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FIXED POINT THEOREMS IN NORMED LINEAR SPACES USING A GENERALIZED Z-TYPE CONDITION

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Abstract. In this paper, a strong convergence theorem is proved for a generalized Mann type iteration scheme in normed linear spaces. We also consider two, two-step iteration schemes and prove the strong convergence of these iterations in normed linear spaces. We use a generalized Z-type condition to prove our results. Our results extend and improve upon, among others, the corresponding results proved by Berinde [1], Yildirim et al. [12] and Bosede [4].

1. Introduction and Preliminary Definitions

The following iterative scheme was introduced by Mann [8] in 1953:
Let K be a nonempty, closed, convex subset of a normed linear space E and $T : K \to K$ be a self map. For $x_0 \in K$, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n T x_n, \qquad n = 0, 1, 2, \dots, \tag{1.1}$$

where $\{\alpha_n\}$ is a real sequence in $[0, 1]$, is called the Mann iteration process. Many convergence theorems and approximation results have been proved using the Mann iteration process. A two-step iterative process was introduced by Ishikawa [6] in 1974 which is defined as follows:
For $x_0 \in K$, the sequence $\{x_n\}_{n=0}^{\infty}$ given by

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T y_n \\ y_n &= (1-\beta_n)x_n + \beta_n T x_n, \qquad n = 0, 1, 2, \dots, \end{aligned} \tag{1.2}$$

where K is a nonempty, closed, convex subset of a normed linear space E, $T : K \to K$ is a self map and $\{\alpha_n\}, \{\beta_n\}$ are real sequences in $[0, 1]$. This iteration process is called as Ishikawa iteration process.

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Here we note that when $\beta_n = 0$, this iteration process reduces to Mann iteration scheme given by (1.1).

We consider an iteration scheme introduced by Owojori and Imoru [9] consisting of two nonlinear operators in normed linear spaces. The scheme is defined as follows: Let K be a nonempty, closed, convex subset of a normed linear space. Let $T : K \to K$ and $S : K \to K$ be two maps. Let $\{x_n\}_{n=0}^{\infty}$ be the sequence defined for arbitrary $x_0 \in K$ as

$$x_{n+1} = a_n x_n + b_n T x_n + c_n S x_n, \qquad n = 0, 1, 2, \ldots, \tag{1.3}$$

where $\{a_n\}, \{b_n\}$ and $\{c_n\}$ are real sequences in $[0, 1]$ with $a_n + b_n + c_n = 1$ and $b_{n+}c_n = \alpha_n$. The sequence $\{x_n\}_{n=0}^{\infty}$ generated by (1.3) is called the generalized Mann type iteration procedure.

We note that when $T = S$, the iteration defined by (1.3) reduces to the Mann iterative scheme given by (1.1).

In 2008, Thianwan [11] introduced the following two-step iteration,

$$\begin{aligned} x_{n+1} &= (1 - \alpha_n) y_n + \alpha_n T y_n \\ y_n &= (1 - \beta_n) x_n + \beta_n T x_n, \qquad n = 0, 1, 2, \ldots, \end{aligned} \tag{1.4}$$

where K is a nonempty, closed, convex subset of a normed linear space E, $T : K \to K$ is a mapping and $\{\alpha_n\}, \{\beta_n\} \subset (0, 1)$. He proved strong and weak convergence of the iterative scheme given in (1.4) to a common fixed point for two asymptotically nonexpansive mappings in uniformly convex Banach spaces. Making use of the two-step iteration defined by (1.4), Yildirim et al. [12] proved the strong convergence of this iteration process to a fixed point of Zamfirescu operators in arbitrary Banach spaces.

In this paper, we introduce another two-step iteration scheme in normed linear spaces which is given as follows:

$$\begin{aligned} x_{n+1} &= (1 - \alpha_n) y_n + \alpha_n S y_n \\ y_n &= (1 - \beta_n) x_n + \beta_n T x_n, \qquad n = 0, 1, 2, \ldots, \end{aligned} \tag{1.5}$$

where K is a nonempty, closed, convex subset of a normed linear space E, $T, S : K \to K$ are two mappings and $\{\alpha_n\}$, $\{\beta_n\}$ are sequences of positive numbers in $[0, 1]$.

Here we note that when $S = T$, this iteration reduces to iteration given by (1.4).

We need the following definitions in a metric space (X, d):

A mapping $T : X \to X$ is called an a-**contraction** if there exists $a \in [0, 1)$ such that

$$d(Tx, Ty) \leq a d(x, y), \qquad \text{for all } x, y \in X. \tag{z_1}$$

The map T is called a **Kannan mapping** [7] if there exists $b \in [0, \frac{1}{2})$ such that

$$d(Tx, Ty) \leq b[d(x, Tx) + d(y, Ty)], \qquad \text{for all } x, y \in X. \tag{z_2}$$

A similar definition is due to **Chatterjea** [5] : there exists $c \in [0, \frac{1}{2})$ such that

$$(z_3) \qquad d(Tx, Ty) \leq c[d(x, Ty) + d(y, Tx)], \qquad \text{for all } x, y \in X.$$

The conditions (z_1), (z_2) and (z_3) are independent contractive conditions [10].

Combining these three definitions, in 1972 Zamfirescu [13] obtained the following important fixed point theorem:

Theorem 1.1. *Let (X, d) be a complete metric space and $T : X \to X$ be a mapping for which there exist real numbers a, b and c with $a \in [0, 1), b \in [0, \frac{1}{2})$ and $c \in [0, \frac{1}{2})$ such that for all $x, y \in X$, at least one of the following conditions holds:*
(z_1) $d(Tx, Ty) \leq ad(x, y)$,
(z_2) $d(Tx, Ty) \leq b[d(x, Tx) + d(y, Ty)]$,
(z_3) $d(Tx, Ty) \leq c[d(x, Ty) + d(y, Tx)]$.
Then T has a unique fixed point x^* and the Picard iteration $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = Tx_n, \quad n \in \mathbb{N}$$

converges to x^* for any arbitrary $x_0 \in X$.

An operator T which satisfies at least one of the contractive conditions (z_1), (z_2) and (z_3) is called a **Zamfirescu operator** or a Z-operator.

In 2004, Berinde [3] proved the strong convergence of Ishikawa iterative process given by (1.2) to approximate fixed points of Zamfirescu operators in an arbitrary Banach space E. While proving the theorem, he made use of the condition,

$$(1.6) \qquad \|Tx - Ty\| \leq \delta \|x - y\| + 2\delta \|x - Tx\|$$

which holds for any $x, y \in E$ where $0 \leq \delta < 1$.

In this paper, we employ a condition introduced in [4] which is more general than condition (1.6) and establish fixed point theorems in normed linear spaces. The condition is defined as follows:
Let K be a nonempty, closed, convex subset of a normed linear space E and $T : K \to K$ a selfmap of K. There exists a constant $L \geq 0$ such that for all $x, y \in K$, we have

$$(1.7) \qquad \|Tx - Ty\| \leq e^{L\|x - Tx\|}(2\delta \|x - Tx\| + \delta \|x - y\|),$$

where $0 \leq \delta < 1$ and e^x denotes the exponential function of $x \in K$.

Throughout this paper, we call this condition as generalized Z-type condition.

Remark 1.1. If $L = 0$, in the above condition, we obtain

$$\|Tx - Ty\| \leq \delta \|x - y\| + 2\delta \|x - Tx\|,$$

which is the Zamfirescu condition used by Berinde in [3] where

$$\delta = \max\left\{a, \frac{b}{1-b}, \frac{c}{1-c}\right\}, \qquad 0 \leq \delta < 1,$$

while constants a, b and c are as defined in Theorem 1.1.

In this paper, we prove a strong convergence theorem of a generalized Mann type iteration scheme for the self mappings satisfying generalized Z-type condition in normed linear spaces. Making use of the same condition, convergence theorems are proved for two, two-step iteration schemes in normed linear spaces.

In order to prove our main results, we need the following lemma:

Lemma 1.1. [2] *Let $\{a_n\}_{n=0}^{\infty}, \{b_n\}_{n=0}^{\infty}, \{c_n\}_{n=0}^{\infty}$ be sequences of nonnegative numbers satisfying*

$$a_{n+1} \leq (1-\omega_n)a_n + b_n + c_n, \qquad \text{for all } n \geq 0,$$

where $\{\omega_n\}_{n=0}^{\infty} \subset [0,1]$. If $\sum_{n=0}^{\infty} \omega_n = \infty, b_n = O(\omega_n)$ and $\sum_{n=0}^{\infty} c_n < \infty$, then $\lim_{n\to\infty} a_n = 0$.

2. Main Results

Theorem 2.1. *Let K be a nonempty, closed, convex subset of a normed linear space. Let $T : K \to K$ and $S : K \to K$ be two self mappings satisfying generalized Z-type condition given by (1.7) with $F(T) \bigcap F(S) \neq \phi$ where $F(T)$ and $F(S)$ are the sets of fixed points of T and S respectively. For any $x_0 \in K$, let $\{x_n\}_{n=0}^{\infty}$ be the sequence defined by (1.3) where $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ are real sequences in $[0,1]$ with $a_n + b_n + c_n = 1$, $b_{n+}c_n = \alpha_n$ and $\sum_{n=0}^{\infty} \alpha_n = \infty$. Then $\{x_n\}_{n=0}^{\infty}$ converges strongly to a common fixed point of T and S.*

Proof. It follows from the assumption $F(T) \bigcap F(S) \neq \phi$ that T and S have a common fixed point in K, say x^*. Since T and S satisfy generalized Z-type condition given by (1.7), we have that for all $x, y \in K$, the following inequalities hold:

$$\|Tx - Ty\| \leq e^{L\|x-Tx\|}(2\delta \|x - Tx\| + \delta \|x - y\|) \tag{2.1}$$

and

$$\|Sx - Sy\| \leq e^{L\|x-Sx\|}(2\delta \|x - Sx\| + \delta \|x - y\|) \tag{2.2}$$

where $L \geq 0$, $0 \leq \delta < 1$ and $\delta = \max\left\{a, \dfrac{b}{1-b}, \dfrac{c}{1-c}\right\}$.

Now let $\{x_n\}_{n=0}^{\infty}$ be the iteration defined by (1.3) and $x_0 \in K$ be arbitrary. Then

$$\begin{aligned}
\|x_{n+1} - x^*\| &= \|a_n x_n + b_n Tx_n + c_n Sx_n - x^*\| \\
&= \|(1-\alpha_n)x_n + b_n Tx_n + c_n Sx_n - (a_n + b_n + c_n)x^*\| \\
&= \|(1-\alpha_n)x_n + b_n Tx_n + c_n Sx_n - (1-\alpha_n)x^* - b_n x^* - c_n x^*\| \\
&= \|(1-\alpha_n)(x_n - x^*) + b_n(Tx_n - x^*) + c_n(Sx_n - x^*)\| \\
&\leq (1-\alpha_n)\|x_n - x^*\| + b_n \|Tx_n - x^*\| + c_n \|Sx_n - x^*\| . \qquad (2.3)
\end{aligned}$$

Taking $x = x^*$ and $y = x_n$ in (2.1), we get

$$\begin{aligned}
\|Tx^* - Tx_n\| &\leq e^{L\|x^*-Tx^*\|}(2\delta \|x^* - Tx^*\| + \delta \|x^* - x_n\|) \\
&= e^{L\|x^*-x^*\|}(2\delta \|x^* - x^*\| + \delta \|x^* - x_n\|) \\
&= e^{L(0)}(2\delta(0) + \delta \|x_n - x^*\|),
\end{aligned}$$

which implies that

$$\|Tx_n - x^*\| \leq \delta \|x_n - x^*\|. \tag{2.4}$$

Similarly by taking $x = x^*$ and $y = x_n$ in (2.2), we obtain

$$\|Sx_n - x^*\| \leq \delta \|x_n - x^*\|. \tag{2.5}$$

Now using (2.4) and (2.5) in (2.3), we have

$$\begin{aligned} \|x_{n+1} - x^*\| &\leq (1-\alpha_n)\|x_n - x^*\| + b_n\delta\|x_n - x^*\| + c_n\delta\|x_n - x^*\| \\ &= (1-\alpha_n + b_n\delta + c_n\delta)\|x_n - x^*\| \\ &= (1-\alpha_n + \alpha_n\delta)\|x_n - x^*\|. \end{aligned}$$

Thus we have the inequality,

$$\|x_{n+1} - x^*\| \leq [1 - \alpha_n(1-\delta)]\|x_n - x^*\|, \qquad n = 0, 1, 2, \ldots.$$

Since $\{\alpha_n\} \in [0,1], 0 \leq \delta < 1$ and $\sum\limits_{n=0}^{\infty} \alpha_n = \infty$, by applying Lemma 1.1 we get

$$\lim_{n\to\infty} \|x_{n+1} - x^*\| = 0.$$

Thus $\{x_n\}_{n=0}^{\infty}$ converges strongly to x^* which is the common fixed point of T and S. ☐

Corollary 2.1. *Let K be a nonempty, closed, convex subset of a normed linear space. Let $T, S : K \to K$ be two Zamfirescu operators with $F(T) \bigcap F(S) \neq \phi$ where $F(T)$ and $F(S)$ are the sets of fixed points of T and S respectively. For any $x_0 \in K$, let $\{x_n\}_{n=0}^{\infty}$ be the sequence defined by (1.3) where $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ are real sequences in $[0,1]$ with $a_n + b_n + c_n = 1$, $b_n + c_n = \alpha_n$ and $\sum_{n=0}^{\infty} \alpha_n = \infty$. Then $\{x_n\}_{n=0}^{\infty}$ converges strongly to a common fixed point of T and S.*

Theorem 2.2. *Let K be a nonempty, closed, convex subset of a normed linear space E. Let $T : K \to K$ be a self map satisfying generalized Z-type condition given by (1.7) with $F(T) \neq \phi$ where $F(T)$ is the set of fixed points of T. For any $x_0 \in K$, let $\{x_n\}_{n=0}^{\infty}$ be the sequence defined by (1.4) where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences of positive numbers in $[0,1]$ satisfying $\sum_{n=0}^{\infty} \alpha_n = \infty$. Then $\{x_n\}_{n=0}^{\infty}$ converges strongly to the unique fixed point of T.*

Proof. From the assumption $F(T) \neq \phi$, it follows that T has a fixed point in K, say x^*. Since T satisfies generalized Z-type condition given by (1.7), we have the inequality (2.1) holds for all $x, y \in K$. Now using the sequence defined by (1.4), we get for any arbitrary $x_0 \in K$,

$$\begin{aligned} \|x_{n+1}-x^*\| &= \|(1-\alpha_n)y_n+\alpha_n Ty_n - x^*\| \\ &= \|(1-\alpha_n)(y_n-x^*)+\alpha_n(Ty_n-x^*)\| \\ &\leq (1-\alpha_n)\|y_n-x^*\|+\alpha_n\|Ty_n-x^*\| \\ &= (1-\alpha_n)\|y_n-x^*\|+\alpha_n\|Tx^*-Ty_n\|. \end{aligned}$$

Taking $x = x^*$ and $y = x_n$ in (2.1), the above inequality becomes

$$\begin{aligned} \|x_{n+1}-x^*\| &\leq (1-\alpha_n)\|y_n-x^*\|+\alpha_n\left[e^{L\|x^*-Tx^*\|}(2\delta\|x^*-Tx^*\|+\delta\|x^*-y_n\|)\right] \\ &= (1-\alpha_n)\|y_n-x^*\|+\alpha_n\left[e^{L\|x^*-x^*\|}(2\delta\|x^*-x^*\|+\delta\|x^*-y_n\|)\right] \\ &= (1-\alpha_n)\|y_n-x^*\|+\alpha_n\left[e^{L(0)}(2\delta(0)+\delta\|x^*-y_n\|)\right] \\ &= (1-\alpha_n)\|y_n-x^*\|+\alpha_n\delta\|y_n-x^*\|, \end{aligned}$$

which gives

$$\|x_{n+1}-x^*\| \leq (1-\alpha_n+\alpha_n\delta)\|y_n-x^*\|. \tag{2.6}$$

From (1.4) it follows that

$$\begin{aligned} \|y_n-x^*\| &= \|(1-\beta_n)x_n+\beta_n Tx_n-x^*\| \\ &= \|(1-\beta_n)(x_n-x^*)+\beta_n(Tx_n-x^*)\| \\ &\leq (1-\beta_n)\|x_n-x^*\|+\beta_n\|Tx_n-x^*\|. \end{aligned} \tag{2.7}$$

Combining inequalities (2.4), (2.6) and (2.7), we get

$$\begin{aligned} \|x_{n+1}-x^*\| &\leq (1-\alpha_n+\alpha_n\delta)\|y_n-x^*\| \\ &\leq (1-\alpha_n+\alpha_n\delta)[(1-\beta_n)\|x_n-x^*\|+\beta_n\|Tx_n-x^*\|] \\ &\leq (1-\alpha_n+\alpha_n\delta)[(1-\beta_n)\|x_n-x^*\|+\beta_n\delta\|x_n-x^*\|] \\ &= (1-\alpha_n+\alpha_n\delta)(1-\beta_n+\beta_n\delta)\|x_n-x^*\|. \end{aligned}$$

Now consider

$$\begin{aligned} (1-\alpha_n+\alpha_n\delta)(1-\beta_n+\beta_n\delta) &= 1-\alpha_n(1-\delta)-\beta_n(1-\delta)[1-\alpha_n(1-\delta)] \\ &\leq 1-\alpha_n(1-\delta). \end{aligned}$$

Hence we have the inequality,

$$\|x_{n+1}-x^*\| \leq [1-\alpha_n(1-\delta)]\|x_n-x^*\|, \qquad n=0,1,2,\ldots.$$

Since $0 \leq \delta < 1, \alpha_n \in [0,1]$ and $\sum_{n=0}^{\infty}\alpha_n = \infty$, setting $a_n = \|x_n - x^*\|$, $\omega_n = \alpha_n(1-\delta)$ and by applying Lemma 1.1, it results that

$$\lim_{n\to\infty}\|x_{n+1}-x^*\| = 0.$$

Therefore $\{x_n\}_{n=0}^{\infty}$ converges strongly to a fixed point of T.

To show uniqueness of the fixed point x^*, assume that $x_1^*, x_2^* \in F(T)$ and $x_1^* \neq x_2^*$.

Applying generalized Z-type condition given by (1.7) and using the fact that $0 \leq \delta < 1$, we obtain

$$\begin{aligned} \|x_1^* - x_2^*\| &= \|Tx_1^* - Tx_2^*\| \\ &\leq e^{L\|x_1^* - Tx_1^*\|}(2\delta\|x_1^* - Tx_1^*\| + \delta\|x_1^* - x_2^*\|) \\ &= e^{L\|x_1^* - x_1^*\|}(2\delta\|x_1^* - x_1^*\| + \delta\|x_1^* - x_2^*\|) \\ &= e^{L(0)}(2\delta(0) + \delta\|x_1^* - x_2^*\|) \\ &= \delta\|x_1^* - x_2^*\| \\ &< \|x_1^* - x_2^*\| \end{aligned}$$

which is a contradiction. Therefore $x_1^* = x_2^*$. Thus $\{x_n\}_{n=0}^{\infty}$ converges strongly to the unique fixed point of T. □

Corollary 2.2. [12, Theorem 2.1] *Let E be an arbitrary Banach space, K a closed, convex subset of E and $T : K \to K$ a Zamfirescu operator. Let $\{x_n\}_{n=0}^{\infty}$ be the two-step iteration defined by (1.4) and $x_0 \in K$, where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences of positive numbers in $[0, 1]$ satisfying $\sum_{n=0}^{\infty} \alpha_n = \infty$. Then $\{x_n\}_{n=0}^{\infty}$ converges strongly to the fixed point of T.*

Theorem 2.3. *Let K be a nonempty, closed, convex subset of a normed linear space E. Let $T, S : K \to K$ be two self mappings of K satisfying generalized Z-type condition given by (1.7) with $F(T) \bigcap F(S) \neq \phi$ where $F(T)$ and $F(S)$ are the sets of fixed points of T and S respectively. For any $x_0 \in K$, let $\{x_n\}_{n=0}^{\infty}$ be the sequence defined by (1.5) where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences of positive numbers in $[0, 1]$ satisfying $\sum_{n=0}^{\infty} \alpha_n = \infty$. Then $\{x_n\}_{n=0}^{\infty}$ converges strongly to a common fixed point of T and S.*

Proof. Since S satisfies generalized Z-type condition given by (1.7), we get that the inequality (2.2) holds for all $x, y \in K$. As $F(T) \bigcap F(S) \neq \phi$, let x^* be a common fixed point of S and T in K. Now for any arbitrary $x_0 \in K$,

$$\begin{aligned} \|x_{n+1} - x^*\| &= \|(1 - \alpha_n)y_n + \alpha_n Sy_n - x^*\| \\ &= \|(1 - \alpha_n)(y_n - x^*) + \alpha_n(Sy_n - x^*)\| \\ &\leq (1 - \alpha_n)\|y_n - x^*\| + \alpha_n\|Sy_n - x^*\| \\ &= (1 - \alpha_n)\|y_n - x^*\| + \alpha_n\|Sx^* - Sy_n\|. \end{aligned}$$

Taking $x = x^*$ and $y = y_n$ in (2.2), we obtain

$$\begin{aligned} \|x_{n+1} - x^*\| &\leq (1 - \alpha_n)\|y_n - x^*\| + \alpha_n\left[e^{L\|x^* - Sx^*\|}(2\delta\|x^* - Sx^*\| + \delta\|x^* - y_n\|)\right] \\ &= (1 - \alpha_n)\|y_n - x^*\| + \alpha_n\left[e^{L\|x^* - x^*\|}(2\delta\|x^* - x^*\| + \delta\|x^* - y_n\|)\right] \\ &= (1 - \alpha_n)\|y_n - x^*\| + \alpha_n\left[e^{L(0)}(2\delta(0) + \delta\|x^* - y_n\|)\right] \\ &= (1 - \alpha_n)\|y_n - x^*\| + \alpha_n\delta\|y_n - x^*\|, \end{aligned}$$

which gives

$$\|x_{n+1} - x^*\| \leq (1 - \alpha_n + \alpha_n \delta) \|y_n - x^*\|.$$

Now, applying iteration process (1.5) and proceeding with the arguments similar to those in the proof of Theorem 2.2, we obtain

$$\lim_{n \to \infty} \|x_{n+1} - x^*\| = 0.$$

Hence $\{x_n\}_{n=0}^{\infty}$ converges strongly to x^* which is a common fixed point of T and S. □

Corollary 2.3. *Let K be a nonempty, closed, convex subset of a normed linear space E. Let $T, S : K \to K$ be two Z-operators with $F(T) \bigcap F(S) \neq \phi$ where $F(T)$ and $F(S)$ are the sets of fixed points of T and S respectively. For any $x_0 \in K$, let $\{x_n\}_{n=0}^{\infty}$ be the sequence defined by* (1.5) *where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences of positive numbers in $[0, 1]$ satisfying $\sum_{n=0}^{\infty} \alpha_n = \infty$. Then $\{x_n\}_{n=0}^{\infty}$ converges strongly to a common fixed point of T and S.*

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