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A NOTE ON HOMOGENEOUS PRODUCTION MODELS

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Abstract. This short note corrects an error appeared in Theorem 3.1 of [1].

1. Statement and its proof

Theorem 3.1 of [1] is incorrect. The correct statement shall read as follows.

Theorem A. Let $Q = f(x_1, \dots, x_n)$ be a homogeneous production function of degree $d \neq 0$. Then the production hypersurface of Q has null Gauss-Kronecker curvature if and only if either

(a) the production function has constant return to scale, or
(b) the production function is of form:

$$f(x_1, \dots, x_n) = \left(x_1 \phi \left(\frac{x_2}{x_1}, \dots, \frac{x_n}{x_1} \right) \right)^d,$$

where $\phi(u_1, \dots, u_{n-1})$ is an $(n-1)$-input function satisfying the homogeneous Monge-Ampère equation: $\det(\phi_{ij}) = 0$.

Proof. Let $Q = f(x_1, \dots, x_n)$ be a homogeneous production function of degree $d \neq 0$. Then Q is a positive function which satisfies

$$f(tx_1, \dots, tx_n) = t^d f(x_1, \dots, x_n) \tag{1.1}$$

for some constant d.

If $d = 1$, the production function exhibits constant return to scale.

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If $d \neq 1$, then $Q^{1/d}$ is a linearly homogeneous function. Thus we may express the production function Q as

$$f(x_1,\dots,x_n) = \left(x_1\phi\left(\frac{x_2}{x_1},\dots,\frac{x_n}{x_1}\right)\right)^d \tag{1.2}$$

for some $(n-1)$-input function $\phi(u_1,\dots,u_{n-1})$. It follows from (1.2) after long computation that

$$\det(f_{ij}) = (d-1)d^n x_1^{n(d-2)}\left(\phi\left(\frac{x_2}{x_1},\dots,\frac{x_n}{x_1}\right)\right)^{n(d-1)+1}\det(\phi_{ij}), \tag{1.3}$$

where (ϕ_{ij}) is the Hessian matrix of $\phi(u_1,\dots,u_{n-1})$. Therefore, we may conclude from (1.3) that if the homogenous production function f satisfies the homogenous Monge-Ampère equation $\det(f_{ij}) = 0$, then the corresponding $(n-1)$-input function $\phi(u_1,\dots,u_{n-1})$ satisfies its homogeneous Monge-Ampère equation $\det(\phi_{ij}) = 0$.

Conversely, if the homogenous production function $f(x_1,\dots,x_n)$ has constant return to scale, then $d = 1$. Thus according to the Euler Homogeneous Function Theorem we have

$$x_1f_1 + x_2f_2 + \cdots + x_nf_n = f. \tag{1.4}$$

After taking the partial derivatives of (1.4) with respect to $x_1,\dots,x_n$, respectively, we obtain

$$\begin{aligned} x_1f_{11} + x_2f_{12} + \cdots + x_nf_{1n} &= 0,\\ x_1f_{12} + x_2f_{22} + \cdots + x_nf_{2n} &= 0,\\ &\vdots\\ x_1f_{1n} + x_2f_{2n} + \cdots + x_nf_{nn} &= 0, \end{aligned} \tag{1.5}$$

which implies that $\det(f_{ij}) = 0$. Consequently, the production hypersurface defined by f has null Gauss-Kronecker curvature.

Now, suppose that the homogenous production function is of the form

$$f(x_1,\dots,x_n) = \left(x_1\phi\left(\frac{x_2}{x_1},\dots,\frac{x_n}{x_1}\right)\right)^d, \tag{1.6}$$

where ϕ is an $(n-1)$-input function satisfying the homogeneous Monge-Ampère equation $\det(\phi_{ij}) = 0$. Then it follows from (1.3) that $\det(f_{ij}) = 0$ holds. Consequently, the production hypersurface given by f has null Gauss-Kronecker curvature as well. □

Remark 1.1. The $(n-1)$-input function ϕ appeared in case (2) of this theorem is not necessary homogeneous.

Remark 1.2. In the original statement of Theorem 3.1 appeared in [1], case (2) of the theorem was missing.

References

[1] B.-Y. Chen, *On some geometric properties of h-homogeneous production function in microeconomics*, Kragujevac J. Math. **35**(3) (2011), 343–357.

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