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ON A NEW MODEL OF NONLOCAL MODIFIED GRAVITY

JELENA GRUJIC[1]

Abstract. Nonlocal modified gravity without matter, where nonlocality is of the form $R^{-p}\mathcal{F}(\Box)R$ is considered from the cosmological point of view. Equations of motion are derived. Cosmological solutions of the form $a(t) = a_0|t-t_0|^\alpha$, for the FLRW metric and curvature constant $k = 0, \pm 1$, are found.

1. Introduction

General Relativity is the Einstein theory of gravity usually given in the form of the equation of motion for gravitational (metric) field $g_{\mu\nu}$, i.e. $R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$, where $R_{\mu\nu}$ is the Ricci curvature tensor, R is scalar curvature, $T_{\mu\nu}$ is the energy-momentum tensor, and speed of light is taken $c = 1$. This Einstein equation can be derived from the Einstein-Hilbert action $S = \frac{1}{16\pi G}\int \sqrt{-g}R d^4x + \int \sqrt{-g} L_m d^4x$, where $g = \det(g_{\mu\nu})$ and L_m is Lagrangian of matter.

Despite its theoretical beauty and many phenomenological evidences, general relativity is not a complete theory and should be modified. There are many its modifications, which are motivated by quantum gravity, string theory, astrophysics and cosmology (for a review, see [1]). One of very promising directions of research is *nonlocal modified gravity* and its applications to cosmology (as a review, see [2], [3] and [4]). It usually contains an infinite number of spacetime derivatives in the form of some power expansions of the d'Alembert operator $\Box = \frac{1}{\sqrt{-g}}\partial_\mu \sqrt{-g} g^{\mu\nu}\partial_\nu$ or of its inverse $\Box^{-1}$, or a combination of both. We are mainly interested in nonlocality expressed in the form of an analytic function $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$. For nonlocal gravity with $\Box^{-1}$ see also [17, 3, 18, 19]. Nonlocality also improves renormalizability of gravity, see [20, 21]

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and references therein. Note that there have been some investigations with R^{-1} modification of gravity, but they are not nonlocal and they have problems to be confirmed for the Solar System [16].

To solve cosmological Big Bang singularity, nonlocal gravity with replacement $R \to R + R\mathcal{F}(\Box)R$ in the Einstein-Hilbert action was proposed in [5]. This nonlocal model is further elaborated in the series of papers [6, 7, 8, 9, 10, 11, 12].

We are interesting here in nonlocal gravity action without matter in the form

$$S = \int \left(\frac{R - 2\Lambda}{16\pi G} + R^{-p}\mathcal{F}(\Box)R \right) \sqrt{-g}\, d^4x, \tag{1.1}$$

where R is scalar curvature, $p \in \mathbb{N}$, $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$ is an analytic function of the d'Alembert-Beltrami operator $\Box = \frac{1}{\sqrt{-g}}\partial_\mu \sqrt{-g} g^{\mu\nu}\partial_\nu$, $g = \det(g_{\mu\nu})$.

The action (1.1) for $p = 1$ was introduced in [13] as a new approach to nonlocal gravity. Our intention is to present some power-law cosmological solutions as a part of a systematic investigation of nonlocal gravity (1.1). For $p = 1$, in [13, 14] similar power-law cosmological solutions were obtained.

2. Equations of Motion

By variation of action (1.1) with respect to metric $g^{\mu\nu}$ one obtains the equations of motion for $g_{\mu\nu}$

$$\begin{aligned} &\frac{1}{16\pi G}(G_{\mu\nu} + \Lambda g_{\mu\nu}) - \frac{1}{2} g_{\mu\nu} R^{-p}\mathcal{F}(\Box)R + (R_{\mu\nu}\Phi - K_{\mu\nu}\Phi) \\ &+ \frac{1}{2}\sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \big(g_{\mu\nu}\partial^\alpha \Box^l R^{-p} \partial_\alpha \Box^{n-1-l} R + g_{\mu\nu}\Box^l R^{-p}\Box^{n-l}R \\ &- 2\partial_\mu \Box^l R^{-p}\partial_\nu \Box^{n-1-l}R \big) = 0, \end{aligned} \tag{2.1}$$

where $K_{\mu\nu} = \nabla_\mu\nabla_\nu - g_{\mu\nu}\Box$, $\Phi = -pR^{-p-1}\mathcal{F}(\Box)R + \mathcal{F}(\Box)R^{-p}$.

In the case of the FLRW metric, equation (2.1) is equivalent to its trace and 00 component, respectively:

$$\begin{aligned} &\frac{1}{16\pi G}(4\Lambda - R) - 2R^{-p}\mathcal{F}(\Box)R + (R\Phi + 3\Box\Phi) \\ &+ \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \big(\partial^\mu \Box^l R^{-p}\partial_\mu \Box^{n-1-l}R + 2\Box^l R^{-p}\Box^{n-l}R \big) = 0, \end{aligned} \tag{2.2}$$

$$
\begin{aligned}
&\frac{1}{16\pi G}(G_{00}+\Lambda g_{00})-\frac{1}{2}g_{00}R^{-p}\mathcal{F}(\Box)R+(R_{00}\Phi-K_{00}\Phi)\\
&+\frac{1}{2}\sum_{n=1}^{\infty}f_n\sum_{l=0}^{n-1}\big(g_{00}\partial^{\alpha}\Box^{l}R^{-p}\partial_{\alpha}\Box^{n-1-l}R+g_{00}\Box^{l}R^{-p}\Box^{n-l}R\\
&-2\partial_0\Box^{l}R^{-p}\partial_0\Box^{n-1-l}R\big)=0.
\end{aligned}
\tag{2.3}
$$

Equations (2.2) and (2.3) are more suitable for further investigation than (2.1).

In the FLRW metric $ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2d\theta^2 + r^2\sin^2\theta d\phi^2\right)$ one has $R = 6\left(\frac{\ddot a}{a} + \frac{\dot a^2}{a^2} + \frac{k}{a^2}\right)$ and $\Box h(t) = -\ddot h(t) - 3H\dot h(t)$, where $H = \frac{\dot a}{a}$ is the Hubble parameter. In the sequel we solve equations of motions (2.2) and (2.3) for cosmological scale factor $a(t)$ and the corresponding R:

$$a(t) = a_0|t-t_0|^{\alpha}, \tag{2.4}$$

$$R(t) = 6\left(\alpha(2\alpha-1)(t-t_0)^{-2} + \frac{k}{a_0^2}(t-t_0)^{-2\alpha}\right). \tag{2.5}$$

3. Case $k = 0$, $\alpha \neq 0$ and $\alpha \neq \frac{1}{2}$

In this case, there is the following dependence on parameter α :

$$
\begin{aligned}
a &= a_0|t-t_0|^{\alpha}, & H &= \frac{\alpha}{t-t_0},\\
R &= Q(t-t_0)^{-2}, & Q &= 6\alpha(2\alpha-1),\\
R_{00} &= 3\alpha(1-\alpha)(t-t_0)^{-2}, & G_{00} &= 3\alpha^2(t-t_0)^{-2}.
\end{aligned}
$$

Now expressions $\Box^n R^{-p}$ and $\Box^n R$ become

$$
\begin{aligned}
&\Box^n R^{-p} = B(n,p)(t-t_0)^{2p-2n}, \quad \Box^n R = B(n,-1)(t-t_0)^{-2-2n},\\
&B(n,p) = Q^{-p}2^n\prod_{l=1}^{n}(p-l+1)(-2p-3\alpha+2l-1),\ n\geq 1,\\
&B(n,-1) = Q2^n\prod_{l=1}^{n}(-l)(1-3\alpha+2l),\ n\geq 1,\\
&B(0,p) = Q^{-p}, \quad B(0,-1) = Q.
\end{aligned}
\tag{3.1}
$$

Also, we have

$$\mathcal{F}(\Box)(R^{-p}) = \sum_{n=0}^{\infty} f_n B(n,p)(t-t_0)^{2p-2n}.$$

Coefficients $B(n,s)$ we can transform in the following way:

$$
\begin{aligned}
B(n,s) &= Q^{-s}(-4)^n \frac{\Gamma(s+1)\Gamma(s+\eta+1)}{\Gamma(s+1-n)\Gamma(s+\eta+1-n)}, \quad s \neq -1,\\
B(p,-1) &= Q 4^p p! \frac{\Gamma(\eta)}{\Gamma(\eta-p)},
\end{aligned} \tag{3.2}
$$

where $\eta = \frac{3\alpha-1}{2}$. Substituting these equations into (2.2) and (2.3) one has

$$
\begin{aligned}
&\sum_{n=0}^{\infty} f_n B(n,p)\left(Q - 6(p-n)(2p-2n-1) - 18\alpha(p-n)\right)\tau\\
&\quad + Q^{-p-1}\sum_{n=0}^{\infty} f_n B(n,-1)\left((-2-p)Q + 6p(p-n)(2p-2n-1+3\alpha)\right)\tau\\
&\quad + 2\sum_{n=1}^{\infty} f_n A_n \tau = \frac{1}{16\pi G}(Q(t-t_0)^{-2} - 4\Lambda)
\end{aligned} \tag{3.3}
$$

and

$$
\begin{aligned}
&3\alpha\sum_{n=0}^{\infty} f_n B(n,p)\left(1-\alpha+2(p-n)\right)\tau\\
&\quad + Q^{-p}\sum_{n=0}^{\infty} f_n B(n,-1)\left(\frac{1}{2} - 3\alpha(1-\alpha)pQ^{-1} - 6\alpha p Q^{-1}(p-n)\right)\tau\\
&\quad - \frac{1}{2}\sum_{n=1}^{\infty} f_n B_n \tau = \frac{1}{16\pi G}(\Lambda - 3\alpha^2(t-t_0)^{-2}),
\end{aligned} \tag{3.4}
$$

where

$$
A_n = \sum_{l=0}^{n-1} B(l,p)(B(n-l,-1) - 2(p-l)(l-n)B(n-l-1,-1)), \tag{3.5}
$$

$$
B_n = \sum_{l=0}^{n-1} B(l,p)(B(n-l,-1) + 4(p-l)(l-n)B(n-l-1,-1)), \tag{3.6}
$$

$$
\tau = (t-t_0)^{2p-2n-2}.
$$

From equations (3.3) and (3.4) it follows that if $i \neq p, p-1$ we have $f_i = 0$. Coefficients f_p and f_{p-1} have to satisfy the following systems:

$$
\begin{aligned}
f_p(K_1 + S_1) &= \frac{Q}{16\pi G}, \\
f_p(K_2 + \frac{1}{2}S_2) &= \frac{-3\alpha^2}{16\pi G}, \qquad (3.7) \\
f_{p-1}(K_3 + S_3) &= \frac{-\Lambda}{4\pi G}, \\
f_{p-1}(K_4 + \frac{1}{2}S_4) &= \frac{\Lambda}{16\pi G}, \qquad (3.8)
\end{aligned}
$$

where

$$
\begin{aligned}
K_1 &= (-p-2)Q^{-p}B(p,-1) + QB(p,p), \\
S_1 &= \sum_{l=0}^{p-1} 2B(l,p)\left(2(l-p)^2 B(-l+p-1,-1) + B(p-l,-1)\right), \\
K_2 &= \left(3(\alpha-1)\alpha\, p\, Q^{-1} + \frac{1}{2}\right) Q^{-p}B(p,-1) + 3(1-\alpha)\alpha B(p,p), \\
S_2 &= \sum_{l=0}^{p-1} B(l,p)\left(4(p-l)^2 B(-l+p-1,-1) - B(p-l,-1)\right), \\
K_3 &= \left(6(3\alpha+1)p\, Q^{-1} - p - 2\right) Q^{-p}B(p-1,-1) + QB(p-1,p) + 3B(p,p), \\
S_3 &= \sum_{l=0}^{p-2} 2B(l,p)\left(2(l-p)(l-p+1)B(-l+p-2,-1) + B(-l+p-1,-1)\right), p \neq 1, \\
K_4 &= \left(3(\alpha-1)\alpha p - 6\alpha p + \frac{Q}{2}\right) Q^{-p-1}B(p-1,-1) + 3(3-\alpha)\alpha B(p-1,p), \\
S_4 &= \sum_{l=0}^{p-2} B(l,p)\left(4(l-p)(l-p+1)B(-l+p-2,-1) - B(-l+p-1,-1)\right), p \neq 1.
\end{aligned}
$$

For $p = 1$ we have $S_3 = S_4 = 0$.

The systems (3.7) and (3.8) have a solution if and only if

$$
\begin{aligned}
-3\alpha^2 K_1 - QK_2 &= 3\alpha^2 S_1 + \frac{1}{2}QS_2, \\
K_3 + 4K_4 &= -(S_3 + 2S_4).
\end{aligned}
$$

Using (3.2) we can show that last conditions are satisfied for all $\alpha \in \mathbb{R} \setminus \{0, \frac{1}{2}\}$ and $p \in \mathbb{N}$.

4. Case $k=0$, $\alpha \to 0$ (Minkowski Space)

Substituting (3.1) and (3.5) into trace equation (3.3) we obtain

$$
\begin{aligned}
&(6\alpha(2\alpha-1))^{-p}\Bigg(\sum_{n=0}^{\infty} f_n\widetilde{B}(n,p)\,(6\alpha(2\alpha-1)-6(p-n)(2p-2n-1)-18\alpha(p-n))\,\tau\\
&+\sum_{n=0}^{\infty} f_n\widetilde{B}(n,-1)\,((-2-p)6\alpha(2\alpha-1)+6p(p-n)(2p-2n-1)+18\alpha p(p-n))\,\tau\\
&+12\alpha(2\alpha-1)\sum_{n=1}^{\infty} f_n\sum_{l=0}^{n-1}\widetilde{B}(l,p)(\widetilde{B}(n-l,-1)-2(p-l)(l-n)\widetilde{B}(n-l-1,-1))\tau\Bigg)
\end{aligned}
\tag{4.1}
$$
$$
=\frac{1}{16\pi G}(6\alpha(2\alpha-1)(t-t_0)^{-2}-4\Lambda),
$$

where

$$
\begin{aligned}
\widetilde{B}(n,p)&=2^n\prod_{l=1}^{n}(p-l+1)(-2p-3\alpha+2l-1),\ n\geq 1,\quad \widetilde{B}(0,p)=1,\\
\widetilde{B}(n,-1)&=2^n\prod_{l=1}^{n}(-l)(1-3\alpha+2l),\ n\geq 1,\quad \widetilde{B}(0,-1)=1,\\
\tau&=(t-t_0)^{2p-2n-2}.
\end{aligned}
$$

Now, if $\alpha\to 0$ from (4.1) we get

$$
\sum_{n=1}^{\infty} f_n D_n 2^{n+1}(p-n)(2p-2n-1)(t-t_0)^{2p-2n-2}=0,
$$

where

$$
D_n=p\prod_{l=1}^{n}(-l)(1+2l)-\prod_{l=1}^{n}(p-l+1)(-2p+2l-1).
$$

From this we conclude

$$
f_{p-1},f_p\in\mathbb{R},\quad f_i=0,\ i\neq p-1,\ p.
$$

Substituting $f_i=0,\ i\neq p-1,\ p$ into equation (3.4) we obtain the following equations:

$$\Big((-1-p)\widetilde{B}(p,-1)+\widetilde{B}(p,p)$$
$$+\sum_{l=0}^{p-1}\widetilde{B}(l,p)(\widetilde{B}(p-l,-1)-4(p-l)^2\widetilde{B}(p-l-1,-1))\Big)f_p(t-t_0)^{-2}=0,$$
$$\Big((-1-3p)\widetilde{B}(p-1,-1)+3\widetilde{B}(p-1,p)$$
$$+\sum_{l=0}^{p-2}\widetilde{B}(l,p)(\widetilde{B}(p-l-1,-1)+4(p-l)(l-p+1)\widetilde{B}(p-l-2,-1))\Big)f_{p-1}=0.$$

When $\alpha\to 0$ the corresponding solution is

$$f_p\in\mathbb{R},\quad f_i=0,\ i\neq p.$$

Since all $f_n=0$, $n\neq p$ this is not nonlocal gravity model (1.1). It follows that the above power-law cosmological solutions have not Minkowski space as their background, or in other words, they cannot be obtained as perturbations on Minkowski space.

Remark. For $p\neq 1$, case $k=0$, $\alpha\to\frac{1}{2}$ does not yield solution which satisfies the equations of motion. For $p=1$ we have solution (see [14]).

5. Case $k\neq 0$

In order to simplify expression (2.5) there are three possibilities: $\alpha=0$, $\alpha=\frac{1}{2}$ and $\alpha=1$. For $\alpha=\frac{1}{2}$ we have no solutions which satisfy the equations of motion.

5.1. **Case $\alpha=0$.**
In the case $\alpha=0$ we obtain

$$a=a_0,\quad R=\frac{6k}{a_0^2},\qquad H=0,\quad \Box R=0. \tag{5.1}$$

We have $\Phi=(1-p)R^{-p}f_0$.

Substituting (5.1) into equations of motion we get the following system of equations:

$$(-p-1)R^{1-p}f_0=\frac{R}{16\pi G}-\frac{\Lambda}{4\pi G},$$
$$R^{1-p}f_0=\frac{\Lambda}{8\pi G}-\frac{R}{16\pi G},$$
$$f_i\in\mathbb{R},\ i\neq 0.$$

The previous system has solution if and only if

$$32\pi G\Lambda(1-p)=-pR.$$

Obviously, for $p=1$ the last condition is not satisfied.

5.2. **Case $\alpha = 1$.** In the case $\alpha = 1$ we obtain

$$a = a_0|t - t_0|, \quad R = 6(1 + \frac{k}{a_0^2})(t - t_0)^{-2},$$
$$H = \frac{1}{t - t_0}, \quad \Box R = 0,$$
$$R_{00} = 0, \quad G_{00} = 3(1 + \frac{k}{a_0^2})(t - t_0)^{-2},$$

and

$$\Box^n R^{-p} = E(n,p)(t - t_0)^{2p-2n},$$
$$E(n,p) = \left(6 + \frac{6k}{a_0^2}\right)^{-p} 2^n \prod_{l=1}^{n}(p - l + 1)(-2p + 2l - 4), \ n \geq 1,$$
$$E(0,p) = \left(6 + \frac{6k}{a_0^2}\right)^{-p},$$
$$\Box^n R = E(n, -1)(t - t_0)^{-2-2n} = 0, \quad n \geq 1.$$

Also, we have

$$\mathcal{F}(\Box)(R^{-p}) = \sum_{n=0}^{\infty} f_n E(n,p)(t - t_0)^{2p-2n}.$$

Using the above expressions, from equations of motion it follows that coefficients f_p and f_{p-1} have to satisfy the following systems of equations:

$$(4E(p-1,p) + E(p,p))f_p = \frac{1}{16\pi G},$$
$$4E(p-1,p)f_p = -\frac{1}{16\pi G} \tag{5.2}$$

and

$$(48(1 + \frac{k}{a_0^2})E(p-2,p) + 6(1 + \frac{k}{a_0^2})E(p-1,p) + 3E(p,p))f_{p-1} = -\frac{\Lambda}{4\pi G},$$
$$6(4(1 + \frac{k}{a_0^2})E(p-2,p) + E(p-1,p))f_{p-1} = \frac{\Lambda}{16\pi G}, \quad p \neq 1. \tag{5.3}$$

We obtain

$$f_n = 0, \quad 0 \leq n \leq p - 2 \quad (p \neq 1),$$
$$f_n \in \mathbb{R}, \quad n \geq p + 1.$$

The systems (5.2) and (5.3) have solution if and only if

$$8E(p-1,p) = -E(p,p),$$
$$48(1 + \frac{k}{a_0^2})E(p-2,p) + (10 + \frac{2k}{a_0^2})E(p-1,p) + E(p,p) = 0.$$

After short calculation we obtain that the last two conditions are satisfied for all $p \neq 1$.
For $p = 1$ system (5.3) (which corresponds to coefficient f_0) becomes:

$$f_0 = \frac{\Lambda}{8\pi G}.$$

Therefore, equations of motion are satisfied for all $p \in \mathbb{N}$.

6. Concluding Remarks

In this paper we have presented some power-law cosmological solutions of the form $a(t) = a_0|t - t_0|^\alpha$, which are derived from modified gravity with nonlocal term $R^{-p}\mathcal{F}(\Box)R$. These solutions do not have Minkowski space background. However, for $p = 1$ there is the de Sitter bounce solution $a(t) = a_0 \exp(\lambda t)$, which in the limit $\lambda \to 0$ leads to the Minkowski space (see [14, 15]).

It is worth noting that there is solution $a(t) = |t - t_0|$ which corresponds to the Milne universe for $k = -1$.

Note also that all the above presented power-law solutions $a(t) = a_0|t - t_0|^\alpha$ have scalar curvature $R(t) = 6\left(\alpha(2\alpha - 1)(t - t_0)^{-2} + \frac{k}{a_0^2}(t - t_0)^{-2\alpha}\right)$ (2.5), which satisfies relation $\Box R = qR^2$, where parameter q depends on α. This quadratic relation $\Box R = qR^2$ was used in [13] as an Ansatz to solve equations of motion for $p = 1$.

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[1]Teacher Education Faculty,
University of Belgrade,
Kraljice Natalije 43, Belgrade, Serbia
E-mail address: jelenagg@gmail.com