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SUPER MEAN LABELING OF SOME SUBDIVISION GRAPHS

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Abstract. Let G be a graph and $f : V(G) \to \{1, 2, 3, \ldots, p + q\}$ be an injection. For each edge $e = uv$, the induced edge labeling f^* is defined as follows:

$$f^*(e) = \begin{cases} \frac{f(u)+f(v)}{2}, & \text{if } f(u) + f(v) \text{ is even,} \\ \frac{f(u)+f(v)+1}{2}, & \text{if } f(u) + f(v) \text{ is odd.} \end{cases}$$

Then f is called super mean labeling if $f(V(G)) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \ldots, p + q\}$. A graph that admits a super mean labeling is called super mean graph. In this paper, we have studied the super meanness property of the subdivision of the H-graph H_n, $H_n \odot K_1$, $H_n \odot S_2$, slanting ladder, $T_n \odot K_1$, $C_n \odot K_1$ and $C_n @ C_m$.

1. Introduction

Throughout this paper, by a graph we mean a finite, undirected and simple graph. Let $G(V, E)$ be a graph with p vertices and q edges. For notations and terminology we follow [2].

The path on n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n. A triangular snake is obtained from a path by identifying each edge of the path with an edge of the cycle C_3. The graph $C_m @ C_n$ is obtained by identifying an edge of C_m with an edge of C_n. The slanting ladder SL_n is a graph obtained from two paths $u_1u_2 \ldots u_n$ and $v_1v_2 \ldots v_n$ by joining each u_i with v_{i+1}, $1 \leq i \leq n-1$. The H-graph of a path P_n, denoted by H_n is the graph obtained from two copies of P_n with vertices $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd and the vertices $v_{\frac{n}{2}+1}$ and $u_{\frac{n}{2}}$ if n is even. The corona of a graph G on p vertices $v_1, v_2, \ldots, v_p$ is the graph obtained from G by adding p new vertices $u_1, u_2, \ldots, u_p$

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and the new edges u_iv_i for $1 \leq i \leq p$. The corona of G is denoted by $G \odot K_1$. The 2-corona of a graph G, denoted by $G \odot S_2$ is a graph obtained from G by identifying the center vertex of the star S_2 at each vertex of G. A graph which can be obtained from a given graph by breaking up each edge into one or more segments by inserting intermediate vertices between its two ends. If each edge of a graph G is broken into two by exactly one vertex, then the resultant graph is taken as $S(G)$.

A vertex labeling of G is an assignment $f : V(G) \to \{1, 2, \ldots, p+q\}$ be an injection. For a vertex labeling f, the induced edge labeling $f^*(e = uv)$ is defined by

$$f^*(e) = \begin{cases} \frac{f(u)+f(v)}{2}, & \text{if } f(u) + f(v) \text{ is even,} \\ \frac{f(u)+f(v)+1}{2}, & \text{if } f(u) + f(v) \text{ is odd.} \end{cases}$$

Then f is called super mean labeling if

$$f(V(G)) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \ldots, p + q\}.$$

Clearly f^* is injective. A graph that admits a super mean labeling is called super mean graph.

A super mean labeling of the graph P_7^2 is shown in Figure 1.

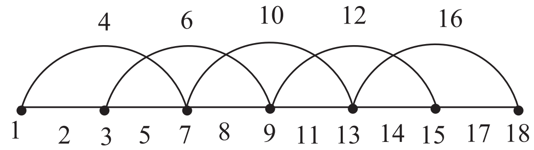


FIGURE 1

The concept of mean labeling was introduced and studied by S. Somasundaram and R. Ponraj [5]. Some new families of mean graphs are discussed in [10, 11].

The concept of super mean labeling was introduced and studied by D. Ramya et al. [4]. Further some more results on super mean graphs are discussed in [1, 3, 6–9].

In this paper, we have studied the super meanness of the subdivision of the graphs H-graph H_n, $H_n \odot K_1$, $H_n \odot S_2$, slanting ladder, $T_n \odot K_1$, $C_n \odot K_1$ and $C_n@C_m$.

2. SUPER MEAN GRAPHS

Theorem 2.1. *The graph $S(H_n)$ is a super mean graph, for $n \geq 3$.*

Proof. Let $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ be the vertices of the paths of length $n - 1$. Each edge u_iu_{i+1} is subdivided by a vertex x_i, $1 \leq i \leq n - 1$ and each edge v_iv_{i+1} is subdivided by a vertex y_i, $1 \leq i \leq n - 1$. The edge $u_{\frac{n+1}{2}}v_{\frac{n+1}{2}}$ is divided by a vertex z when n is odd. The edge $u_{\frac{n+2}{2}}v_{\frac{n}{2}}$ is divided by a vertex z when n is even. The graph $S(H_n)$ has $4n - 1$ vertices and $4n - 2$ edges.

Define $f : V(S(H_n)) \to \{1, 2, 3, \ldots, p + q = 8n - 3\}$ as follows:

$$f(u_i) = 4i - 3, \quad 1 \leq i \leq n,$$

$$f(v_i) = \begin{cases} 4(n+i)-5, & 1 \le i \le \left\lfloor \frac{n-1}{2} \right\rfloor, \\ 4(n+i)-3, & \left\lfloor \frac{n+1}{2} \right\rfloor \le i \le n, \end{cases}$$

$$f(x_i) = 4i-1, \quad 1 \le i \le n-1,$$

$$f(y_i) = \begin{cases} 4(n+i)-3, & 1 \le i \le \left\lfloor \frac{n-1}{2} \right\rfloor, \\ 4(n+i)-1, & \left\lfloor \frac{n+1}{2} \right\rfloor \le i \le n-1, \end{cases}$$

$$\text{and } f(z) = \begin{cases} 6n-4, & \text{if } n \text{ is odd}, \\ 6n-6, & \text{if } n \text{ is even}. \end{cases}$$

For the vertex labeling f, the induced edge labeling is given as follows:

$$f^*(u_i x_i) = 4i-2, \quad 1 \le i \le n-1,$$

$$f^*(x_i u_{i+1}) = 4i, \quad 1 \le i \le n-1,$$

$$f^*(v_i y_i) = \begin{cases} 4(n+i)-4, & 1 \le i \le \left\lfloor \frac{n-1}{2} \right\rfloor, \\ 4(n+i)-2, & \left\lfloor \frac{n+1}{2} \right\rfloor \le i \le n-1, \end{cases}$$

$$f^*(y_i v_{i+1}) = \begin{cases} 4(n+i)-2, & 1 \le i \le \left\lfloor \frac{n-3}{2} \right\rfloor, \\ 6n-3, & i = \frac{n-1}{2} \text{ and } n \text{ is odd}, \\ 6n-5, & i = \frac{n-2}{2} \text{ and } n \text{ is even}, \\ 4(n+i), & \left\lfloor \frac{n+1}{2} \right\rfloor \le i \le n-1, \end{cases}$$

$$f^*\left(u_{\frac{n+1}{2}} z\right) = 4n-2, \quad \text{if } n \text{ rm is odd},$$

$$f^*\left(z v_{\frac{n+1}{2}}\right) = 6n-2, \quad \text{if } n \text{ is odd},$$

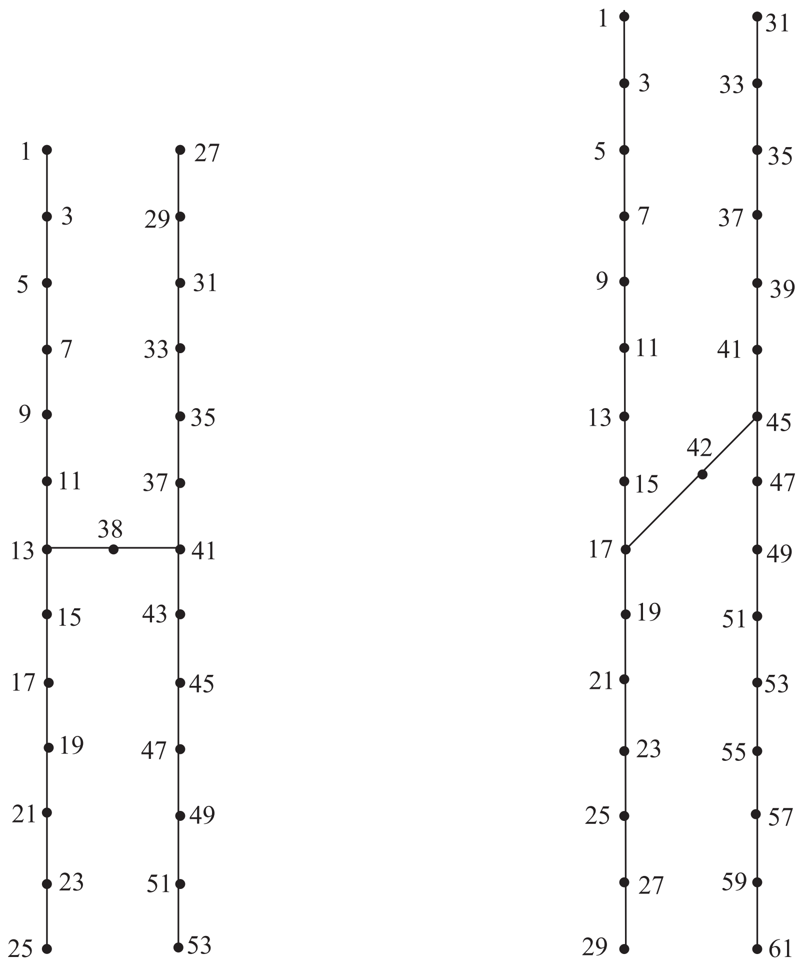
$$f^*\left(u_{\frac{n+2}{2}} z\right) = 4n-2, \quad \text{if } n \text{ is even},$$

$$\text{and } f^*\left(z v_{\frac{n}{2}}\right) = 6n-4, \quad \text{if } n \text{ is even}.$$

Thus, f is a super mean labeling and hence $S(H_n)$ is a super mean graph.
For example, a super mean labeling of $S(H_7)$ and $S(H_8)$ are shown in Figure 2. □

Theorem 2.2. *The graph $S(H_n \odot K_1)$ is a super mean graph, for $n \ge 3$.*

Proof. Let $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ be the vertices of the paths of length $n-1$. Let $a_{1,i}a_{2,i}u_i$ be the path attached at each $u_i, 1 \le i \le n$ and $b_{1,i}b_{2,i}v_i$ be the path attached at each $v_i, 1 \le i \le n$. Each edge $u_i u_{i+1}$ is subdivided by a vertex x_i, $1 \le i \le n-1$ and each edge $v_i v_{i+1}$ is subdivided by a vertex y_i, $1 \le i \le n-1$. The edge $u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}$ is divided by a vertex z when n is odd. The edge $u_{\frac{n+2}{2}} v_{\frac{n}{2}}$ is divided by a vertex z when n is even. The graph $S(H_n \odot K_1)$ has $8n-1$ vertices and $8n-2$ edges.



(a) $S(H_7)$ (b) $S(H_8)$

FIGURE 2

Define $f : V(S(H_n \odot K_1)) \to \{1, 2, 3, \ldots, p + q = 16n - 3\}$ as follows:

$$f(u_i) = \begin{cases} 5, & i = 1, \\ 8i - 7, & 2 \leq i \leq n, \end{cases}$$

$$f(v_i) = \begin{cases} 8n + 3, & i = 1, \\ 8(n + i) - 9, & 2 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor, \\ 8(n + i) - 7, & \left\lfloor \frac{n+1}{2} \right\rfloor \leq i \leq n, \end{cases}$$

$$f(a_{1,i}) = \begin{cases} 1, & i = 1, \\ 8i - 2, & 2 \leq i \leq n, \end{cases}$$

$$f(a_{2,i}) = 8i - 5, \quad 1 \leq i \leq n.$$

$$f(b_{1,i}) = \begin{cases} 8n-1, & i=1, \\ 8(n+i)-4, & 2 \le i \le \left\lfloor \frac{n-1}{2} \right\rfloor, \\ 8(n+i)-2, & \left\lfloor \frac{n+1}{2} \right\rfloor \le i \le n-1, \\ 16n-3, & i=n, \end{cases}$$

$$f(b_{2,i}) = \begin{cases} 8(n+i)-7, & 1 \le i \le \left\lfloor \frac{n-1}{2} \right\rfloor, \\ 8(n+i)-5, & \left\lfloor \frac{n+1}{2} \right\rfloor \le i \le n, \end{cases}$$

$$f(x_i) = 8i-1, \quad 1 \le i \le n-1,$$

$$f(y_i) = \begin{cases} 8(n+i)-3, & 1 \le i \le \left\lfloor \frac{n-1}{2} \right\rfloor, \\ 8(n+i)-1, & \left\lfloor \frac{n+1}{2} \right\rfloor \le i \le n-1, \end{cases}$$

$$\text{and } f(z) = \begin{cases} 12n-6, & \text{if } n \text{ is odd,} \\ 12n-10, & \text{if } n \text{ is even.} \end{cases}$$

The induced edge labeling is obtained as follows:

$$f^*(u_i x_i) = \begin{cases} 6, & i=1, \\ 8i-4, & 2 \le i \le n-1, \end{cases}$$

$$f^*(x_i u_{i+1}) = 8i, \quad 1 \le i \le n-1,$$

$$f^*(v_i y_i) = \begin{cases} 8n+4, & i=1, \\ 8(n+i)-6, & 2 \le i \le \left\lfloor \frac{n-1}{2} \right\rfloor, \\ 8(n+i)-4, & \left\lfloor \frac{n+1}{2} \right\rfloor \le i \le n-1, \end{cases}$$

$$f^*(y_i v_{i+1}) = \begin{cases} 8(n+i)-2, & 1 \le i \le \left\lfloor \frac{n-3}{2} \right\rfloor, \\ 12n-5, & i = \frac{n-1}{2} \text{ and } n \text{ is odd,} \\ 12n-9 & i = \frac{n-2}{2} \text{ and } n \text{ is even,} \\ 8(n+i), & \left\lfloor \frac{n+1}{2} \right\rfloor \le i \le n-1, \end{cases}$$

$$f^*(a_{1,i} a_{2,i}) = \begin{cases} 2, & i=1, \\ 8i-3, & 2 \le i \le n, \end{cases}$$

$$f^*(a_{2,i} u_i) = \begin{cases} 4, & i=1, \\ 8i-6, & 2 \le i \le n, \end{cases}$$

$$f^*(b_{1,i} b_{2,i}) = \begin{cases} 8n, & i=1 \\ 8(n+i)-5, & 2 \le i \le \left\lfloor \frac{n-1}{2} \right\rfloor, \\ 8(n+i)-3, & \left\lfloor \frac{n+1}{2} \right\rfloor \le i \le n-1, \\ 16n-4, & i=n, \end{cases}$$

$$f^*(b_{2,i}v_i) = \begin{cases} 8n+2, & i=1, \\ 8(n+i)-8, & 2 \le i \le \left\lfloor \frac{n-1}{2} \right\rfloor, \\ 8(n+i)-6, & \left\lfloor \frac{n+1}{2} \right\rfloor \le i \le n, \end{cases}$$

$$f^*\left(u_{\frac{n+1}{2}}z\right) = 8n-4, \quad \text{if } n \text{ is odd,}$$

$$f^*\left(zv_{\frac{n+1}{2}}\right) = 12n-4, \quad \text{if } n \text{ is odd.}$$

$$f^*\left(u_{\frac{n+2}{2}}z\right) = 8n-4, \quad \text{if } n \text{ is even,}$$

$$f^*\left(zv_{\frac{n}{2}}\right) = 12n-8, \quad \text{if } n \text{ is even.}$$

Thus, f is a super mean labeling and hence $S(H_n \odot K_1)$ is a super mean graph.

For example, a super mean labeling of $S(H_9 \odot K_1)$ and $S(H_{10} \odot K_1)$ are shown in Figure 3. □

Theorem 2.3. *The graph $S(H_n \odot S_2)$ is a super mean graph, for $n \ge 3$.*

Proof. Let $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ be the vertices of the paths of length $n-1$. Let $a_{1,i}a_{2,i}u_i$ and $a_{3,i}a_{4,i}u_i$ be the paths attached at each u_i, $1 \le i \le n$ and $b_{1,i}b_{2,i}v_i$ and $b_{3,i}b_{4,i}v_i$ be the paths attached at each v_i, $1 \le i \le n$. Each edge u_iu_{i+1} is subdivided by a vertex x_i, $1 \le i \le n-1$ and each edge v_iv_{i+1} is subdivided by a vertex y_i, $1 \le i \le n-1$. The edge $u_{\frac{n+1}{2}}v_{\frac{n+1}{2}}$ is divided by a vertex z when n is odd. The edge $u_{\frac{n+2}{2}}v_{\frac{n}{2}}$ is divided by a vertex z when n is even. The graph $S(H_n \odot S_2)$ has $12n-1$ vertices and $12n-2$ edges.

Define $f : V(S(H_n \odot S_2)) \to \{1, 2, 3, \ldots, p+q = 24n-3\}$ as follows:

$$f(u_i) = 12i-7, \quad 1 \le i \le n,$$

$$f(v_i) = \begin{cases} 12(n+i)-9, & 1 \le i \le \left\lfloor \frac{n-1}{2} \right\rfloor, \\ 12(n+i)-7, & \left\lfloor \frac{n+1}{2} \right\rfloor \le i \le n, \end{cases}$$

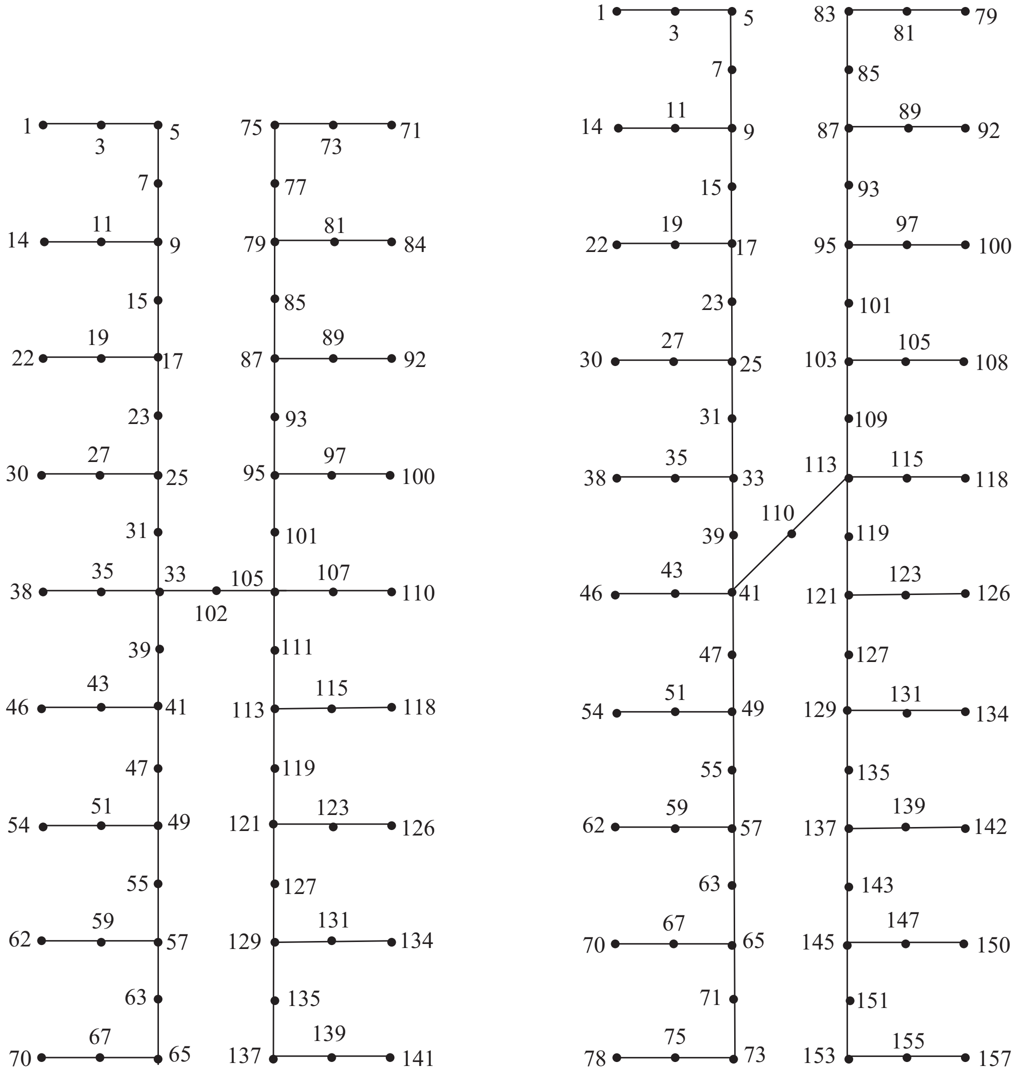
$$f(a_{1,i}) = \begin{cases} 1, & i=1, \\ 12i-13, & 2 \le i \le n, \end{cases}$$

$$f(a_{2,i}) = \begin{cases} 3, & i=1, \\ 12i-11, & 2 \le i \le n, \end{cases}$$

$$f(a_{3,i}) = 12i-3, \quad 1 \le i \le n,$$

$$f(a_{4,i}) = 12i-5, \quad 1 \le i \le n,$$

$$f(x_i) = 12i+2, \quad 1 \le i \le n-1,$$



(a) $S(H_9 \odot K_1)$ (b) $S(H_10 \odot K_1)$

FIGURE 3

$$f(b_{1,i}) = \begin{cases} 12n-1, & i=1, \\ 12(n+i)-15, & 2 \le i \le \left\lfloor \frac{n-1}{2} \right\rfloor, \\ 18n-8, & i=\frac{n+1}{2} \text{ and } n \text{ is odd}, \\ 18n-14, & i=\frac{n}{2} \text{ and } n \text{ is even}, \\ 12(n+i)-13, & \left\lfloor \frac{n+3}{2} \right\rfloor \le i \le n, \end{cases}$$

$$f(b_{2,i}) = \begin{cases} 12n+1, & i=1, \\ 12(n+i)-13, & 2 \le i \le \lfloor \frac{n-1}{2} \rfloor, \\ 18n-6, & i=\frac{n+1}{2} \text{ and } n \text{ is odd}, \\ 18n-12, & i=\frac{n}{2} \text{ and } n \text{ is even}, \\ 12(n+i)-11, & \lfloor \frac{n+3}{2} \rfloor \le i \le n, \end{cases}$$

$$f(b_{3,i}) = \begin{cases} 12(n+i)-5, & 1 \le i \le \lfloor \frac{n-3}{2} \rfloor, \\ 18n-10, & i=\frac{n-1}{2} \text{ and } n \text{ is odd}, \\ 18n-16, & i=\frac{n-2}{2} \text{ and } n \text{ is even}, \\ 12(n+i)-3, & \lfloor \frac{n+1}{2} \rfloor \le i \le n, \end{cases}$$

$$f(b_{4,i}) = \begin{cases} 12(n+i)-7, & 1 \le i \le \lfloor \frac{n-1}{2} \rfloor, \\ 12(n+i)-5, & \lfloor \frac{n+1}{2} \rfloor \le i \le n, \end{cases}$$

$$f(z) = \begin{cases} 18n-4, & \text{if } n \text{ is odd}, \\ 18n-10, & \text{if } n \text{ is even}, \end{cases}$$

$$\text{and } f(y_i) = \begin{cases} 12(n+i), & 1 \le i \le \lfloor \frac{n-3}{2} \rfloor, \\ 18n-9, & i=\frac{n-1}{2} \text{ and } n \text{ is odd}, \\ 18n-15, & i=\frac{n-2}{2} \text{ and } n \text{ is even}, \\ 12(n+i)+2, & \lfloor \frac{n+1}{2} \rfloor \le i \le n-1. \end{cases}$$

For the vertex labeling f, the induced edge labels are obtained as follows:

$$f^*(a_{1,i}a_{2,i}) = \begin{cases} 2, & i=1, \\ 12(i-1), & 2 \le i \le n, \end{cases}$$

$$f^*(a_{2,i}u_i) = \begin{cases} 4, & i=1, \\ 12i-9, & 2 \le i \le n, \end{cases}$$

$$f^*(a_{3,i}a_{4,i}) = 12i-4, \quad 1 \le i \le n,$$

$$f^*(a_{4,i}u_i) = 12i-6, \quad 1 \le i \le n,$$

$$f^*(u_ix_i) = 12i-2, \quad 1 \le i \le n-1,$$

$$f^*(x_iu_{i+1}) = 12i+4, \quad 1 \le i \le n-1,$$

$$f^*(b_{1,i}b_{2,i}) = \begin{cases} 12n, & i=1, \\ 12(n+i)-14, & 2 \le i \le \lfloor \frac{n-1}{2} \rfloor, \\ 18n-7, & i=\frac{n+1}{2} \text{ and } n \text{ is odd}, \\ 18n-13, & i=\frac{n}{2} \text{ and } n \text{ is even}, \\ 12(n+i)-12, & \lfloor \frac{n+3}{2} \rfloor \le i \le n, \end{cases}$$

$$f^*(b_{2,i}v_i) = \begin{cases} 12n+2, & i=1, \\ 12(n+i)-11, & 2 \le i \le \left\lfloor \frac{n-1}{2} \right\rfloor, \\ 12(n+i)-9, & \left\lfloor \frac{n+1}{2} \right\rfloor \le i \le n, \end{cases}$$

$$f^*(b_{3,i}b_{4,i}) = \begin{cases} 12(n+i)-6, & 1 \le i \le \left\lfloor \frac{n-3}{2} \right\rfloor, \\ 18n-11, & i = \frac{n-1}{2} \text{ and } n \text{ is odd}, \\ 18n-17, & i = \frac{n-2}{2} \text{ and } n \text{ is even}, \\ 12(n+i)-4, & \left\lfloor \frac{n+1}{2} \right\rfloor \le i \le n, \end{cases}$$

$$f^*(b_{4,i}v_i) = \begin{cases} 12(n+i)-8, & 1 \le i \le \left\lfloor \frac{n-1}{2} \right\rfloor, \\ 12(n+i)-6, & \left\lfloor \frac{n+1}{2} \right\rfloor \le i \le n, \end{cases}$$

$$f^*(v_iy_i) = \begin{cases} 12(n+i)-4, & 1 \le i \le \left\lfloor \frac{n-3}{2} \right\rfloor, \\ 18n-12, & i = \frac{n-1}{2} \text{ and } n \text{ is odd}, \\ 18n-18, & i = \frac{n-2}{2} \text{ and } n \text{ is even}, \\ 12(n+i)-2, & \left\lfloor \frac{n+1}{2} \right\rfloor \le i \le n-1, \end{cases}$$

$$f^*(y_iv_{i+1}) = \begin{cases} 12(n+i)+2, & 1 \le i \le \left\lfloor \frac{n-3}{2} \right\rfloor, \\ 18n-5, & i = \frac{n-1}{2} \text{ and } n \text{ is odd}, \\ 18n-11, & i = \frac{n-2}{2} \text{ and } n \text{ is even}, \\ 12(n+i)+4, & \left\lfloor \frac{n+1}{2} \right\rfloor \le i \le n-1, \end{cases}$$

$$f^*\left(u_{\frac{n+1}{2}}z\right) = 12n-2 \text{ if } n \text{ is odd},$$

$$f^*\left(zv_{\frac{n+1}{2}}\right) = 18n-2 \text{ if } n \text{ is even},$$

$$f^*\left(u_{\frac{n+2}{2}}z\right) = 12n-2 \text{ if } n \text{ is odd},$$

$$\text{and } f^*\left(zv_{\frac{n}{2}}\right) = 18n-8 \text{ if } n \text{ is even}.$$

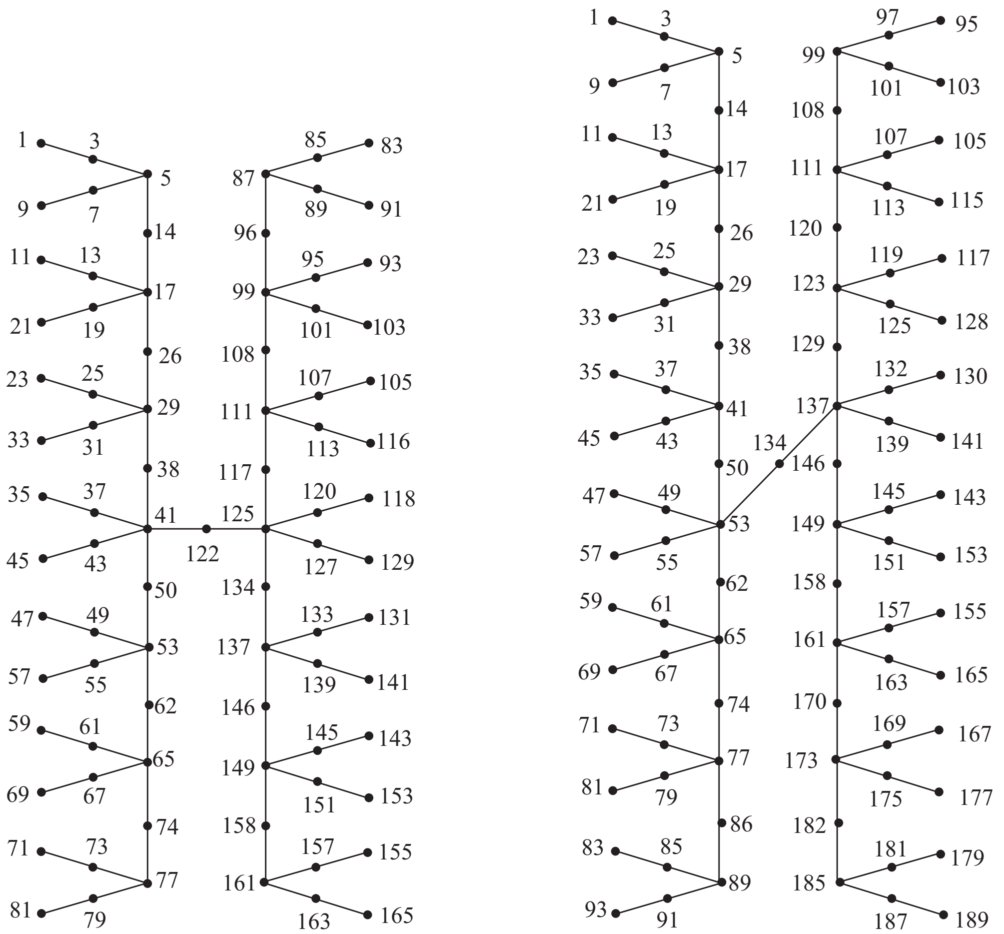
Thus, f is a super mean labeling and hence $S(H_n \odot S_2)$ is a super mean graph.

For example, a super mean labeling of $S(H_7 \odot S_2)$ and $S(H_8 \odot S_2)$ are shown in Figure 4. □

Theorem 2.4. *The graph $S(SL_n)$ is a super mean graph, for $n \ge 2$.*

Proof. Let $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ be the vertices on the paths of length $n-1$. Let x_i, y_i and z_i be the vertices subdivided the edges u_iu_{i+1}, v_iv_{i+1} and v_iu_{i+1} respectively for each i, $1 \le i \le n-1$. The graph $S(SL_n)$ has $5n-3$ vertices and $6n-6$ edges.

Case (i): n is odd.



(a) $S(H_7 \odot S_2)$ (b) $S(H_8 \odot S_2)$

FIGURE 4

Define $f : V(S(SL_n)) \to \{1, 2, \ldots, p+q = 11n-9\}$ as follows:

$$f(u_i) = \begin{cases} 1, & i = 1, \\ 5, & i = 2, \\ 13, & i = 3, \\ 11i-13, & 4 \le i \le n \text{ and } i \text{ is even}, \\ 11i-19, & 4 \le i \le n \text{ and } i \text{ is odd}, \end{cases}$$

$$f(v_i) = \begin{cases} 11, & i = 1, \\ 11i-2, & 2 \le i \le n-1 \text{ and } i \text{ is even}, \\ 11i-8, & 2 \le i \le n-1 \text{ and } i \text{ is odd}, \\ 11n-9, & i = n, \end{cases}$$

$$f(x_i) = \begin{cases} 3, & i = 1, \\ 10, & i = 2, \\ 11i - 5, & 3 \leq i \leq n-1 \text{ and } i \text{ is odd}, \\ 11i - 10, & 3 \leq i \leq n-1 \text{ and } i \text{ is even}, \end{cases}$$

$$f(y_i) = \begin{cases} 11i + 6, & 1 \leq i \leq n-2 \text{ and } i \text{ is odd}, \\ 11i + 1, & 1 \leq i \leq n-2 \text{ and } i \text{ is even}, \end{cases}$$

$$f(y_{n-1}) = 11(n-1),$$

$$f(z_i) = \begin{cases} 7, & i = 1, \\ 11i - 6, & 2 \leq i \leq n-1. \end{cases}$$

For the vertex labeling f, the induced edge labeling f^* is given follows:

$$f^*(u_i x_i) = \begin{cases} 2, & i = 1, \\ 8, & i = 2, \\ 11i - 12, & 3 \leq i \leq n-2 \text{ and } i \text{ is odd}, \\ 11i - 11, & 3 \leq i \leq n-2 \text{ and } i \text{ is even}, \end{cases}$$

$$f^*(x_i u_{i+1}) = \begin{cases} 4, & i = 1, \\ 12, & i = 2, \\ 11i - 3, & 3 \leq i \leq n-1 \text{ and } i \text{ is odd}, \\ 11i - 9, & 3 \leq i \leq n-1 \text{ and } i \text{ is even}, \end{cases}$$

$$f^*(v_i y_i) = \begin{cases} 14, & i = 1, \\ 11i, & 2 \leq i \leq n-2 \text{ and } i \text{ is even}, \\ 11i - 1, & 2 \leq i \leq n-2 \text{ and } i \text{ is odd}, \\ 11n - 12, & i = n-1, \end{cases}$$

$$f^*(y_i v_{i+1}) = \begin{cases} 11i + 8, & 1 \leq i \leq n-2 \text{ and } i \text{ is odd}, \\ 11i + 2, & 1 \leq i \leq n-2 \text{ and } i \text{ is even}, \\ 11n - 10, & i = n-1, \end{cases}$$

$$f^*(v_i z_i) = \begin{cases} 9, & i = 1, \\ 11i - 4, & 2 \leq i \leq n-1 \text{ and } i \text{ is even}, \\ 11i - 6, & 2 \leq i \leq n-1 \text{ and } i \text{ is odd}, \end{cases}$$

$$f^*(z_i u_{i+1}) = \begin{cases} 6, & i = 1, \\ 11i - 7, & 2 \leq i \leq n-1 \text{ and } i \text{ is even}, \\ 11i - 4, & 2 \leq i \leq n-1 \text{ and } i \text{ is odd}. \end{cases}$$

Case (ii): n is even, $n \geq 4$.

Define $f : V(S(SL_n)) \to \{1, 2, \ldots, p+q = 11n-9\}$ as follows:

$$f(u_i) = \begin{cases} 1, & i = 1, \\ 5, & i = 2, \\ 13, & i = 3, \\ 11i-13, & 4 \leq i \leq n-1 \text{ and } i \text{ is even}, \\ 11i-19, & 4 \leq i \leq n-1 \text{ and } i \text{ is odd}, \\ 11n-11, & i = n, \end{cases}$$

$$f(v_i) = \begin{cases} 11, & i = 1, \\ 11i-2, & 2 \leq i \leq n-1 \text{ and } i \text{ is even}, \\ 11i-8, & 2 \leq i \leq n-1 \text{ and } i \text{ is odd}. \\ 11n-9, & i = n, \end{cases}$$

$$f(x_i) = \begin{cases} 3, & i = 1, \\ 10, & i = 2, \\ 11i-5, & 3 \leq i \leq n \text{ and } i \text{ is odd}, \\ 11i-10, & 3 \leq i \leq n \text{ and } i \text{ is even}, \end{cases}$$

$$f(y_i) = \begin{cases} 11i+6, & 1 \leq i \leq n-2 \text{ and } i \text{ is odd}, \\ 11i+1, & 1 \leq i \leq n-2 \text{ and } i \text{ is even}, \\ 11n-12, & i = n-1, \end{cases}$$

$$f(z_i) = \begin{cases} 7, & i = 1, \\ 11i-6, & 2 \leq i \leq n-1. \end{cases}$$

The induced edge labeling is obtained as follows:

$$f^*(u_i x_i) = \begin{cases} 2, & i = 1, \\ 8, & i = 2, \\ 11i-12, & 3 \leq i \leq n-1 \text{ and } i \text{ is odd}, \\ 11i-10, & 3 \leq i \leq n-1 \text{ and } i \text{ is even}, \end{cases}$$

$$f^*(x_i u_{i+1}) = \begin{cases} 4, & i = 1, \\ 12, & i = 2, \\ 11i-3, & 3 \leq i \leq n-2 \text{ and } i \text{ is odd}, \\ 11i-9, & 3 \leq i \leq n-2 \text{ and } i \text{ is even}, \\ 11n-13, & i = n-1, \end{cases}$$

$$f^*(v_i y_i) = \begin{cases} 14, & i = 1, \\ 11i, & 2 \leq i \leq n-2 \text{ and } i \text{ is even}, \\ 11i-1, & 2 \leq i \leq n-2 \text{ and } i \text{ is odd}, \\ 11n-15, & i = n-1, \end{cases}$$

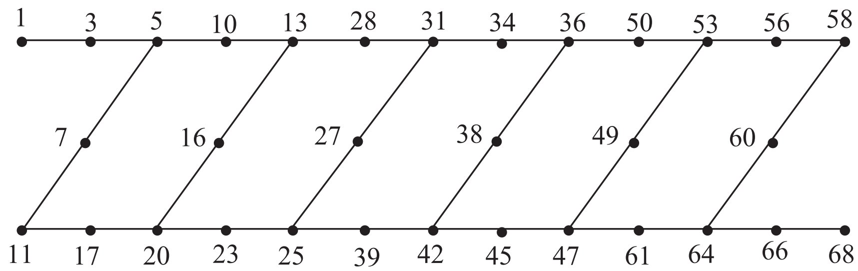
$$f^*(y_iv_{i+1}) = \begin{cases} 11i+8, & 1 \le i \le n-2 \text{ and } i \text{ is odd,} \\ 11i+2, & 1 \le i \le n-2 \text{ and } i \text{ is even,} \\ 11n-10, & i = n-1, \end{cases}$$

$$f^*(v_iz_i) = \begin{cases} 9, & i = 1, \\ 11i-4, & 2 \le i \le n-1 \text{ and } i \text{ is even,} \\ 11i-7, & 2 \le i \le n-1 \text{ and } i \text{ is odd,} \end{cases}$$

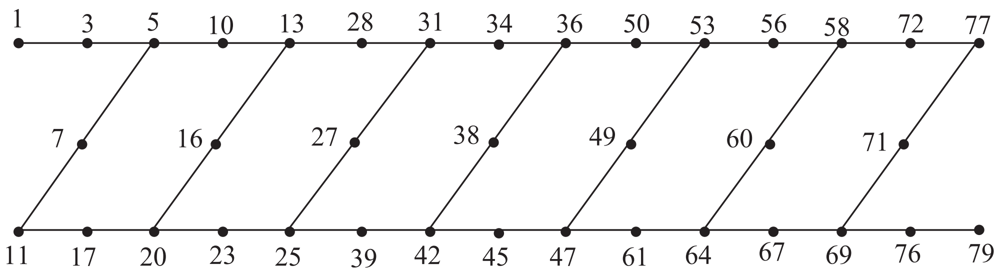
$$f^*(z_iu_{i+1}) = \begin{cases} 6, & i = 1, \\ 11i-7, & 2 \le i \le n-2 \text{ and } i \text{ is even,} \\ 11i+7, & 2 \le i \le n-2 \text{ and } i \text{ is odd,} \\ 11n-14, & i = n-1. \end{cases}$$

Thus, f is a super mean labeling of $S(SL_n)$ and hence $S(SL_n)$ is a super mean graph.

For example, a super mean labeling of $S(SL_7)$ and $S(SL_8)$ are shown in Figure 5.



(a) $S(SL_7)$



(b) $S(SL_8)$

FIGURE 5

When $n = 2$, a super mean labeling of the graph is shown in Figure 6. □

Theorem 2.5. *The graph $S(T_n \odot K_1)$ is a super mean graph for any n.*

Proof. Let $u_1, u_2, \ldots, u_n, u_{n+1}$ be the vertices on the path of length n in T_n and let v_i, $1 \le i \le n$ be the vertices of T_n in which v_i is adjacent to u_i and u_{i+1}. Let $v_i'a_iv_i$ be the path attached at each v_i, $1 \le i \le n$ and $u_i'b_iu_i$ be the path attached at each u_i, $1 \le i \le n+1$. Let x_i, y_i and z_i be the vertices which subdivided the edges u_iu_{i+1}, u_iv_i

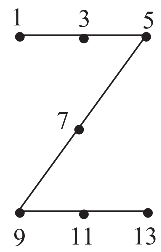


FIGURE 6

and v_iu_{i+1} respectively for each i, $1 \leq i \leq n$. The graph $S(T_n \odot K_1)$ has $9n+3$ vertices and $10n+2$ edges.

Define $f : V(S(T_n \odot K_1)) \to \{1, 2, \ldots, p+q = 19n+5\}$ as follows:

$$f(u_i) = 19i - 14, \quad 1 \leq i \leq n+1,$$
$$f(v_i) = 19i - 8, \quad 1 \leq i \leq n,$$
$$f(v_i') = 19i - 4, \quad 1 \leq i \leq n,$$
$$f(a_i) = 19i - 6, \quad 1 \leq i \leq n,$$
$$f(u_i') = \begin{cases} 1, & i = 1, \\ 19i - 20, & 2 \leq i \leq n+1, \end{cases}$$
$$f(b_i) = \begin{cases} 3, & i = 1, \\ 19i - 18, & 2 \leq i \leq n+1, \end{cases}$$
$$f(x_i) = 19i - 9, \quad 1 \leq i \leq n,$$
$$f(y_i) = 19i - 12, \quad 1 \leq i \leq n,$$
$$f(z_i) = 19i + 2, \quad 1 \leq i \leq n.$$

The induced edge labeling is defined as follows:

$$f^*(u_ix_i) = 19i - 11, \quad 1 \leq i \leq n,$$
$$f^*(x_iu_{i+1}) = 19i - 2, \quad 1 \leq i \leq n,$$
$$f^*(u_iy_i) = 19i - 13, \quad 1 \leq i \leq n,$$
$$f^*(y_iv_i) = 19i - 10, \quad 1 \leq i \leq n,$$
$$f^*(v_iz_i) = 19i - 3, \quad 1 \leq i \leq n,$$
$$f^*(z_iu_{i+1}) = 19i + 4, \quad 1 \leq i \leq n,$$
$$f^*(v_ia_i) = 19i - 7, \quad 1 \leq i \leq n,$$
$$f^*(a_iv_i') = 19i - 5, \quad 1 \leq i \leq n,$$
$$f^*(u_ib_i) = \begin{cases} 4, & i = 1, \\ 19i - 16, & 2 \leq i \leq n+1, \end{cases}$$

$$f^*(b_iu_i') = \begin{cases} 2, & i = 1, \\ 19(i-1), & 2 \leq i \leq n+1. \end{cases}$$

Thus, f is a super mean labeling of $S(T_n \odot K_1)$. □

For example, a super mean labeling of $S(T_6 \odot K_1)$ is shown in Figure 7.

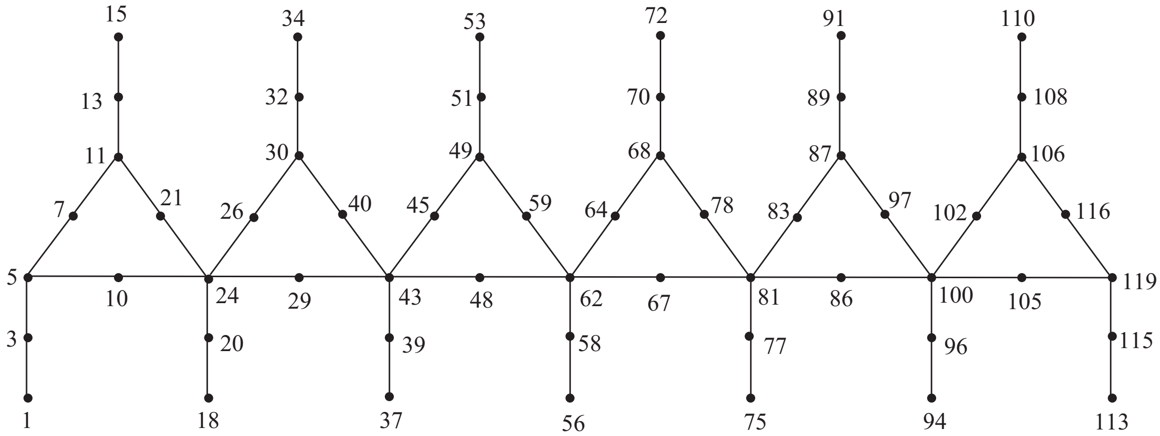


FIGURE 7. $S(T_6 \odot K_1)$

Theorem 2.6. *The graph $S(C_n \odot K_1)$ is a super mean graph, for $n \geq 3$.*

Proof. Let $u_1, u_2, \ldots, u_n$ be the vertices of the cycle C_n. Let $v_iy_iu_i$ be the path attached at each u_i, $1 \leq i \leq n$. Each edge u_iu_{i+1} is subdivided by a vertex x_i, $1 \leq i \leq n-1$ and the edge u_nu_1 is subdivided by a vertex x_n.

Case(i): n is odd.

Define $f : V(S(C_n \odot K_1)) \to \{1, 2, \ldots, 8n\}$ as follows:

$$f(u_i) = \begin{cases} 5, & i = 1, \\ 16i - 21, & 2 \leq i \leq \frac{n+1}{2}, \\ 8n, & i = \frac{n+3}{2}, \\ 16(n-i) + 22, & \frac{n+5}{2} \leq i \leq n, \end{cases}$$

$$f(v_i) = \begin{cases} 1, & i = 1, \\ 16i - 17, & 2 \leq i \leq \frac{n+1}{2}, \\ 16(n-i) + 18, & \frac{n+3}{2} \leq i \leq n, \end{cases}$$

$$f(x_i) = \begin{cases} 16i - 9, & 1 \leq i \leq \frac{n-1}{2}, \\ 8n - 3, & i = \frac{n+1}{2}, \\ 16(n-i) + 10, & \frac{n+3}{2} \leq i \leq n, \end{cases}$$

$$f(y_i) = \begin{cases} 3, & i = 1, \\ 16i - 19, & 2 \le i \le \frac{n+1}{2}, \\ 16(n-i) + 20, & \frac{n+3}{2} \le i \le n. \end{cases}$$

The induced edge labeling is defined as follows:

$$f^*(u_i x_i) = \begin{cases} 6, & i = 1, \\ 16i - 15, & 2 \le i \le \frac{n-1}{2}, \\ 8(n-1), & i = \frac{n+1}{2}, \\ 8n - 7, & i = \frac{n+3}{2}, \\ 16(n-i) + 16, & \frac{n+5}{2} \le i \le n, \end{cases}$$

$$f^*(x_i u_{i+1}) = \begin{cases} 16i - 7, & 1 \le i \le \frac{n-1}{2}, \\ 8n - 1, & i = \frac{n+1}{2}, \\ 16(n-i) + 8, & \frac{n+3}{2} \le i \le n-1, \end{cases}$$

$$f^*(x_n u_1) = 8,$$

$$f^*(v_i y_i) = \begin{cases} 2, & i = 1, \\ 16i - 18, & 2 \le i \le \frac{n+1}{2}, \\ 16(n-i) + 19, & \frac{n+3}{2} \le i \le n, \end{cases}$$

$$\text{and } f^*(y_i u_i) = \begin{cases} 4, & i = 1, \\ 16i - 20, & 2 \le i \le \frac{n+1}{2}, \\ 8n - 2, & i = \frac{n+3}{2}, \\ 16(n-i) + 21, & \frac{n+5}{2} \le i \le n. \end{cases}$$

Case (ii): n is even.

$$f(u_i) = \begin{cases} 5, & i = 1, \\ 16i - 21, & 2 \le i \le \frac{n}{2}, \\ 8n - 4, & i = \frac{n+2}{2}, \\ 16(n-i) + 22, & \frac{n+4}{2} \le i \le n, \end{cases}$$

$$f(v_i) = \begin{cases} 1, & i = 1, \\ 16i - 17, & 2 \le i \le \frac{n}{2}, \\ 8n, & i = \frac{n+2}{2}, \\ 16(n-i) + 18, & \frac{n+4}{2} \le i \le n, \end{cases}$$

$$f(x_i) = \begin{cases} 16i - 9, & 1 \le i \le \frac{n}{2}, \\ 8n - 7, & i = \frac{n+2}{2}, \\ 16(n-i) + 10, & \frac{n+4}{2} \le i \le n, \end{cases}$$

$$f(y_i) = \begin{cases} 3, & i = 1, \\ 16i - 19, & 2 \leq i \leq \frac{n}{2}, \\ 8n - 2, & i = \frac{n+2}{2}, \\ 16(n-i) + 20, & \frac{n+4}{2} \leq i \leq n. \end{cases}$$

For the vertex labeling f, the induced edge labeling f^* is given as follows:

$$f^*(u_ix_i) = \begin{cases} 6, & i = 1, \\ 16i - 15, & 2 \leq i \leq \frac{n}{2}, \\ 8n - 5, & i = \frac{n+2}{2}, \\ 16(n-i+1), & \frac{n+4}{2} \leq i \leq n, \end{cases}$$

$$f^*(x_iu_{i+1}) = \begin{cases} 16i - 7, & 1 \leq i \leq \frac{n-2}{2}, \\ 8n - 6, & i = \frac{n}{2}, \\ 16(n-i) + 8, & \frac{n+2}{2} \leq i \leq n-1, \end{cases}$$

$$f^*(x_nu_1) = 8,$$

$$f^*(v_iy_i) = \begin{cases} 2, & i = 1, \\ 16i - 18, & 2 \leq i \leq \frac{n}{2}, \\ 8n - 1, & i = \frac{n+2}{2}, \\ 16(n-i) + 19, & \frac{n+4}{2} \leq i \leq n, \end{cases}$$

$$\text{and } f^*(y_iu_i) = \begin{cases} 4, & i = 1, \\ 16i - 20, & 2 \leq i \leq \frac{n}{2}, \\ 8n - 3, & i = \frac{n+2}{2}, \\ 16(n-i) + 21, & \frac{n+4}{2} \leq i \leq n. \end{cases}$$

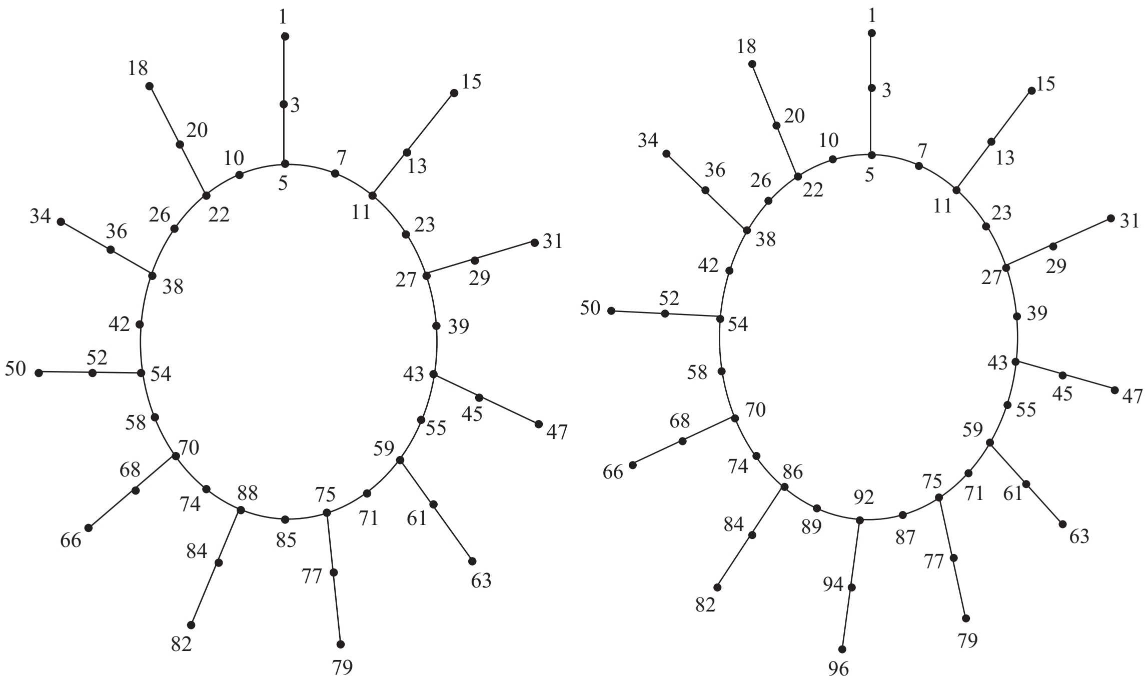
Thus, f is a super mean labeling and hence $S(C_n \odot K_1)$ is a super mean graph. □

For example, a super mean labeling of $S(C_{11} \odot K_1)$ and $S(C_{12} \odot K_1)$ are shown in Figure 8.

Theorem 2.7. *The graph $S(C_m @ C_n)$ is a super mean graph for $m, n \geq 3$.*

Proof. $C_m @ C_n$ is a graph obtained by identifying an edge of two cycles C_m and C_n. $C_m @ C_n$ has $m + n - 2$ vertices and $m + n - 1$ edges. In $S(C_m @ C_n)$, $2(m + n - 2)$ vertices lies on the circle and one vertex lies on a chord. Then, the graph $S(C_m @ C_n)$ has $2m + 2n - 3$ vertices and $2(m + n - 1)$ edges.

Let us assume that $m \leq n$.



(a) $S(C_{11} \odot K_1)$ (b) $S(C_{12} \odot K_1)$

FIGURE 8

Case (i): m is odd and n is odd.

Let $m = 2k+1$, $k \geq 1$ and $n = 2l+1$, $l \geq 1$. We denote the vertices of $S(C_m @ C_n)$ is shown in Figure 9.

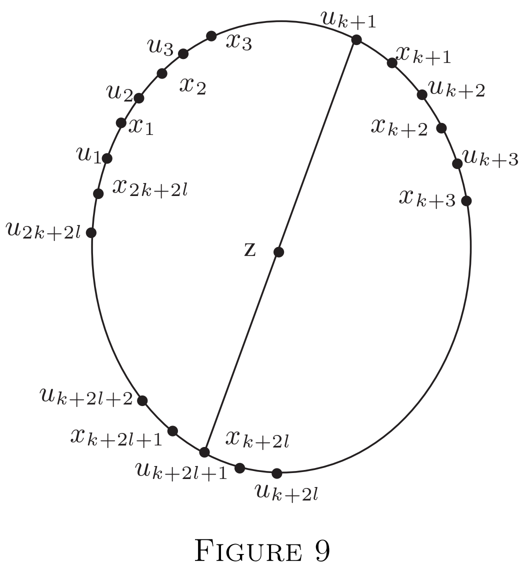


FIGURE 9

Define $f : V(S(C_m @ C_n)) \to \{1, 2, 3, \ldots, p+q = 4(m+n)-5\}$ as follows:

$$f(u_i) = \begin{cases} 1, & i = 1, \\ 8i-9, & 2 \leq i \leq k, \\ 4m-6, & i = k+1, \\ 8i, & k+2 \leq i \leq k+l, \\ 8(m+n-i)-9, & k+l+1 \leq i \leq k+2l-1, \\ 4m+5, & i = k+2l, \\ 4m, & i = k+2l+1, \\ 8(m+n-i)-6, & k+2l+2 \leq i \leq 2k+2l, \end{cases}$$

$$f(x_i) = \begin{cases} 8i-5, & 1 \leq i \leq k, \\ 8i+4, & k+1 \leq i \leq k+l-1, \\ 8(m+n-i)-13, & k+l \leq i \leq k+2l-1, \\ 4m+3, & i = k+2l, \\ 4m-5, & i = k+2l+1, \\ 8(m+n-i)-10, & k+2l+2 \leq i \leq 2k+2l, \end{cases}$$

and $f(z) = 4m-3$.

The induced edge labeling f^* is obtained as follows:

$$f^*(u_i x_i) = \begin{cases} 2, & i = 1, \\ 8i-7, & 2 \leq i \leq k, \\ 4m+1, & i = k+1, \\ 8i+2, & k+2 \leq i \leq k+l, \\ 8(m+n-i)-11, & k+l+1 \leq i \leq k+2l-1, \\ 4m+4, & i = k+2l, \\ 4m-2, & i = k+2l+1, \\ 8(m+n-1-i), & k+2l+2 \leq i \leq 2k+2l-2, \end{cases}$$

$$f^*(x_i u_{i+1}) = \begin{cases} 8i-3, & 1 \leq i \leq k, \\ 8i+6, & k+1 \leq i \leq k+l-1, \\ 8(m+n-i)-15, & k+l \leq i \leq k+2l-2, \\ 4m+6, & i = k+2l-1, \\ 4m+2, & i = k+2l, \\ 8(m+n-i)-12, & k+2l+1 \leq i \leq 2k+2l-1, \end{cases}$$

$$f^*(x_{2k+2l} u_1) = 4,$$

$$f^*(u_{k+1} z) = 4m-4,$$

and $f^*(z u_{k+2l+1}) = 4m-1$.

Thus, f is a super mean labeling. A super mean labeling of $S(C_7@C_9)$ is shown in Figure 10.

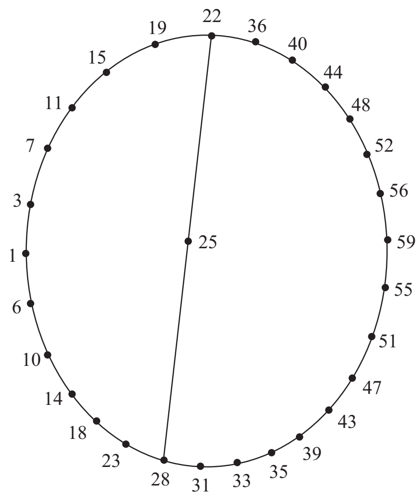


FIGURE 10

Case (ii): m is odd and n is even.

Let $m = 2k+1$, $k \geq 1$ and $n = 2l$, $l \geq 2$.

Define $f : V(S(C_m@C_n)) \to \{1, 2, 3, \ldots, p+q = 4(m+n)-5\}$ as follows:

$$f(u_i) = \begin{cases} 1, & i = 1, \\ 8i-9, & 2 \leq i \leq k, \\ 4m-6, & i = k+1, \\ 8i, & k+2 \leq i \leq k+l-1, \\ 8(m+n-i)-9, & k+l \leq i \leq k+2l-2, \\ 4m+5, & i = k+2l-1, \\ 4m, & i = k+2l, \\ 8(m+n-i)-6, & k+2l+1 \leq i \leq 2k+2l-1, \end{cases}$$

$$f(x_i) = \begin{cases} 8i-5, & 1 \leq i \leq k, \\ 8i+4, & k+1 \leq i \leq k+l-1, \\ 8(m+n-i)-13, & k+l \leq i \leq k+2l-2, \\ 4m+3, & i = k+2l-1, \\ 4m-5, & i = k+2l, \\ 8(m+n-i)-10, & k+2l+1 \leq i \leq 2k+2l-1, \end{cases}$$

and $f(z) = 4m-3$.

For the vertex labeling f, the induced edge labeling f^* is given as follows:

$$f^*(u_ix_i) = \begin{cases} 2, & i = 1, \\ 8i-7, & 2 \le i \le k, \\ 4m+1, & i = k+1, \\ 8i+2, & k+2 \le i \le k+l-1, \\ 8(m+n-i)-11, & k+l \le i \le k+2l-2, \\ 4m+4, & i = k+2l-1, \\ 4m-2, & i = k+2l, \\ 8(m+n-i)-8, & k+2l+1 \le i \le 2k+2l-1, \end{cases}$$

$$f^*(x_iu_{i+1}) = \begin{cases} 8i-3, & 1 \le i \le k, \\ 8i+6, & k+1 \le i \le k+l-1, \\ 8(m+n-i)-15, & k+l \le i \le k+2l-3, \\ 4m+6, & i = k+2l-2, \\ 4m+2, & i = k+2l-1, \\ 8(m+n-i)-12, & k+2l \le i \le 2k+2l-2, \end{cases}$$

$$f^*(x_{2k+2l-1}u_1) = 4,$$

$$f^*(u_{k+1}z) = 4m-4,$$

and $f^*(zu_{k+2l}) = 4m-1.$

Thus, f is a super mean labeling. A super mean labeling of $S(C_7@C_{10})$ is shown in Figure 11.

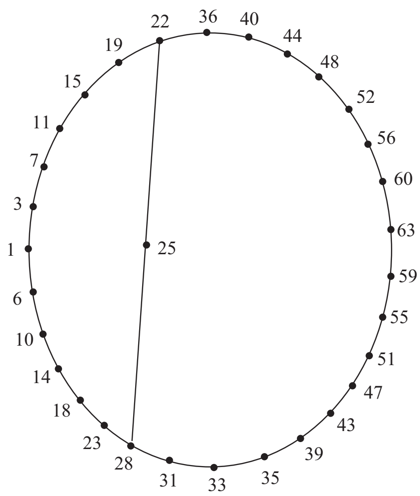


FIGURE 11

Case (iii): m is even and n is even.

Let $m = 2k$, $k \ge 2$ and $n = 2l$, $l \ge 2$.

Define $f : V(S(C_m @ C_n)) \to \{1, 2, 3, \ldots, p + q = 4(m + n) - 5\}$ as follows:

$$f(u_i) = \begin{cases} 1, & i = 1, \\ 8i - 9, & 2 \le i \le k, \\ 4m, & i = k + 1, \\ 4m + 5, & i = k + 2, \\ 8i - 13, & k + 3 \le i \le k + l + 1, \\ 8(m + n - i) + 4, & k + l + 2 \le i \le k + 2l - 1, \\ 8(m + n - i) - 6, & k + 2l \le i \le 2k + 2l - 2, \end{cases}$$

$$f(x_i) = \begin{cases} 8i - 5, & 1 \le i \le k + 1, \\ 8i - 9, & k + 2 \le i \le k + l, \\ 8(m + n - i), & k + l + 1 \le i \le k + 2l - 1, \\ 8(m + n - i) - 10, & k + 2l \le i \le 2k + 2l - 2, \end{cases}$$

and $f(z) = 4m - 3$.

For the vertex labeling f, the induced edge labeling f^* is obtained as follows:

$$f^*(u_i x_i) = \begin{cases} 2, & i = 1, \\ 8i - 7, & 2 \le i \le k, \\ 4m + 2, & i = k + 1, \\ 4m + 6, & i = k + 2, \\ 8i - 11, & k + 3 \le i \le k + l, \\ 8(m + n - i) + 2, & k + l + 1 \le i \le k + 2l - 1, \\ 8(m + n - i) - 8, & k + 2l \le i \le 2k + 2l - 2, \end{cases}$$

$$f^*(x_i u_{i+1}) = \begin{cases} 8i - 3, & 1 \le i \le k - 1, \\ 4m - 2, & i = k, \\ 4m + 4, & i = k + 1, \\ 8i - 7, & k + 2 \le i \le k + l, \\ 8(m + n - i) - 2, & k + l + 1 \le i \le k + 2l - 2, \\ 4m + 1, & i = k + 2l - 1, \\ 8(m + n - i) - 12, & k + 2l \le i \le 2k + 2l - 3, \end{cases}$$

$$f^*(x_{2k+2l-2} u_1) = 4,$$
$$f^*(u_{k+1} z) = 4m - 1,$$
and $f^*(z u_{k+2l}) = 4m - 4.$

Thus, f is a super mean labeling. A super mean labeling of $S(C_6 @ C_8)$ is shown in Figure 12.

Hence, the graph $S(C_m @ C_n)$ is a super mean graph for $m, n \ge 3$. □

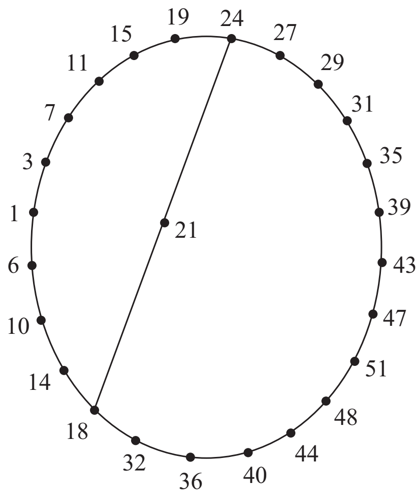


FIGURE 12

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