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FUNCTIONS AND BAIRE SPACES

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Abstract. Some results concerning functions that preserve Baire spaces in the context of images and preimages are obtained in *Section IV* of [R. C. Haworth, R. A. McCoy, *Baire spaces*, Dissertationes Math., **151**, PWN, Warszawa 1977]. In Section 2 of the present paper further results in this direction are offered. In Sections 3 and 4 a few theorems related to some other problems on baireness of topological spaces are proposed.

1. Preliminaries

Throughout, topological spaces are denoted by (X, τ) or (Y, σ). Let S be a subset of a space (X, τ). The ***interior*** and the ***closure*** of S in (X, τ) will be denoted by $\mathrm{int}_\tau(S)$ (or $\mathrm{int}\,(S)$) and $\mathrm{cl}_\tau(S)$ (or $\mathrm{cl}\,(S)$) respectively. The set S is said to be ***semi-open*** [12] (resp. ***preopen*** [13], ***semi-closed*** [3], ***preclosed***, ***regular open***) in (X, τ), if $S \subset \mathrm{cl}\,(\mathrm{int}\,(S))$ (resp. $S \subset \mathrm{int}\,(\mathrm{cl}\,(S))$, $S \supset \mathrm{int}\,(\mathrm{cl}\,(S))$, $S \supset \mathrm{cl}\,(\mathrm{int}\,(S))$, $S = \mathrm{int}\,(\mathrm{cl}\,(S))$). The collection of all semi-open (resp. preopen, semi-closed, pre-closed, closed, clopen) subsets of (X, τ) shall be denoted by $\mathrm{SO}\,(X, \tau)$ (resp. $\mathrm{PO}\,(X, \tau)$, $\mathrm{SC}\,(X, \tau)$, $\mathrm{PC}\,(X, \tau)$, $\mathrm{c}\,(X, \tau)$, $\mathrm{CO}\,(X, \tau)$). A set $S \in \mathrm{SO}\,(X, \tau)$ if and only if there exists an $O \in \tau$ such that $O \subset S \subset \mathrm{cl}\,(S)$ [12]. Every nonvoid semi-open set contains a nonvoid open subset [3, Remark 1.2]. The intersection of all $F \in \mathrm{SC}\,(X, \tau)$ with $S \subset F$ is called the ***semi-closure*** [3] of S in (X, τ). The union of all $U \in \mathrm{SO}\,(X, \tau)$ with $U \subset S$ is called the ***semi-interior*** [3] of S in (X, τ). The semi-closure and semi-interior of S are denoted respectively by $\mathrm{scl}\,(S)$ (or $\mathrm{scl}_\tau(S)$) and $\mathrm{sint}\,(S)$ (or $\mathrm{sint}_\tau(S)$). We have $S \in \mathrm{SO}\,(X, \tau)$

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(resp. $S \in \mathrm{SC}(X,\tau)$) iff $S = \mathrm{sint}(S)$ (resp. $S = \mathrm{scl}(S)$) [3, Theorem 1.4]. Also, $\mathrm{scl}(S) = \mathrm{int}(\mathrm{cl}(S))$ if and only if $S \in \mathrm{PO}(X,\tau)$ [11, Proposition 2.7(a)]. If $O \in \tau$ and $S \in \mathrm{SO}(X,\tau)$ then $O \cap S \in \mathrm{SO}(X,\tau)$ [3, Theorem 1.9]. A space (X,τ) is ***Baire*** if each nonempty set $S \in \tau$ is of the 2nd category (equivalently: $\mathrm{cl}(\bigcap_{n\in\mathbb{N}} O_n) = X$ for any family $\{O_n\}_{n\in\mathbb{N}} \subset \tau$ with $\mathrm{cl}(O_n) = X$ for each n; *or*: $\mathrm{cl}(\bigcap_{n\in\mathbb{N}} A_n) = X$ for any family $\{A_n\}_{n\in\mathbb{N}} \subset \mathrm{SO}(X,\tau)$ with $\mathrm{cl}(A_n) = X$ for each n, see [8]). A space (X,τ) is said to be $\mathcal{S}$-***connected*** [16] if there are no nonempty sets $A_1, A_2 \in \mathrm{SO}(X,\tau)$ such that $A_1 \cap A_2 = \varnothing$ and $A_1 \cup A_2 = X$. In the opposite case (X,τ) is called $\mathcal{S}$-***disconnected***.

2. Continuous and semi-open injections

Lemma 2.1. *If an injection $f : (X,\tau) \to (Y,\sigma)$ is continuous and closed, then for any subset $S \subset X$ we have*

$$\mathrm{int}_\sigma\Big(\mathrm{cl}_\sigma\big(f(S)\big)\Big) \subset f\Big(\mathrm{int}_\tau(\mathrm{cl}_\tau(S))\Big). \tag{2.1}$$

Proof. As f is closed we have $f^{-1}\Big(\mathrm{int}(\mathrm{cl}(f(S)))\Big) \subset f^{-1}\Big(\mathrm{int}(f(\mathrm{cl}(S)))\Big)$. But f is also continuous, so $f^{-1}\Big(\mathrm{int}(\mathrm{cl}(f(S)))\Big) \subset \mathrm{int}\Big(f^{-1}(f(\mathrm{cl}(S)))\Big) = \mathrm{int}(\mathrm{cl}(S))$. Consequently,

$$f\Big(f^{-1}\Big(\mathrm{int}(\mathrm{cl}(f(S)))\Big)\Big) \subset f\Big(\mathrm{int}(\mathrm{cl}(S))\Big).$$

Since f is closed, $\mathrm{int}(\mathrm{cl}(f(S))) \subset f(X)$. Thus, by injectivity of f the inclusion (2.1) holds. □

A function $f : (X,\tau) \to (Y,\sigma)$ is said to be ***semi-open*** [2] if $f(S) \in \mathrm{SO}(Y,\sigma)$ for every $S \in \tau$.

Theorem 2.1. *Let an injection $f : (X,\tau) \to (Y,\sigma)$ be continuous, semi-open, and closed. If (Y,σ) is Baire, then so is (X,τ).*

Proof. Suppose $G \in \tau$ is of first category in (X,τ). Then $f(G) = \bigcup_{k=1}^{\infty} f(N_k)$ where for each $k \in \mathbb{N}$ the sets N_k are nowhere dense in (X,τ). Using Lemma 2.1 we get that the images $f(N_k)$ are nowhere dense in (Y,σ), $k \in \mathbb{N}$. Hence the semi-open set $f(G) \subset Y$ is of first category. Thus there exists a nonempty set $O \in \sigma$, $O \subset f(G)$, which is of first category. It contradicts the assumption that (Y,σ) is Baire. □

Recall the following result: if a function $f : (X, \tau) \to (Y, \sigma)$ is continuous and open, then for any subset $S \subset X$ we have

$$\operatorname{int}_\sigma\Big(\operatorname{cl}_\sigma\big(f(S)\big)\Big) \supset f\Big(\operatorname{int}_\tau(\operatorname{cl}_\tau(S))\Big).$$

So, in view of Lemma 2.1 we get what follows.

Proposition 2.1. *If an injection $f : (X, \tau) \to (Y, \sigma)$ is continuous, open, and closed, then for any subset $S \subset X$*

$$\operatorname{int}_\sigma\Big(\operatorname{cl}_\sigma\big(f(S)\big)\Big) = f\Big(\operatorname{int}_\tau(\operatorname{cl}_\tau(S))\Big).$$

Corollary 2.1. *Let an injection $f : (X, \tau) \to (Y, \sigma)$ be continuous, open, and closed. Then, for any subset $S \subset X$,*

(a) *S is nowhere dense in (X, τ) if and only if $f(S)$ is nowhere dense in (Y, σ),*
(b) *if S is regular open in (X, τ), then $f(S)$ is regular open in (Y, σ).*

The following continuity-like property will be useful in the sequel.

Definition 2.1. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be ***s-perfectly continuous*** if the preimage $f^{-1}(V) \in \operatorname{CO}(X, \tau)$ for each $V \in \operatorname{SO}(Y, \sigma)$.

Every s-perfectly continuous function is perfectly continuous ($f^{-1}(V) \in \operatorname{CO}(X, \tau)$ for each $V \in \sigma$, see [1]), but the converse does not hold in general.

Example 2.1.

(a) Let $X = \{a, b, c\} = Y$, $\tau = \big\{\varnothing, X, \{a\}, \{b, c\}\big\}$, $\sigma = \big\{\varnothing, Y, \{a\}\big\}$. The function $\operatorname{id}_X : (X, \tau) \to (X, \sigma)$ is perfectly continuous, but it is not s-perfectly continuous: consider $\{a, b\} \in \operatorname{SO}(X, \sigma)$.
(b) If we put in (a) $Y = \{a, b, c, d\}$, then injective but not surjective id_X is perfectly continuous and not s-perfectly continuous, as well.

Recall that every perfectly continuous function is continuous, but the converse is not true, see [15, p.249]. Next, we shall establish

Theorem 2.2. *Let an injection $f : (X, \tau) \to (Y, \sigma)$ be s-perfectly continuous and semi-open. If (X, τ) is compact, (Y, σ) is Hausdorff and Baire, then (X, τ) is Baire.*

In order to prove this result we shall need some lemmas. The second one is well-known.

Lemma 2.2. [14, Theorem 5] *Let, for a* (Y,σ), $Y_0 \in \mathrm{SO}\,(Y,\sigma)$. *Then for every* $A \subset Y_0$, $A \in \mathrm{SO}\,(Y,\sigma)$ *if and only if* $A \in \mathrm{SO}\,(Y_0,\sigma_{Y_0})$.

Lemma 2.3. *If a function* $f : (X,\tau) \to (Y,\sigma)$ *is continuous,* (X,τ) *is compact, and* (Y,σ) *is Hausdorff, then for each subset* $S \subset X$ *one has*

$$f\Big(\mathrm{cl}_\tau(S)\Big) = \mathrm{cl}_\sigma(f(S)).$$

Recall that a collection of subsets of a space (X,τ) is called a ***pseudo-cover*** [9, p.13], if its union is dense in (X,τ). A pseudo-cover is said to be semi-open (resp. open) if all its members are semi-open (resp. open) in (X,τ).

Lemma 2.4 ([8] *for semi-open and* [9, Theorem 1.21] *for open case*)**.** *If there exists a semi-open (or open) pseudo-cover of* (X,τ) *whose members are Baire subspaces of* (X,τ)*, then* (X,τ) *is a Baire space as well.*

Lemma 2.5 ([8], [9, Proposition 1.14], [7, p.256 Problem 2])**.** *Any semi-open (hence open) subset of a Baire space is Baire too.*

Proof of Theorem 2.2. In virtue of Lemma 2.4 it is enough to show that each point of (X,τ) has an open neighbourhood which is Baire in (X,τ). So, let $x \in X$ be arbitrarily chosen. Then $f(x) \in f(X) \in \mathrm{SO}\,(Y,\sigma)$. Since (Y,σ) is Hausdorff, there exists $G_x \subsetneq Y$ with $f(x) \in G_x \in \sigma$. Hence $f(x) \in G_x \cap f(X) = V_x \in \mathrm{SO}\,(Y,\sigma)$, consequently by Lemma 2.2, $V_x \in \mathrm{SO}\,\big(f(X),\sigma_{f(X)}\big)$. Now consider a subset $f^{-1}(V_x) \in \tau$. Since f is continuous, then the restriction $f \restriction f^{-1}(V_x) = f_x\colon f^{-1}(V_x) \to V_x$ is continuous too. Thus $f_x\Big(\mathrm{cl}_{\tau_{f^{-1}(V_x)}}(S)\Big) \subset \mathrm{cl}_{\sigma_{V_x}}(f_x(S))$ for any $S \subset f^{-1}(V_x)$. Let $\{O_n\}_{n\in\mathbb{N}} \subset \tau_{f^{-1}(V_x)}$ be an arbitrary family of sets dense in $\Big(f^{-1}(V_x),\tau_{f^{-1}(V_x)}\Big)$. We shall show that

$$\mathrm{cl}_{\tau_{f^{-1}(V_x)}}\left(\bigcap_{n\in\mathbb{N}} O_n\right) = f^{-1}(V_x). \tag{2.2}$$

By continuity of f_x each $f_x(O_n) \in \mathrm{SO}\,(Y,\sigma)$, $n \in \mathbb{N}$, is dense in (V_x,σ_{V_x}). Obviously (by Lemma 2.2), $f_x(O_n) \in \mathrm{SO}\,(V_x,\sigma_{V_x})$ for each n. Since, by assumption, (Y,σ) is Baire, then (V_x,σ_{V_x}) is Baire as well (see Lemma 2.5). So, $\mathrm{cl}_{\tau_{V_x}}\Big(\bigcap_{n\in\mathbb{N}} f_x(O_n)\Big) = V_x$. But $f^{-1}(V_x) \in \mathrm{c}\,(X,\tau)$ and hence it is compact. Applying now Lemma 2.3 we get

$$f_x\left(\mathrm{cl}_{\tau_{f^{-1}(V_x)}}\left(\bigcap_{n\in\mathbb{N}} O_n\right)\right) = \mathrm{cl}_{\sigma_{V_x}}\left(f_x\left(\bigcap_{n\in\mathbb{N}} O_n\right)\right) = \mathrm{cl}_{\sigma_{V_x}}\left(\bigcap_{n\in\mathbb{N}} f_x(O_n)\right) = V_x.$$

Therefore (2.2) follows. □

A brief insight into the foregoing proof leads to the next result.

Theorem 2.3. *Let an injection $f : (X, \tau) \to (Y, \sigma)$ be perfectly continuous and open. If (X, τ) is compact, (Y, σ) is Hausdorff and Baire, then (X, τ) is Baire.*

Recall that a function $f : (X, \tau) \to (Y, \sigma)$ is called a ***semi-homeomorphism*** [4] if it is bijective, pre-semi-open (i.e., $f(U) \in \mathrm{SO}\,(Y, \sigma)$ for every $U \in \mathrm{SO}\,(X, \tau)$), and irresolute ($f^{-1}(V) \in \mathrm{SO}\,(X, \tau)$ for every $V \in \mathrm{SO}\,(Y, \sigma)$). Each homeomorphism is a semi-homeomorphism, but not conversely [4, Theorem 1.9 and Example 1.2]. The following interesting result is a consequence of [9, Theorem 4.1 and Proposition 4.3]; the details are left to the reader (we make use of [3, Remark 1.2]).

Proposition 2.2. *Let $f : (X, \tau) \to (Y, \sigma)$ be a semi-homeomorphism. Then (X, τ) is a Baire space if and only if (Y, σ) is Baire.*

3. Contra-continuity

A function $f : (X, \tau) \to (Y, \sigma)$ is said to be ***contra-continuous*** [5] if $f^{-1}(V) \in \mathrm{c}\,(X, \tau)$ for every $V \in \sigma$ (or, equivalently, $f^{-1}(F) \in \tau$ for every $F \in \mathrm{c}\,(Y, \sigma)$).

Theorem 3.1. *Let a surjection $f : (X, \tau) \to (Y, \sigma)$ be contra-continuous and open. Then (Y, σ) is a Baire space.*

Proof. Suppose $B \in \sigma$, $B \neq \varnothing$, is of first category in (Y, σ). So, $B = \bigcup_{n\in\mathbb{N}} B_n$ where each B_n is nowhere dense in (Y, σ). By contra-continuity and openness of f we calculate as follows.

$$\begin{aligned} f^{-1}(B) = f^{-1}\left(\bigcup_{n\in\mathbb{N}} B_n\right) &= \bigcup_{n\in\mathbb{N}} f^{-1}(B_n) \subset \bigcup_{n\in\mathbb{N}} f^{-1}(\mathrm{cl}\,(B_n)) = \\ &= \bigcup_{n\in\mathbb{N}} \mathrm{int}\left(f^{-1}(\mathrm{cl}\,(B_n))\right) \subset \bigcup_{n\in\mathbb{N}} f^{-1}(\mathrm{int}\,(\mathrm{cl}\,(B_n))) = \varnothing. \end{aligned}$$

A contradiction completes the proof. □

Let us remark that from the above proof it follows, under assumptions of Theorem 3.1, that each nonempty subset of a space (Y, σ) is of second category.

Definition 3.1. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be ***s-contra-precontinuous*** if the preimage $f^{-1}(V) \in \mathrm{PC}\,(X, \tau)$ for each $V \in \mathrm{SO}\,(Y, \sigma)$ (equivalently $f^{-1}(F) \in \mathrm{PO}\,(X, \tau)$ for each $F \in \mathrm{SC}\,(Y, \sigma)$).

Each s-contra-precontinuous function is contra-precontinuous ($f^{-1}(V) \in \mathrm{PC}\,(X,\tau)$ for each $V \in \sigma$ [10]) and not conversely.

Example 3.1. Let $X = \{a,b,c\}$, $\tau = \big\{\varnothing, X, \{b\}\big\}$, $\sigma = \big\{\varnothing, X, \{a\}\big\}$. Then $\mathrm{id}_X : (X,\tau) \to (X,\sigma)$ is contra-precontinuous, but not s-contra-precontinuous: consider $V = \{a,b\} \in \mathrm{SO}\,(X,\sigma)$.

Theorem 3.2. *Let a surjection $f : (X,\tau) \to (Y,\sigma)$ be s-contra-continuous, continuous and open. Then (Y,σ) is a Baire space.*

Proof. Let $\varnothing \neq B = \bigcup_{n\in\mathbb{N}} B_n \in \sigma$, where B_n is nowhere dense in (Y,σ) for every $n \in \mathbb{N}$. Since each nowhere dense set is semi-closed [3, Theorem 1.3], using our assumption we calculate as follows:

$$f^{-1}(B) = \bigcup_{n\in\mathbb{N}} f^{-1}(B_n) \subset \bigcup_{n\in\mathbb{N}} \mathrm{int}\left(\mathrm{cl}\left(f^{-1}(B_n)\right)\right) \subset$$
$$\subset \bigcup_{n\in\mathbb{N}} \mathrm{int}\left(f^{-1}(\mathrm{cl}\,(B_n))\right) \subset \bigcup_{n\in\mathbb{N}} f^{-1}(\mathrm{int}\,(\mathrm{cl}\,(B_n))) = \varnothing.$$

Thus $\varnothing \neq f^{-1}(B) = \varnothing$, a contradiction. □

Observe also that under assumptions of Theorem 3.2 every nonempty subset of (Y,σ) is of second category. As an example of a Baire space with such a property, it is enough to consider $X = \{a,b,c,d\}$ with

$$\tau = \big\{\varnothing, X, \{a\}, \{b\}, \{a,b\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}\big\}.$$

A function $f : (X,\tau) \to (Y,\sigma)$ is called ***contra-semicontinuous*** [6] if $f^{-1}(V) \in \mathrm{SC}\,(X,\tau)$ for every $V \in \sigma$ (equivalently, $f^{-1}(F) \in \mathrm{SO}\,(X,\tau)$ for every $F \in \mathrm{c}\,(Y,\sigma)$). In order to prove our next result we need the following improvement of Baire category theorem.

Lemma 3.1. [8] *Let (X,τ) be a Baire space. If $\{A_n\}_{n\in\mathbb{N}} \subset \mathrm{SC}\,(X,\tau)$ is a family covering X, then at least one A_n must contain a set from τ; i.e., have a nonvoid interior.*

Theorem 3.3. *Let (X,τ) be a Baire space and let (Y,σ) be Lindelöf. If there is a contra-semicontinuous bijection $f : (X,\tau) \to (Y,\sigma)$, then (X,τ) is $\mathcal{S}$-disconnected.*

Proof. As (Y,σ) is Lindelöf, by definition, it is Hausdorff. So, for every $y \in Y$ there is a $U_y \in \sigma$ with $y \in U_y \subsetneq Y$ and in turn (lindelöfness) $Y = \bigcup_{k=1}^{\infty} U_{y_k}$ for a certain countable set $\{y_k\}_{k=1}^{\infty} \subset Y$. Then $X = \bigcup_{k=1}^{\infty} f^{-1}(U_{y_k})$, where $f^{-1}(U_{y_k}) \in \mathrm{SC}\,(X,\tau)$, $k \in \mathbb{N}$. Applying Lemma 3.1 we pick a k_0 such that $\mathrm{int}_\tau\left(U_{y_{k_0}}\right) \neq \varnothing$. Hence $\varnothing \neq \mathrm{scl}_\tau\left(\mathrm{int}_\tau\left(U_{y_{k_0}}\right)\right) \subset f^{-1}\left(U_{y_{k_0}}\right) \subsetneq X$. By [11, Proposition 2.7(a)], $\mathrm{scl}\left(\mathrm{int}\left(U_{y_{k_0}}\right)\right) = \mathrm{int}\left(\mathrm{cl}\left(\mathrm{int}\left(U_{y_{k_0}}\right)\right)\right) = A_1$. Obviously, $\varnothing \neq A_2 = X \setminus A_1 \in \mathrm{SO}\,(X,\tau)$ and therefore (X,τ) is $\mathcal{S}$-disconnected. □

4. δ-open and δ^*-open functions

A function $f : (X,\tau) \to (Y,\sigma)$ is said to be ***δ-open*** [9] if $f^{-1}(N)$ is nowhere dense in (X,τ) for every nowhere dense subset N of (Y,σ).

Lemma 4.1. [8] *For every subset S of a space (X,τ), $\mathrm{cl}\,(\mathrm{sint}\,(S)) = \mathrm{cl}\,(\mathrm{int}\,(S))$.*

Theorem 4.1. *Let a surjective contra-semicontinuous function $f : (X,\tau) \to (Y,\sigma)$ be δ-open. If (Y,σ) is a $\mathcal{T}_1$-space having a dense subset $Y_0 \in \mathrm{SO}\,(Y,\sigma)$ with the property that $f^{-1}(\{y\})$ is a Baire subspace of (X,τ) for each $y \in Y$, then (X,τ) is Baire.*

Proof. We have $Y = \mathrm{cl}\,(Y_0) = \mathrm{cl}\,(\mathrm{sint}\,(Y_0)) = \mathrm{cl}\,(\mathrm{int}\,(Y_0))$. Consequently $\mathrm{int}\,(\mathrm{cl}\,(Y \setminus Y_0)) = \varnothing$ and, since f is δ-open, $\mathrm{int}\left(\mathrm{cl}\left(f^{-1}(Y \setminus Y_0)\right)\right) = \varnothing$. Hence, it may be easily seen that $\mathrm{cl}\left(\mathrm{int}\left(f^{-1}(Y_0)\right)\right) = X$. Then $f^{-1}(Y_0)$ is dense in (X,τ). By hypothesis, $\bigcup_{y \in Y_0} f^{-1}(\{y\})$ is also dense in (X,τ). Since, moreover, each preimage $f^{-1}(\{y\}) \in \mathrm{SO}\,(X,\tau)$ is Baire, by Lemma 2.4 the whole space (X,τ) is Baire. □

Definition 4.1. A function $f : (X,\tau) \to (Y,\sigma)$ will be called ***δ*-open*** if $f(N)$ is nowhere dense in (Y,σ) for every nowhere dense subset N of (X,τ).

In our last proof we will follow an idea due to Haworth and McCoy [9, p.48], despite their proof is far from being clear enough at some points.

Theorem 4.2. *Let a space (X,τ) satisfy the second axiom of countability, a space (Y,σ) be of second category in itself, and a surjection $f : (X,\tau) \to (Y,\sigma)$ be δ^*-open. If there exists a residual subset Z of (Y,σ) such that for each $z \in Z$ the preimage $f^{-1}(z)$ is of second category in itself, then (X,τ) is of second category.*

Proof. The notation is the same as in the proof of [9, Theorem 4.11]. Suppose (X,τ) is of first category in itself; i.e., $X = \bigcup_{n\in\mathbb{N}} F_n$ for some nowhere dense closed sets F_n in (X,τ), $n \in \mathbb{N}$. We set $M(F_n) = \left\{y \in Y \colon \mathrm{int}_{f^{-1}(y)}(f^{-1}(y) \cap F_n) \neq \varnothing\right\}$, $n \in \mathbb{N}$. Fix a countable base $\{U_i\}_{i\in\mathbb{N}}$ for (X,τ) and set also $M_i^n = \left\{y \in Y \colon \varnothing \neq f^{-1}(y) \cap U_i \subset F_n\right\}$,

$n, i \in \mathbb{N}$. We have $M(F_n) = \bigcup_{i\in\mathbb{N}} M_i^n$, $n \in \mathbb{N}$. Choose arbitrary n and consider a nonempty M_i^n, $i \in \mathbb{N}$. For any $y \in M_i^n$ we have $f^{-1}(y) \cap U_i \subset F_n$, so $\{y\} \cap f(U_i) \subset f(F_n)$. Consequently, $M_i^n \cap f(U_i) \subset f(F_n)$. Therefore, $\bigcup_{i\in\mathbb{N}} M_i^n \cap \bigcup_{i\in\mathbb{N}} f(U_i) = M(F_n) \cap Y = M(F_n) \subset f(F_n)$. By assumption the set $M(F_n)$ is nowhere dense and so $M = \bigcup_{n\in\mathbb{N}} M(F_n)$ has the first category in (Y, σ). The remaining steps of the proof are the same as at the end of the proof of [9, Theorem 4.1]. □

Corollary 4.1. *Theorem* 4.2 *is true if to assume all* $f^{-1}(z)$, $z \in Z$, *are Baire subspaces* (*instead of the second category assumption*).

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