\varphi(a^2, b^2) = -\varphi(a, b)^2, \quad (a, b) \in \mathcal{U}.$$

By a same method as Theorem 2.1, we have the following result for co-bi-Jordan homomorphisms.

Theorem 2.2. *Every co-bi-Jordan homomorphism from commutative Banach algebra $\mathcal{U}$ into a semisimple commutative Banach algebra $\mathcal{D}$ is a co-bi-homomorphism.*

We say that the Banach algebra $\mathcal{A}$ is weakly commutative if

$$(ax)^2 = a^2x^2 \quad \text{and} \quad ax^2a = x^2a^2,$$

for all $a, x, \in \mathcal{A}$. Clearly, every commutative Banach algebra is weakly commutative, but in general, the converse is false. For example, let

$$\mathcal{A} = \left\{ \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} : \quad a, b \in \mathbb{R} \right\}.$$

Then it is obvious to check that with the usual matrix product for all $x, y \in \mathcal{A}$,

$$(xy)^2 = x^2y^2 \quad \text{and} \quad xy^2x = y^2x^2.$$

Thus, $\mathcal{A}$ is weakly commutative, but it is neither unital nor commutative.

Note that a unital Banach algebra is weakly commutative if and only if it is commutative.

Lemma 2.1. *Let $\mathcal{U}$ be a weakly commutative Banach algebra, and $\varphi : \mathcal{U} \to \mathbb{C}$ be a bi-Jordan homomorphism. Then*

$$\varphi(ax, by) = \varphi(ax, yb) = \varphi(xa, by),$$

for all $(a, b), (x, y) \in \mathcal{U}$.

Proof. By Lemma 1 of [13],

$$\varphi(a^2, by + yb) = 2\varphi(a, b)\varphi(a, y), \quad (a, b), (a, y) \in \mathcal{U}. \tag{2.6}$$

Replacing a by ax in (2.6) we get

$$\varphi((ax)^2, by + yb) = 2\varphi(ax, b)\varphi(ax, y). \tag{2.7}$$

Replacing b by by and y by yb in (2.7), gives

$$\varphi((ax)^2, by^2b + yb^2y) = 2\varphi(ax, by)\varphi(ax, yb). \tag{2.8}$$

Since $\mathcal{U}$ is weakly commutative, by (2.8) we have

$$\begin{aligned}2\varphi(ax,by)\varphi(ax,yb) &= \varphi((ax)^2, by^2b + yb^2y)\\ &= \varphi((ax)^2, y^2b^2 + b^2y^2)\\ &= \varphi((ax)^2, b^2y^2) + \varphi((ax)^2, y^2b^2)\\ &= \varphi(ax,by)^2 + \varphi(ax,yb)^2.\end{aligned}$$

Thus,

$$\Big(\varphi(ax,by) - \varphi(ax,yb)\Big)^2 = 0,$$

which proves that

$$\varphi(ax,by) = \varphi(ax,yb),$$

for all $(a,b),(x,y) \in \mathcal{U}$. In a similar way, we can prove that $\varphi(ax,by) = \varphi(xa,by)$. This complete the proof. □

The next result is the generalization of Theorem 2.1.

Theorem 2.3. *Suppose that φ is a bi-Jordan homomorphism from weakly commutative Banach algebra $\mathcal{U}$ into a semisimple commutative Banach algebra $\mathcal{D}$. Then φ is a bi-homomorphism.*

Proof. We first assume that $\mathcal{D} = \mathbb{C}$ and let $\varphi : \mathcal{U} \to \mathbb{C}$ be a bi-Jordan homomorphism. Then for all $(a,b) \in \mathcal{U}$,

$$\varphi(a^2, b^2) = \varphi(a,b)^2. \tag{2.9}$$

Replacing a by $a+x$ and b by $b+y$ in (2.9), gives

$$\varphi(ax + xa, by + yb) = 2\varphi(a,b)\varphi(x,y) + 2\varphi(a,y)\varphi(x,b), \tag{2.10}$$

for all $(a,b),(x,y) \in \mathcal{U}$. It follows from (2.10) and Lemma 2.1 that

$$\begin{aligned}4\varphi(ax,by) &= \varphi(ax + xa, by + yb)\\ &= 2\varphi(a,b)\varphi(x,y) + 2\varphi(a,y)\varphi(x,b).\end{aligned}$$

Hence,

$$2\varphi(ax,by) = \varphi(a,b)\varphi(x,y) + \varphi(a,y)\varphi(x,b),$$

for all $(a,b),(x,y) \in \mathcal{U}$. Thus, the relation (2.4) in Theorem 2.1 holds. Now the rest of proof is similar to the proof of Theorem 2.1. □

As a consequence of Theorem 2.3 we have the next result.

Corollary 2.1. *Suppose that $\mathcal{U}$ is weakly commutative and $\varphi : \mathcal{U} \to \mathbb{C}$ satisfies*

$$|\varphi(ax,by) - \varphi(a,b)\varphi(x,y)| \le \delta(\|(a,b)\| + \|(x,y)\|), \tag{2.11}$$

for all $(a,b),(x,y) \in \mathcal{U}$ and for some $\delta \ge 0$. Then φ is a bi-homomorphism.

Proof. Replacing (x, y) by (a, b) in (2.11), gives

$$|\varphi(a^2, b^2) - \varphi(a, b)^2| \leq 2\delta(\|a\| + \|b\|), \tag{2.12}$$

for all $(a, b) \in \mathcal{U}$. Take $a = 2^n x$ and $b = 2^n y$ in (2.12), then

$$|\varphi(x^2, y^2) - \varphi(x, y)^2| \leq \frac{2^{n+1}\delta(\|x\| + \|y\|)}{2^{4n}} \to 0,$$

as $n \to \infty$. Hence,

$$\varphi(x^2, y^2) = \varphi(x, y)^2, \quad (x, y) \in \mathcal{U}.$$

Therefore, φ is a bi-Jordan and so it is a bi-homomorphism by Theorem 2.3. □

Example 2.1. Let

$$\mathcal{U} = \left\{ \left(\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} x & y \\ 0 & 0 \end{bmatrix} \right) : \quad a, b, x, y \in \mathbb{R} \right\}.$$

Then $\mathcal{U}$ is a weakly commutative Banach algebra, but it is not commutative. Hence by Theorem 2.3, each bi-Jordan homomorphism from $\mathcal{U}$ into a semisimple commutative Banach algebra $\mathcal{D}$ is a bi-homomorphism and via versa.

The commutativity of Banach algebra $\mathcal{D}$ in Theorem 2.3 is essential. For example, let

$$\mathcal{A} = \left\{ \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} : \quad a, b \in \mathbb{R} \right\},$$

as above and let $\mathcal{A}^\sharp$ be the unitization of $\mathcal{A}$ with the identity matrix as a unit. Set $\mathcal{U} = \mathcal{A} \times \mathcal{A}^\sharp$ and define $\varphi : \mathcal{U} \to \mathcal{A}$ by $\varphi(x, y) = xy$. Then for all $(x, y) \in \mathcal{U}$,

$$\varphi(x^2, y^2) = \varphi(x, y)^2.$$

Hence φ is bi-Jordan homomorphism, but it is not bi-homomorphism. Because, let

$$x = \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}, \quad y = \begin{bmatrix} c & d \\ 0 & 0 \end{bmatrix}, \quad m = \begin{bmatrix} s & t \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad n = I,$$

where I is the identity matrix. Then $(x, y), (m, n) \in \mathcal{U}$, but

$$\varphi(xm, yn) \neq \varphi(x, y)\varphi(m, n).$$

3. Chracterization of 3-bi-Jordan Homomorhisms

Clearly, the Banach algebra $\mathcal{U}$ is unital if and only if both $\mathcal{A}$ and $\mathcal{B}$ are unital. Without any confusion we denote by e, the unit element of both $\mathcal{A}$ and $\mathcal{B}$.

Lemma 3.1. *Let $\mathcal{U}$ be a unital commutative Banach algebra, and $\varphi : \mathcal{U} \to \mathbb{C}$ be a 3-bi-Jordan homomorphism. Then for all $(a, b) \in \mathcal{U}$,*

(a) $\varphi(a^3, b^2 + b) = \varphi(a, b)^2\varphi(a, e) + \varphi(a, b)\varphi(a, e)^2$,
(b) $\varphi(a^2 + a, b^3) = \varphi(a, b)^2\varphi(e, b) + \varphi(a, b)\varphi(e, b)^2$.

Proof. The proof is straightforward. □

Lemma 3.2. *By the hypotheses of above Lemma, for all $(a, b), (x, y) \in \mathcal{U}$,*

(a) $3\varphi(ax^2, b) = \varphi(a, b)\varphi(x, e)^2 + 2\varphi(a, e)\varphi(x, e)\varphi(x, b)$,
(b) $3\varphi(a, by^2) = \varphi(a, b)\varphi(e, y)^2 + 2\varphi(e, b)\varphi(e, y)\varphi(a, y)$.

Proof. We prove (a), that the assertion (b) can be proved similarly. Let $\varphi : \mathfrak{U} \to \mathbb{C}$ be a 3-bi-Jordan homomorphism. Then for all $(a, b) \in \mathfrak{U}$,

$$\varphi(a^3, b^3) = \varphi(a, b)^3. \tag{3.1}$$

Replacing b by $b + y$ in (3.1), gives

$$\varphi(a^3, b^2y + by^2) = \varphi(a, b)^2\varphi(a, y) + \varphi(a, b)\varphi(a, y)^2, \tag{3.2}$$

for all $(a, b), (a, y) \in \mathfrak{U}$. Replacing y by $-y$ in (3.2), we get

$$\varphi(a^3, -b^2y + by^2) = -\varphi(a, b)^2\varphi(a, y) + \varphi(a, b)\varphi(a, y)^2. \tag{3.3}$$

By (3.2) and (3.3) we have

$$\varphi(a^3, by^2) = \varphi(a, b)\varphi(a, y)^2. \tag{3.4}$$

Replacing y by e in (3.4), gives

$$\varphi(a^3, b) = \varphi(a, b)\varphi(a, e)^2. \tag{3.5}$$

Replacing a by $a + x$ in (3.5), to obtain

$$3\varphi(ax^2 + a^2x, b) = I + J, \tag{3.6}$$

where

$$I = \varphi(x, b)\varphi(a, e)^2 + 2\varphi(a, b)\varphi(a, e)\varphi(x, e)$$

and

$$J = \varphi(a, b)\varphi(x, e)^2 + 2\varphi(a, e)\varphi(x, b)\varphi(x, e).$$

Replacing x by $-x$ in (3.6), we get

$$3\varphi(ax^2 - a^2x, b) = -I + J. \tag{3.7}$$

By (3.6) and (3.7) we have

$$3\varphi(ax^2, b) = \varphi(a, b)\varphi(x, e)^2 + 2\varphi(a, e)\varphi(x, b)\varphi(x, e),$$

for all $(a, b), (x, e) \in \mathfrak{U}$, as required. □

Now we state and prove the main Theorem of this section.

Theorem 3.1. *Suppose that φ is a 3-bi-Jordan homomorphism from unital commutative Banach algebra $\mathfrak{U}$ into $\mathbb{C}$. Then φ is a 3-bi-homomorphism.*

Proof. Let $\varphi : \mathfrak{U} \to \mathbb{C}$ be a 3-bi-Jordan homomorphism. Then

$$\varphi(a^3, b^3) = \varphi(a, b)^3, \quad (a, b) \in \mathfrak{U}. \tag{3.8}$$

Replacing both of a and b by e, gives $\varphi(e, e) = \varphi(e, e)^3$. Since $\varphi(e, e) \neq 0$, so $\varphi(e, e) = 1$ or $\varphi(e, e) = -1$. We first assume that $\varphi(e, e) = 1$. Replacing a by $a + e$ and b by $b + e$ in (3.8), and simplifies the result by Lemma 3.1, we get

$$9\varphi(a^2 + a, b^2 + b) = 3\{\varphi(a, b)^2 + \varphi(a, b) + P + Q + R + S\}, \tag{3.9}$$

where

$$P = 2\varphi(a,b)\varphi(a,e) + \varphi(e,b)\varphi(a,e)^2, \qquad Q = 2\varphi(a,b)\varphi(e,b) + \varphi(a,e)\varphi(e,b)^2,$$
$$R = 2\varphi(a,e)\varphi(e,b), \qquad S = 2\varphi(a,e)\varphi(a,b)\varphi(e,b).$$

It follows from preceding Lemma that for all $(a,b) \in \mathcal{U}$,

$$P = 3\varphi(a^2,b), \quad Q = 3\varphi(a,b^2), \quad R = 2\varphi(a,b) \quad \text{and} \quad S = 2\varphi(a,b)^2. \tag{3.10}$$

By (3.9) and (3.10) we obtain

$$\varphi(a^2,b^2) = \varphi(a,b)^2,$$

for all $(a,b) \in \mathcal{U}$. Hence, φ is bi-Jordan homomorphism and so it is bi-homomorphism by Theorem 2.1. Thus, φ is 3-bi-homomorphism.

Now suppose that $\varphi(e,e) = -1$. Then by a similar arqument we have

$$\varphi(a^2,b^2) = -\varphi(a,b)^2, \quad (a,b) \in \mathcal{U}.$$

Therefore by Theorem 2.2, φ is co-bi-homomorphism. That is,

$$\varphi(ax,by) = -\varphi(a,b)\varphi(x,y),$$

for all $(a,b),(x,y) \in \mathcal{U}$. Thus,

$$\begin{aligned}\varphi(axu,byv) &= -\varphi(a,b)[\varphi(xu,yv)]\\ &= -\varphi(a,b)[-\varphi(x,y)\varphi(u,v)]\\ &= \varphi(a,b)\varphi(x,y)\varphi(u,v),\end{aligned}$$

for all $(a,b),(x,y),(u,v) \in \mathcal{U}$. So φ is 3-bi-homomorphism, as claimed. □

As a consequence of Theorem 3.1 we have the next result.

Corollary 3.1. *Suppose that φ is a 3-bi-Jordan homomorphism from unital commutative Banach algebra $\mathcal{U}$ into a semisimple commutative Banach algebra $\mathcal{D}$. Then φ is a 3-bi-homomorphism.*

In view of Theorem 1.1 and Theorem 2.1, the following question suggests itself: does Theorem 2.1 hold without commutativity of $\mathcal{U}$?

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