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REMEMBERING ALESSANDRO OSSICINI

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[†] Alessandro Ossicini was born in Rome, on 9th September, 1921. He died in Rome, on 17th October 1999 at the age of 78 years, after a period of serious illness.

He grew in a ten people family (parents, two brothers and six sisters), but he lost his father in 1936 at the age of 15. He often remarked that the study of Mathematics, was for him, like finding another source of security. Those were the years of the Fascist dictatorship, in Italy, and all the family was strongly involved against it, for their ideals of justice and freedom. For these ideals he fought and later he was awarded a medal for Partisan activities in 1943-44. During the Partisan war he knew Vittoria Borgogno, also involved in the same group of freedom fighters, his future wife. A beautiful report of the pre-war and war period can be found in the book of his elder brother Adriano Ossicini[1].

In December 1944, just after the freeing of Rome in June, he got his first-class honours degree in Mathematics and Physics at the University of Rome "La Sapienza", and immediately later, in the difficult period of the reconstruction after the tragic events of the Second World War , he firstly started his teaching activity at the junior high school, at Veroli and, since 1955 in Rome. However he was still able to continue his academic studies concerning Special Functions, under the guidance of Giovanni Sansone. At that time Sansone was director of UMI (Unione Matematica Italiana) and Professor at the University of Florence.

From the economic point of view they were very difficult years. After the marriage (1946) he became father of a large family (four daughters and four sons), nevertheless all of us, his children, remember these years as years of happiness and serenity.

His studies and consequenty all the obtained results allowed him, in 1958, to get the University teaching qualification in Mathematical Analysis (he discussed a theme involving the Fundamental Theorem of Algebra) and then, in 1966, to be the winner in the competition for the chair of Numerical Analysis at the University of L'Aquila. In 1967 he obtained the transfer to the chair of Complements of Mathematics at the Faculty of Engineering of the University of Rome "La Sapienza", where he carried out an intense teaching and scientific activity up to the end of his tenure, in 1991, collaborating with several colleagues of the Institute of Applied Mathematics (the current Department of Mathematical Methods and Models for Applied Sciences) and in particular with Prof. Aldo Ghizzetti, Prof. Francesco Rosati and Dr. Maria Renata Martinelli.

His long research activity led him to publish over 70 works which can be mainly found in the MathSciNet. Alessandro Ossicini's studies concern various topics, including the following themes (the numbering refers to the list of works, that is included at the end of this document):

- Non trigonometrical Fourier series, orthogonal polynomials and special functions: cfr. [24]-[31]-[33]-[43]-[44]-[45]-[46]-[47]-[48]-[49]-[50]-[51]-[52]-[53]- [54]-

[55]-[56]-[57]-[58].

- Laplace transforms and applications: cfr. [23]-[36].
- Partial differential equations: cfr. [32]-[37]-[38]-[39]-[40]-[41]-[42].
- Singular integral equations: cfr. [21]-[22].
- Quasi-periodic functions: cfr. [29].
- Approximation theory, quadrature rules and applications: cfr. [12]-[15]-[16]-[18]-[19]-[20]-[25]-[26]-[28]-[30]-[34]-[35].
- Moment problems: cfr. [8].
- Evaluation of integrals and asymptotic behavior of integral transforms: cfr. [5]-[27].
- Ipergaussian quadrature rules and s-orthogonal polynomials: cfr. [2]-[3]-[4]-[6]-[7]-[9]-[10]-[11]-[13]-[14]-[17].

He was particularly proud of the two monographs written in collaboration with Aldo Ghizzetti :*Quadrature formulae* (Academic Press, New York, 1970) and *Trasformate di Laplace e calcolo simbolico* (UTET, Torino, 1971).

In 1994 he obtained the early retirement, because of the serious illness, which had worsened his health state. He spent his last years of life constantly nursed by his wife and encircled by his sons, daughters, and grandchildren.

Now we would like to add some words, related to the human qualities of our father. Our father had a reluctant character, but endowed of deep and tenacious feelings and the constant commitment to carry out his institutional tasks at his best. He was good and generous and possessed a deep religious faith. We have encountered and still often encounter old students of his. They all remember and appreciate him for his fatherly understanding and for the enthusiasm he transmitted, regarding studies and Science in general. A great part of his colleagues speak of him as an exemplary model.

We remember his constant presence near us, he took care of us, both in our life and in our studies. We all, sons and daughters, completed our University studies (some in Mathematics or Physics). Our personal story with Science is a story of love. This love was born hearing our father speaking of Nature, Maths, and Life.

‡ It is about 10 years as I work in the theory of quadrature formulas with multiple nodes and corresponding orthogonal polynomials. This part of the paper refers to the research achievements and significance of the work of Professor Alessandro Ossicini in that field of mathematics. During the period of almost 40 years he given many important results. Some of them will be mentioned here.

More than 100 years after Gauss published his famous method of approximate integration there appeared the idea of numerical integration involving multiple nodes. Taking any system of n distinct points $\{\tau_1, \tau_2, \ldots, \tau_n\}$ and n nonnegative integers $m_1, m_2, \ldots, m_n$, and starting from the Hermite interpolation formula, Chakalov in 1948 obtained the quadrature formula

$$\int_{\operatorname{supp} d\lambda} f(t)\, d\lambda(t) = \sum_{\nu=1}^{n} \sum_{i=0}^{m_\nu - 1} A_{i,\nu} f^{(i)}(\tau_\nu) + R(f), \tag{1}$$

with $d\lambda(t) = dt$ on the support $[-1, 1]$, which is exact for all polynomials of degree at most $\sum_{\nu=1}^{n} m_\nu - 1$. In that time it was been interesting how to determine the nodes τ_ν such that (1) be a quadrature of Gaussian type, i. e., a quadrature with the maximum degree of exactness.

For the quadrature formula (1), with $d\lambda(t) = dt$ on $[-1, 1]$ and $m_\nu = 2s + 1,\ \nu = 1, 2, \ldots, n$, the answer is given by P. Turán in 1950. Those formulae are called Gauss-Turán quadratures. The nodes τ_ν are the zeros of the polynomial $\pi_{n,s}$ which is known as s-orthogonal.

The case with the weight function $d\lambda(t) = w(t)\, dt$ on an interval $[a, b]$ has been investigated by Italian mathematicians Ossicini et al., and also by Chakalov, Stroud, Stancu, Ionescu, Pavel, etc.

In the more general case, for a given sequence of nonnegative integers $\sigma = (s_1, s_2, \ldots, s_n)$ the corresponding quadrature formula (1) with $m_\nu = 2s_\nu + 1,\ \nu =$

$1, 2, \ldots, n$, has the maximum degree of exactness $N-1$ ($N = 2\sum_{\nu=1}^{n} s_\nu + 2n$) if and only if the nodes τ_ν, for which

$$\tau_1 < \tau_2 < \ldots < \tau_n, \tag{2}$$

are the zeros of the so called σ-orthogonal polynomial $\pi_{n,\sigma}$. This generalization of the Gauss-Turán quadrature formula (for $d\lambda(t) = dt$ on (a, b)) to rules having nodes with arbitrary multiplicities was derived independently by Chakalov in 1954 and Popoviciu in 1955. Because of that those formulas are called the Chakalov-Popoviciu quadrature formulae. The existence of such quadrature rules was proved by Chakalov, Popoviciu, Morelli and Verna, and existence and uniqueness (the first time!), subject to (2), by Ossicini (with Ghizzetti) [16].

The proof of existence and uniqueness of the s-orthogonal polynomials $\pi_{n,s}$ which have n distinct real zeros which are all contained in the open interval (a, b) was given by Turán. It was also proved by Ossicini using different methods [35] (see also [25]).

In the case $w(t) = 1$ of Legendre s-orthogonal polynomials, where the normalization factor is taken to have $\pi_{n,s}(1) = 1$, Ossicini (with Ghizzetti) [31] proved that $|\pi_{n,s}(t)| \leq 1$ and

$$\int_{-1}^{1} [\pi_{n,s}(t)]^{2s+2}\, dt = \frac{2}{1 + (2s+2)n}\,.$$

It would be interesting to determine this value for other classical weights.

In 1930 Bernstein showed that the Chebyshev polynomials of first kind are s-orthogonal on $[-1, 1]$ for each $s \geq 0$. Ossicini (with Rosati) [14] in 1975 found three other measures for which the s-orthogonal polynomials can be identified as Chebyshev polynomials of the second, third, and fourth kind.

Considering the set of Jacobi polynomials Ossicini (with Rosati) [3] showed that the only Jacobi polynomials which are s-orthogonal for a positive integer s are the Chebyshev polynomials of first kind.

In 1978 Ossicini (with Rosati) [11] given the proof of convergence in the Gauss-Turán quadratures for $f \in C^{2s}[a, b]$.

Some properties of Cotes numbers in the Gauss-Turán quadrature, some inequalities related to zeros of s-orthogonal polynomials, as well as density of the zeros of

s-orthogonal polynomials, were investigated in 1981 and 1992 by Ossicini (with Rosati and Martinelli) [9], [4].

In 1968 Ossicini [28] studied the remainder term in Gauss-Turán quadrature formulas. An influence function is introduced, its relevant properties are investigated, and in classes of functions $AC^{N-1}[a,b]$, $B^N[a,b]$, $C^N[a,b]$ the error estimates are given.

In 1975 Ossicini (with Rosati) [14] found the contour integral representation

$$R(f) = \frac{1}{2\pi i} \oint_\Gamma \frac{\varrho_{n,s}(z)}{[\pi_{n,s}(z)]^{2s+1}} f(z)\, dz, \quad \varrho_{n,s}(z) = \int_a^b \frac{[\pi_{n,s}(t)]^{2s+1}}{z-t}\, d\lambda(t),$$

where $[a,b] \subset int\,\Gamma$ and f is an analytic function in $int\,\Gamma \cup \Gamma$. Taking as Γ confocal ellipses Ossicini et al. [2] considered two special Chebyshev measures and determined estimates for the corresponding remainders, from which they proved the convergence and found the rate of convergence of the corresponding Gauss-Turán quadratures.

Up to now only one book [25] on the results to the theory quadratures with multiples nodes and corresponding orthogonal polynomials has been published. This one was been written by Ossicini (with Ghizzetti) in 1970.

The results of Professor Alessandro Ossicini have been developed, continued, cited, by many known mathematicians. I mention only several of them: B. Bojanov, W. Gautschi, C. Michelli, G. V. Milovanović, Y. G. Shi, D. D. Stancu, etc. His results are cited in numerous papers, books, etc. He left a deep trail in the fields of his researches, and, especially, made the important theoretical progress in the theory of quadrature formulas with multiple nodes.

References

[1] Adriano Ossicini "Un isola sul Tevere" Editori Riuniti (1999).

[2] A. Ossicini, M.R. Martinelli, F. Rosati, *Characteristic functions and s-orthogonal polynomials*. (Italian), Rend. Mat. Appl. (7) **14** (1994), 355–366.

[3] A. Ossicini, F. Rosati, *s-orthogonal Jacobi polynomials*. (Italian), Rend. Mat. Appl. (7) **12** (1992), 399–403.

[4] M.R. Martinelli, A. Ossicini, F. Rosati, *Density of the zeros of a system of s-orthogonal polynomials.* (Italian), Rend. Mat. Appl. (7) **12** (1992), 235–249.

[5] A. Ossicini, F. Rosati, *Calculation of an integral.* (Italian), Rend. Mat. Appl. (7) **8** (1988), 45–59.

[6] M.R. Martinelli, A. Ossicini, F. Rosati, *Trigonometric hyper-Gaussian functionals in the analytic field.* (Italian), Le Matematiche (Catania) **39** (1984), 205–212.

[7] A. Ossicini, F. Rosati, *Trigonometric hyper-Gaussian functionals.* (Italian), Calcolo **20** (1983), 101–109.

[8] A. Ghizzetti, A. Ossicini, *Polinomi ortogonali e problema dei momenti.* (Italian), Pubblicazioni Serie III, 203. Istituto per le Applicazioni del Calcolo "Mauro Picone" (IAC), Roma, 1981, 139 pp.

[9] A. Ossicini, F. Rosati, *Christoffel numbers and s-orthogonal polynomials.* (Italian), E. B. Christoffel (Aachen/Monschau, 1979), pp. 148–157, Birkhäuser, Basel-Boston, Mass., 1981.

[10] A. Ossicini, F. Rosati, *Comparison theorems for the zeros of s-orthogonal polynomials.* (Italian), Calcolo **16** (1979), 371–381.

[11] A. Ossicini, F. Rosati, *The convergence of hyper-Gaussian functionals.* (Italian), Rend. Mat. (6) **11** (1978), no. 1, 97–108.

[12] A. Ossicini, F. Rosati, *Procedimenti interpolatori nella valutazione gaussiana di integrali a valore principale.* (Italian), Le Matematiche (Catania) **31** (1976), 193–213 (1977).

[13] A. Ghizzetti, A. Ossicini, *Generalizzazione dei polinomi s-ortogonali e dei corrispondenti sviluppi in serie.* (Italian), Atti Accad. Sci. Torino Cl. Sci. Fis. Mat. Natur. **109** (1975), 371–379.

[14] A. Ossicini, F. Rosati, *Funzioni caratteristiche nelle formule di quadratura gaussiane con nodi multipli.* (Italian), Collection of articles dedicated to Giovanni Sansone on the occasion of his eighty-fifth birthday. Boll. Un. Mat. Ital. (4) **11** (1975), Suppl., 224–237.

[15] A. Ossicini, F. Rosati, *Sulla valutazione numerica di integrali.* (Italian), Collection of articles dedicated to Mauro Picone on the occasion of his ninetieth birthday. Rend. Mat. (6) **8** (1975), 133–142.

[16] A. Ghizzetti, A. Ossicini, *Sull'esistenza e unicità delle formule di quadratura gaussiane.* (Italian), Collection of articles dedicated to Mauro Picone on the occasion of his ninetieth birthday. Rend. Mat. (6) **8** (1975), 1–15.

[17] A. Ghizzetti, A. Ossicini, *Polinomi s-ortogonali e sviluppi in serie ad essi collegati.* (Italian), Mem. Accad. Sci. Torino Cl. Sci. Fis. Mat. Natur. (4) **18** (1974), ii+16 pp.

[18] A. Ossicini, F. Rosati, *Alcune osservazioni sugli operatori di tipo W di Polya.* (Italian), Le Matematiche (Catania) **28** (1973), 82–92 (1974).

[19] A. Ossicini, F. Rosati, *Operatori differenziali permutabili e loro applicazioni.* (Italian), Rend. Mat. (6) **6** (1973), 865–919.

[20] A. Ghizzetti, A. Ossicini, F. Rosati, *Formule di quadratura con funzione di influenza di segno costante.* (Italian), Calcolo **10** (1973), 87–100.

[21] A. Ghizzetti, A. Ossicini, *Errata: "Studio di una particolare equazione integrale singolare di Wiener-Hopf", (Rend. Mat. (6)* **4** *(1971), 813–853).* (Italian), Rend. Mat. (6) **6** (1973), 491.

[22] A. Ghizzetti, A. Ossicini, *Studio di una particolare equazione integrale singolare di Wiener-Hopf.* (Italian), Rend. Mat. (6) **4** (1971), 813–853 (1972).

[23] A. Ghizzetti, A. Ossicini, *Trasformate di Laplace e calcolo simbolico.* (Italian), Unione Tipografico-Editrice Torinese, Torino, 1971, xi+474 pp.

[24] A. Ossicini, *Su una rappresentazione integrale delle funzioni di Jacobi di seconda specie.* (Italian), Atti Accad. Sci. Torino Cl. Sci. Fis. Mat. Natur. **104** (1970), 779–791.

[25] A. Ghizzetti, A. Ossicini, *Quadrature formulae.* Academic Press, New York 1970, 192 pp.

[26] A. Ossicini, *Alcune formule di quadratura per il calcolo della parte finita e del valore principale di integrali divergenti.* (Italian), Rend. Mat. (6) **2** (1969), 385–401.

[27] A. Ghizzetti, A. Ossicini, *Studio asintotico di una particolare funzione.* (Italian), Rend. Mat. (6) **2** (1969) 1–13.

[28] A. Ossicini, *Le funzioni di influenza nel problema di Gauss sulle formule di quadratura.* (Italian), Le Matematiche (Catania) **23** (1968), 7–30.

[29] A. Ossicini, *Generalizzazione di successioni quasi periodiche.* (Italian), Rend. Mat. (6) **1** (1968), 100–117.

[30] A. Ossicini, *Sulle costanti di Christoffel della formula di quadratura di Gauss-Jacobi.* (Italian), Ist. Lombardo Accad. Sci. Lett. Rend. A **101** (1967), 169–180.

[31] A. Ghizzetti, A. Ossicini, *Su un nuovo tipo di sviluppo di una funzione in serie di polinomi.* (Italian), Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. (8) **43** (1967), 21–29.

[32] A. Ghizzetti, F. Rosati, A. Ossicini, *Sul segno delle funzioni di Green di problemi differenziali ordinari di limiti.* Quaderno n. 9, Pubblicazione dell'Istituto Nazionale per le Applicazioni del Calcolo, **685** (1967).

[33] A. Ossicini, *Risoluzioni di un sistema di equazioni funzionali*. Quaderno n. 7, Pubblicazione dell'Istituto Nazionale per le Applicazioni del Calcolo, **660** (1966).

[34] A. Ossicini, *Sulle formule della quadrature di Tschebycheff*. Quaderno n 7, Pubblicazione dell'Istituto Nazionale per le Applicazioni del Calcolo, **660** (1966).

[35] A. Ossicini, *Costruzione di formule di quadratura di tipo Gaussiano*. (Italian), Ann. Mat. Pura Appl. (4) **72** (1966), 213–237.

[36] A. Ossicini, *Calcolo di un' antitrasformata di Lapace*. Estratto da: Quaderno n. 6, Pubblicazione dell'Istituto Nazionale per le Applicazioni del Calcolo **655** (1965).

[37] A. Ossicini, *Problema singolare di Cauchy, relativo ad una generalizzazione dell'equazione di Eulero-Poisson-Darboux. Il caso* $k \leq -1$. (Italian), Rend. Mat. e Appl. (5) **23** (1964), 40–65.

[38] A. Ossicini, *Problema singolare di Cauchy, relativo ad una generalizzazione dell'equazione di Eulero-Poisson-Darboux*. (Italian), Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. (8) **35** (1963), 454–459.

[39] A. Ossicini, F. Rosati, *Sulla regolarità alla frontiera di soluzioni di equazioni ellittiche*. (Italian), Atti Accad. Sci. Torino Cl. Sci. Fis. Mat. Natur. **98** (1963/1964), 893–907.

[40] A. Ossicini, F. Rosati, *Sugli integrali tripli di espressioni lineari alle derivate parziali del* 4° *ordine*. (Italian), Le Matematiche (Catania) **18** (1963), 40–53.

[41] A. Ossicini, *Sugli integrali doppi di espressioni lineari alle derivate parziali del* 4° *ordine*. (Italian), Rend. Mat. e Appl. (5) **21** (1962), 100–124.

[42] A. Ossicini, F. Rosati, *Sugli integrali tripli di espressioni lineari alle derivate parziali*. (Italian), Le Matematiche (Catania) **17** (1962), 10–37.

[43] A. Ossicini, *Nuova formula asintotica per i polinomi di Jacobi.* (Italian), Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat. (8) **32** (1962), 624–629.

[44] A. Ossicini, *Nuova formula asintotica per i polinomi di Laguerre.* (Italian), Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat. (8) **32** (1962), 488–494.

[45] A. Ossicini, *Sui coefficienti di Gegenbauer-Stieltjes di una funzione non decrescente.* (Italian), Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat. (8) **32** (1962), 313–319.

[46] A. Ossicini, *Sulla sommabilità H_q delle serie di Jacobi.* (Italian), Rend. Mat. e Appl. (5) **19** (1960), 430–442.

[47] A. Ossicini, *Sui coefficienti di Fourier di una funzione rispetto al sistema dei polinomi ultrasferici.* (Italian), Ann. Mat. Pura Appl. (4) **51** (1960), 161–166.

[48] A. Ossicini, *Sulla convergenza di serie multiple.* (Italian), Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) **21** (1956), 382–386.

[49] A. Ossicini, *Sulla sommabilità di Cesaro delle serie di Legendre.* (Italian), Boll. Un. Mat. Ital. (3) **10** (1955), 521–526.

[50] A. Ossicini, *Polinomi associati alle funzioni ultrasferiche di seconda specie.* (Italian), Boll. Un. Mat. Ital. (3) **9** (1954), 170–177.

[51] A. Ossicini, *Funzioni di Legendre di seconda specie e polinomi ultrasferici.* (Italian), Boll. Un. Mat. Ital. (3) **8** (1953), 304–309.

[52] A. Ossicini, *Funzione generatrice dei prodotti di due particolari polinomi di Jacobi.* (Italian), Boll. Un. Mat. Ital. (3) **8** (1953), 45–52.

[53] A. Ossicini, *Funzione generatrice dei prodotti di due polinomi ultrasferici.* (Italian), Boll. Un. Mat. Ital. (3) **7** (1952), 315–320.

[54] A. Ossicini, *Formula e serie de approssimazione asintotica delle funzioni ultrasferiche di seconda specie.* (Italian), Boll. Un. Mat. Ital. (3) **7** (1952), 48–53.

[55] A. Ossicini, *Sulle funzioni ultrasferiche di seconda specie.* (Italian), Boll. Un. Mat. Ital. (3) **6** (1951), 311–315.

[56] A. Ossicini, *Sulla sommabilità delle serie di Legendre.* (Italian), Boll. Un. Mat. Ital. (3) **6** (1951), 218–225.

[57] A. Ossicini, *Immediata limitazione delle derivate di ordine superiore dei polinomi ultra-sferici.* (Italian), Boll. Un. Mat. Ital. (3) **6** (1951), 110–112.

[58] A. Ossicini, *Il calcolo simbolico e la propagazione del calore in una ipersfera dello spazio euclideo ad n dimensioni.* Estratto dagli Annali della Scuola Normale Superiore di Pisa Serie III Vol. V Fasc. III IV. (1951).