

A SURVEY ON STRONGLY REGULAR GRAPHS WITH $m_2 = qm_3$ AND $m_3 = qm_2$

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ABSTRACT. We say that a regular graph G of order n and degree $r \geq 1$ (which is not the complete graph) is strongly regular if there exist non-negative integers τ and θ such that $|S_i \cap S_j| = \tau$ for any two adjacent vertices i and j , and $|S_i \cap S_j| = \theta$ for any two distinct non-adjacent vertices i and j , where S_k denotes the neighborhood of the vertex k . Let $\lambda_1 = r$, λ_2 and λ_3 be the distinct eigenvalues of a connected strongly regular graph. Let $m_1 = 1$, m_2 and m_3 denote the multiplicity of r , λ_2 and λ_3 , respectively. We here survey results related to the parameters n , r , τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 2, 3, \dots, 12$.

1. INTRODUCTION

Let G be a simple graph of order n with vertex set $V(G) = \{1, 2, \dots, n\}$. The spectrum of G consists of the eigenvalues $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$ of its $(0,1)$ adjacency matrix A and is denoted by $\sigma(G)$. We say that a regular graph G of order n and degree $r \geq 1$ (which is not the complete graph K_n) is strongly regular if there exist non-negative integers τ and θ such that $|S_i \cap S_j| = \tau$ for any two adjacent vertices i and j , and $|S_i \cap S_j| = \theta$ for any two distinct non-adjacent vertices i and j , where $S_k \subseteq V(G)$ denotes the neighborhood of the vertex k . We know that a regular connected graph G is strongly regular if and only if it has exactly three distinct eigenvalues [1] (see also [3]). Let $\lambda_1 = r$, λ_2 and λ_3 denote the distinct eigenvalues of a connected strongly regular graph G . Let $m_1 = 1$, m_2 and m_3 denote the multiplicity of r , λ_2 and λ_3 . Further, let $\bar{r} = (n - 1) - r$, $\bar{\lambda}_2 = -\lambda_3 - 1$ and $\bar{\lambda}_3 = -\lambda_2 - 1$ denote the distinct

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eigenvalues of the strongly regular graph $\overline{G}$, where $\overline{G}$ denotes the complement of G . Then $\overline{\tau} = n - 2r - 2 + \theta$ and $\overline{\theta} = n - 2r + \tau$, where $\overline{\tau} = \tau(\overline{G})$ and $\overline{\theta} = \theta(\overline{G})$.

Remark 1.1. (i) If G is a disconnected strongly regular graph of degree r , then $G = mK_{r+1}$, where mH denotes the m -fold union of the graph H .

(ii) G is a disconnected strongly regular graph if and only if $\theta = 0$.

Remark 1.2. (i) A strongly regular graph G of order $n = 4k + 1$ and degree $r = 2k$ with $\tau = k - 1$ and $\theta = k$ is called a conference graph.

(ii) A strongly regular graph is a conference graph if and only if $m_2 = m_3$.

(iii) If $m_2 \neq m_3$, then G is an integral graph.

Note. We say that a connected or disconnected graph G is integral if its spectrum $\sigma(G)$ consists only of integral values.

We have recently started to investigate strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$, where q is a positive integer [4]. In the same work we have described the parameters n , r , τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 2, 3, 4$. In particular, we have described in [5], [6] and [7] the parameters n , r , τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 5, 6, \dots, 12$. We here survey results related to the parameters of strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 2, 3, \dots, 12$, as follows.

Proposition 1.1 ([2]). *Let G be a connected or disconnected strongly regular graph of order n and degree r . Then,*

$$(1.1) \quad r^2 - (\tau - \theta + 1)r - (n - 1)\theta = 0.$$

Proposition 1.2 ([2]). *Let G be a connected strongly regular graph of order n and degree r . Then,*

$$(1.2) \quad 2r + (\tau - \theta)(m_2 + m_3) + \delta(m_2 - m_3) = 0,$$

where $\delta = \lambda_2 - \lambda_3$.

Remark 1.3 ([4]). Using the same procedure applied in [4] we can establish the parameters n , r , τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for any fixed value $q \in \mathbb{N}$, as follows. First, let $m_3 = p$, $m_2 = qp$ and $n = (q + 1)p + 1$, where $q \in \mathbb{N}$. Using (1.2) we obtain $r = p(|\lambda_3| - q\lambda_2)$. Let $|\lambda_3| - q\lambda_2 = t$ where $t = 1, 2, \dots, q$. Let $\lambda_2 = k$ where k is a positive integer. Then, (i) $\lambda_3 = -(qk + t)$; (ii) $\tau - \theta = -((q - 1)k + t)$; (iii) $\delta = (q + 1)k + t$; (iv) $r = pt$ and (v) $\theta = pt - qk^2 - kt$. Using (ii), (iv) and (v) we can easily see that (1.1) is reduced to

$$(1.3) \quad (p + 1)t^2 - ((q + 1)p + 1)t + q(q + 1)k^2 + 2qkt = 0.$$

Second, let $m_2 = p$, $m_3 = qp$ and $n = (q + 1)p + 1$, where $q \in \mathbb{N}$. Using (1.2) we obtain $r = p(q|\lambda_3| - \lambda_2)$. Let $q|\lambda_3| - \lambda_2 = t$, where $t = 1, 2, \dots, q$. Let $\lambda_3 = -k$, where k is a positive integer. Then, (i) $\lambda_2 = qk - t$; (ii) $\tau - \theta = (q - 1)k - t$; (iii) $\delta = (q + 1)k - t$;

(iv) $r = pt$ and (v) $\theta = pt - qk^2 + kt$. Using (ii), (iv) and (v) we can easily see that (1.1) is reduced to

$$(1.4) \quad (p+1)t^2 - ((q+1)p+1)t + q(q+1)k^2 - 2qkt = 0.$$

Using (1.3) and (1.4) we can obtain for $t = 1, 2, \dots, q$ the corresponding classes of strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$, respectively.

2. MAIN RESULTS

Remark 2.1. Since $m_2(\overline{G}) = m_3(G)$ and $m_3(\overline{G}) = m_2(G)$ we note that if $m_2(G) = qm_3(G)$, then $m_3(\overline{G}) = qm_2(\overline{G})$.

Remark 2.2. In Theorems 2.1, 2.2, ..., 2.11 the complements of strongly regular graphs appear in pairs in (k^0) and $(\overline{k}^0)$ classes, where k denotes the corresponding number of a class.

Remark 2.3. $\overline{\alpha K_\beta}$ is a strongly regular graph of order $n = \alpha\beta$ and degree $r = (\alpha-1)\beta$ with $\tau = (\alpha-2)\beta$ and $\theta = (\alpha-1)\beta$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -\beta$ with $m_2 = \alpha(\beta-1)$ and $m_3 = \alpha-1$.

In order to demonstrate a method which is applied to describe the parameters n , r , τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$, we shall establish the parameters of strongly regular graphs with $m_2 = 2m_3$ and $m_3 = 2m_2$, as follows.

Proposition 2.1. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 2m_3$. Then G belongs to the class $(\overline{2}^0)$ or (3^0) represented in Theorem 2.1.*

Proof. Let $m_3 = p$, $m_2 = 2p$ and $n = 3p + 1$ where $p \in \mathbb{N}$. Using (1.2) we obtain $r = p(|\lambda_3| - 2\lambda_2)$. Let $|\lambda_3| - 2\lambda_2 = t$ where $t = 1, 2$. Let $\lambda_2 = k$ where k is a positive integer. Then according to Remark 1.3 we have (i) $\lambda_3 = -(2k+t)$; (ii) $\tau - \theta = -(k+t)$; (iii) $\delta = 3k+t$; (iv) $r = pt$ and (v) $\theta = pt - 2k^2 - kt$. In this case we can easily see that (1.3) is reduced to

$$(2.1) \quad (p+1)t^2 - (3p+1)t + 6k^2 + 4kt = 0.$$

CASE 1. ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(2k+1)$, $\tau - \theta = -(k+1)$, $\delta = 3k+1$, $r = p$ and $\theta = p - 2k^2 - k$. Using (2.1) we find that $p = k(3k+2)$. So, we obtain that G is a strongly regular graph of order $n = (3k+1)^2$ and degree $r = k(3k+2)$ with $\tau = k^2 - 1$ and $\theta = k(k+1)$.

CASE 2. ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = k$ and $\lambda_3 = -(2k+2)$, $\tau - \theta = -(k+2)$, $\delta = 3k+2$, $r = 2p$ and $\theta = 2p - 2k^2 - 2k$. Using (2.1) we find that $p = (k+1)(3k+1)$. Replacing k with $k-1$ we arrive at $p = k(3k-2)$. So, we obtain that G is a strongly regular graph of order $n = (3k-1)^2$ and degree $r = 2k(3k-2)$ with $\tau = (k-1)(4k+1)$ and $\theta = 2k(2k-1)$. $\square$

Proposition 2.2. *Let G be a connected strongly regular graph of order n and degree r with $m_3 = 2m_2$. Then G belongs to the class (2^0) or $(\bar{3}^0)$ represented in Theorem 2.1.*

Proof. Let $m_2 = p$, $m_3 = 2p$ and $n = 3p + 1$ where $p \in \mathbb{N}$. Using (1.2) we obtain $r = p(2|\lambda_3| - \lambda_2)$. Let $2|\lambda_3| - \lambda_2 = t$ where $t = 1, 2$. Let $\lambda_3 = -k$ where k is a positive integer. Then according to Remark 1.3 we have (i) $\lambda_2 = 2k - t$; (ii) $\tau - \theta = k - t$; (iii) $\delta = 3k - t$; (iv) $r = pt$ and (v) $\theta = pt - 2k^2 + kt$. In this case we can easily see that (1.4) is reduced to

$$(2.2) \quad (p+1)t^2 - (3p+1)t + 6k^2 - 4kt = 0.$$

CASE 1. ($t = 1$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 2k - 1$ and $\lambda_3 = -k$, $\tau - \theta = k - 1$, $\delta = 3k - 1$, $r = p$ and $\theta = p - 2k^2 + k$. Using (2.2) we find that $p = k(3k - 2)$. So we obtain that G is a strongly regular graph of order $n = (3k - 1)^2$ and degree $r = k(3k - 2)$ with $\tau = k^2 - 1$ and $\theta = k(k - 1)$.

CASE 2. ($t = 2$). Using (i), (ii), (iii), (iv) and (v) we find that $\lambda_2 = 2k - 2$ and $\lambda_3 = -k$, $\tau - \theta = k - 2$, $\delta = 3k - 2$, $r = 2p$ and $\theta = 2p - 2k^2 + 2k$. Using (2.2) we find that $p = (k - 1)(3k - 1)$. Replacing k with $k + 1$ we arrive at $p = k(3k + 2)$. So we obtain that G is a strongly regular graph of order $n = (3k + 1)^2$ and degree $r = 2k(3k + 2)$ with $\tau = (k + 1)(4k - 1)$ and $\theta = 2k(2k + 1)$. $\square$

Theorem 2.1. *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 2m_3$ or $m_3 = 2m_2$. Then, G is one of the following strongly regular graphs.*

- (1⁰) G is the complete bipartite graph of order $n = 4$ and degree $r = 2$ with $\tau = 0$ and $\theta = 2$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -2$ with $m_2 = 2$ and $m_3 = 1$.
- (2⁰) G is a strongly regular graph of order $n = (3k - 1)^2$ and degree $r = k(3k - 2)$ with $\tau = k^2 - 1$ and $\theta = k(k - 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 2k - 1$ and $\lambda_3 = -k$ with $m_2 = k(3k - 2)$ and $m_3 = 2k(3k - 2)$.
- ($\bar{2}^0$) G is a strongly regular graph of order $n = (3k - 1)^2$ and degree $r = 2k(3k - 2)$ with $\tau = (k - 1)(4k + 1)$ and $\theta = 2k(2k - 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k - 1$ and $\lambda_3 = -2k$ with $m_2 = 2k(3k - 2)$ and $m_3 = k(3k - 2)$.
- (3⁰) G is a strongly regular graph of order $n = (3k + 1)^2$ and degree $r = k(3k + 2)$ with $\tau = k^2 - 1$ and $\theta = k(k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(2k + 1)$ with $m_2 = 2k(3k + 2)$ and $m_3 = k(3k + 2)$.
- ($\bar{3}^0$) G is a strongly regular graph of order $n = (3k + 1)^2$ and degree $r = 2k(3k + 2)$ with $\tau = (k + 1)(4k - 1)$ and $\theta = 2k(2k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 2k$ and $\lambda_3 = -(k + 1)$ with $m_2 = k(3k + 2)$ and $m_3 = 2k(3k + 2)$.

Proof. First, according to Remark 2.3 we have $\alpha(\beta - 1) = 2(\alpha - 1)$, from which we find that $\alpha = 2$, $\beta = 2$. In view of this we obtain the strongly regular graph represented in Theorem 2.1 (1⁰). Next, according to Proposition 2.1 it turns out that G belongs to the class ($\bar{2}^0$) or (3⁰) if $m_2 = 2m_3$. According to Proposition 2.2 it turns out that G belongs to the class (2⁰) or ($\bar{3}^0$) if $m_3 = 2m_2$. $\square$

Theorem 2.2 ([4]). *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 3m_3$ or $m_3 = 3m_2$. Then, G is one of the following strongly regular graphs.*

- (1⁰) G is the strongly regular graph $\overline{3K_3}$ of order $n = 9$ and degree $r = 6$ with $\tau = 3$ and $\theta = 6$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -3$ with $m_2 = 6$ and $m_3 = 2$.
- (2⁰) G is a strongly regular graph of order $n = (4k - 1)^2$ and degree $r = 2k(2k - 1)$ with $\tau = k^2 + k - 1$ and $\theta = k(k - 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 3k - 1$ and $\lambda_3 = -k$ with $m_2 = 2k(2k - 1)$ and $m_3 = 6k(2k - 1)$.
- (2⁰) G is a strongly regular graph of order $n = (4k - 1)^2$ and degree $r = 6k(2k - 1)$ with $\tau = 9k^2 - 5k - 1$ and $\theta = 3k(3k - 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k - 1$ and $\lambda_3 = -3k$ with $m_2 = 6k(2k - 1)$ and $m_3 = 2k(2k - 1)$.
- (3⁰) G is a strongly regular graph of order $n = (4k + 1)^2$ and degree $r = 2k(2k + 1)$ with $\tau = k^2 - k - 1$ and $\theta = k(k + 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(3k + 1)$ with $m_2 = 6k(2k + 1)$ and $m_3 = 2k(2k + 1)$.
- (3⁰) G is a strongly regular graph of order $n = (4k + 1)^2$ and degree $r = 6k(2k + 1)$ with $\tau = 9k^2 + 5k - 1$ and $\theta = 3k(3k + 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 3k$ and $\lambda_3 = -(k + 1)$ with $m_2 = 2k(2k + 1)$ and $m_3 = 6k(2k + 1)$.

Theorem 2.3 ([4]). *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 4m_3$ or $m_3 = 4m_2$. Then, G is one of the following strongly regular graphs.*

- (1⁰) G is the complete bipartite graph $K_{3,3}$ of order $n = 6$ and degree $r = 3$ with $\tau = 0$ and $\theta = 3$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -3$ with $m_2 = 4$ and $m_3 = 1$.
- (2⁰) G is the strongly regular graph $\overline{4K_4}$ of order $n = 16$ and degree $r = 12$ with $\tau = 8$ and $\theta = 12$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -4$ with $m_2 = 12$ and $m_3 = 3$.
- (3⁰) G is a strongly regular graph of order $n = (5k - 1)^2$ and degree $r = k(5k - 2)$ with $\tau = k^2 + 2k - 1$ and $\theta = k(k - 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 4k - 1$ and $\lambda_3 = -k$ with $m_2 = k(5k - 2)$ and $m_3 = 4k(5k - 2)$.
- (3⁰) G is a strongly regular graph of order $n = (5k - 1)^2$ and degree $r = 4k(5k - 2)$ with $\tau = 16k^2 - 7k - 1$ and $\theta = 4k(4k - 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k - 1$ and $\lambda_3 = -4k$ with $m_2 = 4k(5k - 2)$ and $m_3 = k(5k - 2)$.
- (4⁰) G is a strongly regular graph of order $n = (5k + 1)^2$ and degree $r = k(5k + 2)$ with $\tau = k^2 - 2k - 1$ and $\theta = k(k + 1)$, where $k \geq 3$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(4k + 1)$ with $m_2 = 4k(5k + 2)$ and $m_3 = k(5k + 2)$.
- (4⁰) G is a strongly regular graph of order $n = (5k + 1)^2$ and degree $r = 4k(5k + 2)$ with $\tau = 16k^2 + 7k - 1$ and $\theta = 4k(4k + 1)$, where $k \geq 3$. Its eigenvalues are $\lambda_2 = 4k$ and $\lambda_3 = -(k + 1)$ with $m_2 = k(5k + 2)$ and $m_3 = 4k(5k + 2)$.
- (5⁰) G is a strongly regular graph of order $n = 6(5k - 1)^2$ and degree $r = 2(30k^2 - 12k + 1)$ with $\tau = 24k^2 - 15k + 1$ and $\theta = 6k(4k - 1)$, where $k \in \mathbb{N}$. Its

eigenvalues are $\lambda_2 = 3k - 1$ and $\lambda_3 = -(12k - 2)$ with $m_2 = 4(30k^2 - 12k + 1)$ and $m_3 = 30k^2 - 12k + 1$.

- ($\bar{5}^0$) G is a strongly regular graph of order $n = 6(5k - 1)^2$ and degree $r = 3(30k^2 - 12k + 1)$ with $\tau = 18k(3k - 1)$ and $\theta = 3(3k - 1)(6k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 12k - 3$ and $\lambda_3 = -3k$ with $m_2 = 30k^2 - 12k + 1$ and $m_3 = 4(30k^2 - 12k + 1)$.
- (6^0) G is a strongly regular graph of order $n = 6(5k + 1)^2$ and degree $r = 2(30k^2 + 12k + 1)$ with $\tau = 24k^2 + 15k + 1$ and $\theta = 6k(4k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 12k + 2$ and $\lambda_3 = -(3k + 1)$ with $m_2 = 30k^2 + 12k + 1$ and $m_3 = 4(30k^2 + 12k + 1)$.
- ($\bar{6}^0$) G is a strongly regular graph of order $n = 6(5k + 1)^2$ and degree $r = 3(30k^2 + 12k + 1)$ with $\tau = 18k(3k + 1)$ and $\theta = 3(3k + 1)(6k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 3k$ and $\lambda_3 = -(12k + 3)$ with $m_2 = 4(30k^2 + 12k + 1)$ and $m_3 = 30k^2 + 12k + 1$.

Theorem 2.4 ([5]). *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 5m_3$ or $m_3 = 5m_2$. Then, G is one of the following strongly regular graphs.*

- (1^0) G is the strongly regular graph $\overline{5K_5}$ of order $n = 25$ and degree $r = 20$ with $\tau = 15$ and $\theta = 20$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -5$ with $m_2 = 20$ and $m_3 = 4$.
- (2^0) G is a strongly regular graph of order $n = (6k - 1)^2$ and degree $r = 2k(3k - 1)$ with $\tau = k^2 + 3k - 1$ and $\theta = k(k - 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 5k - 1$ and $\lambda_3 = -k$ with $m_2 = 2k(3k - 1)$ and $m_3 = 10k(3k - 1)$.
- ($\bar{2}^0$) G is a strongly regular graph of order $n = (6k - 1)^2$ and degree $r = 10k(3k - 1)$ with $\tau = 25k^2 - 9k - 1$ and $\theta = 5k(5k - 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k - 1$ and $\lambda_3 = -5k$ with $m_2 = 10k(3k - 1)$ and $m_3 = 2k(3k - 1)$.
- (3^0) G is a strongly regular graph of order $n = (6k + 1)^2$ and degree $r = 2k(3k + 1)$ with $\tau = k^2 - 3k - 1$ and $\theta = k(k + 1)$, where $k \geq 4$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(5k + 1)$ with $m_2 = 10k(3k + 1)$ and $m_3 = 2k(3k + 1)$.
- ($\bar{3}^0$) G is a strongly regular graph of order $n = (6k + 1)^2$ and degree $r = 10k(3k + 1)$ with $\tau = 25k^2 + 9k - 1$ and $\theta = 5k(5k + 1)$, where $k \geq 4$. Its eigenvalues are $\lambda_2 = 5k$ and $\lambda_3 = -(k + 1)$ with $m_2 = 2k(3k + 1)$ and $m_3 = 10k(3k + 1)$.

Theorem 2.5 ([5]). *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 6m_3$ or $m_3 = 6m_2$. Then, G is one of the following strongly regular graphs.*

- (1^0) G is the complete bipartite graph $K_{4,4}$ of order $n = 8$ and degree $r = 4$ with $\tau = 0$ and $\theta = 4$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -4$ with $m_2 = 6$ and $m_3 = 1$.
- (2^0) G is the strongly regular graph $\overline{3K_5}$ of order $n = 15$ and degree $r = 10$ with $\tau = 5$ and $\theta = 10$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -5$ with $m_2 = 12$ and $m_3 = 2$.

- (3⁰) G is the strongly regular graph $\overline{6K_6}$ of order $n = 36$ and degree $r = 30$ with $\tau = 24$ and $\theta = 30$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -6$ with $m_2 = 30$ and $m_3 = 5$.
- (4⁰) G is a strongly regular graph of order $n = (7k - 1)^2$ and degree $r = k(7k - 2)$ with $\tau = k^2 + 4k - 1$ and $\theta = k(k - 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 6k - 1$ and $\lambda_3 = -k$ with $m_2 = k(7k - 2)$ and $m_3 = 6k(7k - 2)$.
- (4⁰) G is a strongly regular graph of order $n = (7k - 1)^2$ and degree $r = 6k(7k - 2)$ with $\tau = 36k^2 - 11k - 1$ and $\theta = 6k(6k - 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k - 1$ and $\lambda_3 = -6k$ with $m_2 = 6k(7k - 2)$ and $m_3 = k(7k - 2)$.
- (5⁰) G is a strongly regular graph of order $n = (7k + 1)^2$ and degree $r = k(7k + 2)$ with $\tau = k^2 - 4k - 1$ and $\theta = k(k + 1)$, where $k \geq 5$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(6k + 1)$ with $m_2 = 6k(7k + 2)$ and $m_3 = k(7k + 2)$.
- (5⁰) G is a strongly regular graph of order $n = (7k + 1)^2$ and degree $r = 6k(7k + 2)$ with $\tau = 36k^2 + 11k - 1$ and $\theta = 6k(6k + 1)$, where $k \geq 5$. Its eigenvalues are $\lambda_2 = 6k$ and $\lambda_3 = -(k + 1)$ with $m_2 = k(7k + 2)$ and $m_3 = 6k(7k + 2)$.
- (6⁰) G is a strongly regular graph of order $n = 2(7k - 2)^2$ and degree $r = 3(14k^2 - 8k + 1)$ with $\tau = 18k^2 - 16k + 2$ and $\theta = 6k(3k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 2k - 1$ and $\lambda_3 = -(12k - 3)$ with $m_2 = 6(14k^2 - 8k + 1)$ and $m_3 = 14k^2 - 8k + 1$.
- (6⁰) G is a strongly regular graph of order $n = 2(7k - 2)^2$ and degree $r = 4(14k^2 - 8k + 1)$ with $\tau = 2k(16k - 7)$ and $\theta = 4(2k - 1)(4k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 12k - 4$ and $\lambda_3 = -2k$ with $m_2 = 14k^2 - 8k + 1$ and $m_3 = 6(14k^2 - 8k + 1)$.
- (7⁰) G is a strongly regular graph of order $n = 2(7k + 2)^2$ and degree $r = 3(14k^2 + 8k + 1)$ with $\tau = 18k^2 + 16k + 2$ and $\theta = 6k(3k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 12k + 3$ and $\lambda_3 = -(2k + 1)$ with $m_2 = 14k^2 + 8k + 1$ and $m_3 = 6(14k^2 + 8k + 1)$.
- (7⁰) G is a strongly regular graph of order $n = 2(7k + 2)^2$ and degree $r = 4(14k^2 + 8k + 1)$ with $\tau = 2k(16k + 7)$ and $\theta = 4(2k + 1)(4k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 2k$ and $\lambda_3 = -(12k + 4)$ with $m_2 = 6(14k^2 + 8k + 1)$ and $m_3 = 14k^2 + 8k + 1$.
- (8⁰) G is a strongly regular graph of order $n = 15(7k - 1)^2$ and degree $r = 2(105k^2 - 30k + 2)$ with $\tau = 60k^2 - 35k + 3$ and $\theta = 10k(6k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k - 1$ and $\lambda_3 = -(30k - 4)$ with $m_2 = 6(105k^2 - 30k + 2)$ and $m_3 = 105k^2 - 30k + 2$.
- (8⁰) G is a strongly regular graph of order $n = 15(7k - 1)^2$ and degree $r = 5(105k^2 - 30k + 2)$ with $\tau = 5(5k - 1)(15k - 1)$ and $\theta = 5(5k - 1)(15k - 2)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 30k - 5$ and $\lambda_3 = -5k$ with $m_2 = 105k^2 - 30k + 2$ and $m_3 = 6(105k^2 - 30k + 2)$.
- (9⁰) G is a strongly regular graph of order $n = 15(7k + 1)^2$ and degree $r = 2(105k^2 + 30k + 2)$ with $\tau = 60k^2 + 35k + 3$ and $\theta = 10k(6k + 1)$, where $k \in \mathbb{N}$. Its

eigenvalues are $\lambda_2 = 30k + 4$ and $\lambda_3 = -(5k + 1)$ with $m_2 = 105k^2 + 30k + 2$ and $m_3 = 6(105k^2 + 30k + 2)$.

- ($\bar{9}^0$) G is a strongly regular graph of order $n = 15(7k + 1)^2$ and degree $r = 5(105k^2 + 30k + 2)$ with $\tau = 5(5k + 1)(15k + 1)$ and $\theta = 5(5k + 1)(15 + 2)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k$ and $\lambda_3 = -(30k + 5)$ with $m_2 = 6(105k^2 + 30k + 2)$ and $m_3 = 105k^2 + 30k + 2$.

Theorem 2.6 ([5]). *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 7m_3$ or $m_3 = 7m_2$. Then, G is one of the following strongly regular graphs.*

- (1^0) G is the strongly regular graph $\overline{7K_7}$ of order $n = 49$ and degree $r = 42$ with $\tau = 35$ and $\theta = 42$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -7$ with $m_2 = 42$ and $m_3 = 6$.
- (2^0) G is a strongly regular graph of order $n = (8k - 1)^2$ and degree $r = 2k(4k - 1)$ with $\tau = k^2 + 5k - 1$ and $\theta = k(k - 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 7k - 1$ and $\lambda_3 = -k$ with $m_2 = 2k(4k - 1)$ and $m_3 = 14k(4k - 1)$.
- ($\bar{2}^0$) G is a strongly regular graph of order $n = (8k - 1)^2$ and degree $r = 14k(4k - 1)$ with $\tau = 49k^2 - 13k - 1$ and $\theta = 7k(7k - 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k - 1$ and $\lambda_3 = -7k$ with $m_2 = 14k(4k - 1)$ and $m_3 = 2k(4k - 1)$.
- (3^0) G is a strongly regular graph of order $n = (8k + 1)^2$ and degree $r = 2k(4k + 1)$ with $\tau = k^2 - 5k - 1$ and $\theta = k(k + 1)$, where $k \geq 6$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(7k + 1)$ with $m_2 = 14k(4k + 1)$ and $m_3 = 2k(4k + 1)$.
- ($\bar{3}^0$) G is a strongly regular graph of order $n = (8k + 1)^2$ and degree $r = 14k(4k + 1)$ with $\tau = 49k^2 + 13k - 1$ and $\theta = 7k(7k + 1)$, where $k \geq 6$. Its eigenvalues are $\lambda_2 = 7k$ and $\lambda_3 = -(k + 1)$ with $m_2 = 2k(4k + 1)$ and $m_3 = 14k(4k + 1)$.
- (4^0) G is a strongly regular graph of order $n = 105(8k - 3)^2$ and degree $r = 3(840k^2 - 630k + 118)$ with $\tau = 945k^2 - 765k + 153$ and $\theta = 15(3k - 1)(21k - 8)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 15k - 6$ and $\lambda_3 = -(105k - 39)$ with $m_2 = 7(840k^2 - 630k + 118)$ and $m_3 = 840k^2 - 630k + 118$.
- ($\bar{4}^0$) G is a strongly regular graph of order $n = 105(8k - 3)^2$ and degree $r = 5(840k^2 - 630k + 118)$ with $\tau = 5(525k^2 - 387k + 71)$ and $\theta = 15(5k - 2)(35k - 13)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 105k - 40$ and $\lambda_3 = -(15k - 5)$ with $m_2 = 840k^2 - 630k + 118$ and $m_3 = 7(840k^2 - 630k + 118)$.
- (5^0) G is a strongly regular graph of order $n = 105(8k + 3)^2$ and degree $r = 3(840k^2 + 630k + 118)$ with $\tau = 945k^2 + 765k + 153$ and $\theta = 15(3k + 1)(21k + 8)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 105k + 39$ and $\lambda_3 = -(15k + 6)$ with $m_2 = 840k^2 + 630k + 118$ and $m_3 = 7(840k^2 + 630k + 118)$.
- ($\bar{5}^0$) G is a strongly regular graph of order $n = 105(8k + 3)^2$ and degree $r = 5(840k^2 + 630k + 118)$ with $\tau = 5(525k^2 + 387k + 71)$ and $\theta = 15(5k + 2)(35k + 13)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 15k + 5$ and $\lambda_3 = -(105k + 40)$ with $m_2 = 7(840k^2 + 630k + 118)$ and $m_3 = 840k^2 + 630k + 118$.

Theorem 2.7 ([5]). *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 8m_3$ or $m_3 = 8m_2$. Then, G is one of the following strongly regular graphs.*

- (1⁰) G is the complete bipartite graph $K_{5,5}$ of order $n = 10$ and degree $r = 5$ with $\tau = 0$ and $\theta = 5$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -5$ with $m_2 = 8$ and $m_3 = 1$.
- (2⁰) G is the strongly regular graph $\overline{4K_7}$ of order $n = 28$ and degree $r = 21$ with $\tau = 14$ and $\theta = 21$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -7$ with $m_2 = 24$ and $m_3 = 3$.
- (3⁰) G is the strongly regular graph $\overline{8K_8}$ of order $n = 64$ and degree $r = 56$ with $\tau = 48$ and $\theta = 56$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -8$ with $m_2 = 56$ and $m_3 = 7$.
- (4⁰) G is a strongly regular graph of order $n = (9k - 1)^2$ and degree $r = k(9k - 2)$ with $\tau = k^2 + 6k - 1$ and $\theta = k(k - 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 8k - 1$ and $\lambda_3 = -k$ with $m_2 = k(9k - 2)$ and $m_3 = 8k(9k - 2)$.
- (4⁰) G is a strongly regular graph of order $n = (9k - 1)^2$ and degree $r = 8k(9k - 2)$ with $\tau = 64k^2 - 15k - 1$ and $\theta = 8k(8k - 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k - 1$ and $\lambda_3 = -8k$ with $m_2 = 8k(9k - 2)$ and $m_3 = k(9k - 2)$.
- (5⁰) G is a strongly regular graph of order $n = (9k + 1)^2$ and degree $r = k(9k + 2)$ with $\tau = k^2 - 6k - 1$ and $\theta = k(k + 1)$, where $k \geq 7$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(8k + 1)$ with $m_2 = 8k(9k + 2)$ and $m_3 = k(9k + 2)$.
- (5⁰) G is a strongly regular graph of order $n = (9k + 1)^2$ and degree $r = 8k(9k + 2)$ with $\tau = 64k^2 + 15k - 1$ and $\theta = 8k(8k + 1)$, where $k \geq 7$. Its eigenvalues are $\lambda_2 = 8k$ and $\lambda_3 = -(k + 1)$ with $m_2 = k(9k + 2)$ and $m_3 = 8k(9k + 2)$.
- (6⁰) G is a strongly regular graph of order $n = 4(9k - 4)^2$ and degree $r = 3(2k - 1)(18k - 7)$ with $\tau = 18k(2k - 1)$ and $\theta = (3k - 2)(12k - 5)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 24k - 11$ and $\lambda_3 = -(3k - 1)$ with $m_2 = (2k - 1)(18k - 7)$ and $m_3 = 8(2k - 1)(18k - 7)$.
- (6⁰) G is a strongly regular graph of order $n = 4(9k - 4)^2$ and degree $r = 6(2k - 1)(18k - 7)$ with $\tau = 3(48k^2 - 45k + 10)$ and $\theta = 2(3k - 1)(24k - 11)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 3k - 2$ and $\lambda_3 = -(24k - 10)$ with $m_2 = 8(2k - 1)(18k - 7)$ and $m_3 = (2k - 1)(18k - 7)$.
- (7⁰) G is a strongly regular graph of order $n = 4(9k + 4)^2$ and degree $r = 3(2k + 1)(18k + 7)$ with $\tau = 18k(2k + 1)$ and $\theta = (3k + 2)(12k + 5)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 3k + 1$ and $\lambda_3 = -(24k + 11)$ with $m_2 = 8(2k + 1)(18k + 7)$ and $m_3 = (2k + 1)(18k + 7)$.
- (7⁰) G is a strongly regular graph of order $n = 4(9k + 4)^2$ and degree $r = 6(2k + 1)(18k + 7)$ with $\tau = 3(48k^2 + 45k + 10)$ and $\theta = 2(3k + 1)(24k + 11)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 24k + 10$ and $\lambda_3 = -(3k + 2)$ with $m_2 = (2k + 1)(18k + 7)$ and $m_3 = 8(2k + 1)(18k + 7)$.

- (8⁰) G is a strongly regular graph of order $n = 10(9k - 1)^2$ and degree $r = 4(90k^2 - 20k + 1)$ with $\tau = 160k^2 - 55k + 3$ and $\theta = 20k(8k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k - 1$ and $\lambda_3 = -(40k - 4)$ with $m_2 = 8(90k^2 - 20k + 1)$ and $m_3 = 90k^2 - 20k + 1$.
- (8⁰) G is a strongly regular graph of order $n = 10(9k - 1)^2$ and degree $r = 5(90k^2 - 20k + 1)$ with $\tau = 10k(25k - 4)$ and $\theta = 5(5k - 1)(10k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 40k - 5$ and $\lambda_3 = -5k$ with $m_2 = 90k^2 - 20k + 1$ and $m_3 = 8(90k^2 - 20k + 1)$.
- (9⁰) G is a strongly regular graph of order $n = 10(9k + 1)^2$ and degree $r = 4(90k^2 + 20k + 1)$ with $\tau = 160k^2 + 55k + 3$ and $\theta = 20k(8k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 40k + 4$ and $\lambda_3 = -(5k + 1)$ with $m_2 = 90k^2 + 20k + 1$ and $m_3 = 8(90k^2 + 20k + 1)$.
- (9⁰) G is a strongly regular graph of order $n = 10(9k + 1)^2$ and degree $r = 5(90k^2 + 20k + 1)$ with $\tau = 10k(25k + 4)$ and $\theta = 5(5k + 1)(10k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k$ and $\lambda_3 = -(40k + 5)$ with $m_2 = 8(90k^2 + 20k + 1)$ and $m_3 = 90k^2 + 20k + 1$.
- (10⁰) G is a strongly regular graph of order $n = 28(9k - 1)^2$ and degree $r = 2(252k^2 - 56k + 3)$ with $\tau = 112k^2 - 63k + 5$ and $\theta = 14k(8k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k - 1$ and $\lambda_3 = -(56k - 6)$ with $m_2 = 8(252k^2 - 56k + 3)$ and $m_3 = 252k^2 - 56k + 3$.
- (10⁰) G is a strongly regular graph of order $n = 28(9k - 1)^2$ and degree $r = 7(252k^2 - 56k + 3)$ with $\tau = 14(7k - 1)(14k - 1)$ and $\theta = 7(7k - 1)(28k - 3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 56k - 7$ and $\lambda_3 = -7k$ with $m_2 = 252k^2 - 56k + 3$ and $m_3 = 8(252k^2 - 56k + 3)$.
- (11⁰) G is a strongly regular graph of order $n = 28(9k + 1)^2$ and degree $r = 2(252k^2 + 56k + 3)$ with $\tau = 112k^2 + 63k + 5$ and $\theta = 14k(8k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 56k + 6$ and $\lambda_3 = -(7k + 1)$ with $m_2 = 252k^2 + 56k + 3$ and $m_3 = 8(252k^2 + 56k + 3)$.
- (11⁰) G is a strongly regular graph of order $n = 28(9k + 1)^2$ and degree $r = 7(252k^2 + 56k + 3)$ with $\tau = 14(7k + 1)(14k + 1)$ and $\theta = 7(7k + 1)(28k + 3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k$ and $\lambda_3 = -(56k + 7)$ with $m_2 = 8(252k^2 + 56k + 3)$ and $m_3 = 252k^2 + 56k + 3$.

Theorem 2.8 ([6]). *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 9m_3$ or $m_3 = 9m_2$. Then, G is one of the following strongly regular graphs.*

- (1⁰) G is the strongly regular graph $\overline{3K_7}$ of order $n = 21$ and degree $r = 14$ with $\tau = 7$ and $\theta = 14$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -7$ with $m_2 = 18$ and $m_3 = 2$.
- (2⁰) G is the strongly regular graph $\overline{9K_9}$ of order $n = 81$ and degree $r = 72$ with $\tau = 63$ and $\theta = 72$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -9$ with $m_2 = 72$ and $m_3 = 8$.

- (3⁰) G is a strongly regular graph of order $n = (10k - 1)^2$ and degree $r = 2k(5k - 1)$ with $\tau = k^2 + 7k - 1$ and $\theta = k(k - 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 9k - 1$ and $\lambda_3 = -k$ with $m_2 = 2k(5k - 1)$ and $m_3 = 18k(5k - 1)$.
- (3⁰) G is a strongly regular graph of order $n = (10k - 1)^2$ and degree $r = 18k(5k - 1)$ with $\tau = 81k^2 - 17k - 1$ and $\theta = 9k(9k - 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k - 1$ and $\lambda_3 = -9k$ with $m_2 = 18k(5k - 1)$ and $m_3 = 2k(5k - 1)$.
- (4⁰) G is a strongly regular graph of order $n = (10k + 1)^2$ and degree $r = 2k(5k + 1)$ with $\tau = k^2 - 7k - 1$ and $\theta = k(k + 1)$, where $k \geq 8$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(9k + 1)$ with $m_2 = 18k(5k + 1)$ and $m_3 = 2k(5k + 1)$.
- (4⁰) G is a strongly regular graph of order $n = (10k + 1)^2$ and degree $r = 18k(5k + 1)$ with $\tau = 81k^2 + 17k - 1$ and $\theta = 9k(9k + 1)$, where $k \geq 8$. Its eigenvalues are $\lambda_2 = 9k$ and $\lambda_3 = -(k + 1)$ with $m_2 = 2k(5k + 1)$ and $m_3 = 18k(5k + 1)$.
- (5⁰) G is a strongly regular graph of order $n = 9(10k - 3)^2$ and degree $r = 2(90k^2 - 54k + 8)$ with $\tau = 36k^2 + 4k - 5$ and $\theta = (2k - 1)(18k - 5)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 36k - 11$ and $\lambda_3 = -(4k - 1)$ with $m_2 = 90k^2 - 54k + 8$ and $m_3 = 9(90k^2 - 54k + 8)$.
- (5⁰) G is a strongly regular graph of order $n = 9(10k - 3)^2$ and degree $r = 8(90k^2 - 54k + 8)$ with $\tau = 4(4k - 1)(36k - 13)$ and $\theta = 4(4k - 1)(36k - 11)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 4k - 2$ and $\lambda_3 = -(36k - 10)$ with $m_2 = 9(90k^2 - 54k + 8)$ and $m_3 = 90k^2 - 54k + 8$.
- (6⁰) G is a strongly regular graph of order $n = 9(10k - 3)^2$ and degree $r = 5(90k^2 - 54k + 8)$ with $\tau = 225k^2 - 155k + 25$ and $\theta = (5k - 1)(45k - 14)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k - 2$ and $\lambda_3 = -(45k - 13)$ with $m_2 = 9(90k^2 - 54k + 8)$ and $m_3 = 90k^2 - 54k + 8$.
- (6⁰) G is a strongly regular graph of order $n = 9(10k - 3)^2$ and degree $r = 5(90k^2 - 54k + 8)$ with $\tau = 225k^2 - 115k + 13$ and $\theta = (5k - 2)(45k - 13)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 45k - 14$ and $\lambda_3 = -(5k - 1)$ with $m_2 = 90k^2 - 54k + 8$ and $m_3 = 9(90k^2 - 54k + 8)$.
- (7⁰) G is a strongly regular graph of order $n = 9(10k + 3)^2$ and degree $r = 2(90k^2 + 54k + 8)$ with $\tau = 36k^2 - 4k - 5$ and $\theta = (2k + 1)(18k + 5)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 4k + 1$ and $\lambda_3 = -(36k + 11)$ with $m_2 = 9(90k^2 + 54k + 8)$ and $m_3 = 90k^2 + 54k + 8$.
- (7⁰) G is a strongly regular graph of order $n = 9(10k + 3)^2$ and degree $r = 8(90k^2 + 54k + 8)$ with $\tau = 4(4k + 1)(36k + 13)$ and $\theta = 4(4k + 1)(36k + 11)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 36k + 10$ and $\lambda_3 = -(4k + 2)$ with $m_2 = 90k^2 + 54k + 8$ and $m_3 = 9(90k^2 + 54k + 8)$.
- (8⁰) G is a strongly regular graph of order $n = 9(10k + 3)^2$ and degree $r = 5(90k^2 + 54k + 8)$ with $\tau = 225k^2 + 115k + 13$ and $\theta = (5k + 2)(45k + 13)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 5k + 1$ and $\lambda_3 = -(45k + 14)$ with $m_2 = 9(90k^2 + 54k + 8)$ and $m_3 = 90k^2 + 54k + 8$.
- (8⁰) G is a strongly regular graph of order $n = 9(10k + 3)^2$ and degree $r = 5(90k^2 + 54k + 8)$ with $\tau = 225k^2 + 155k + 25$ and $\theta = (5k + 1)(45k + 14)$, where $k \geq 0$.

- Its eigenvalues are $\lambda_2 = 45k + 13$ and $\lambda_3 = -(5k + 2)$ with $m_2 = 90k^2 + 54k + 8$ and $m_3 = 9(90k^2 + 54k + 8)$.*
- (9^0) *G is a strongly regular graph of order $n = 21(10k - 1)^2$ and degree $r = 3(210k^2 - 42k + 2)$ with $\tau = 189k^2 - 77k + 5$ and $\theta = 21k(9k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k - 1$ and $\lambda_3 = -(63k - 6)$ with $m_2 = 9(210k^2 - 42k + 2)$ and $m_3 = 210k^2 - 42k + 2$.*
- $(\bar{9}^0)$ *G is a strongly regular graph of order $n = 21(10k - 1)^2$ and degree $r = 7(210k^2 - 42k + 2)$ with $\tau = 7(147k^2 - 27k + 1)$ and $\theta = 7(7k - 1)(21k - 2)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 63k - 7$ and $\lambda_3 = -7k$ with $m_2 = 210k^2 - 42k + 2$ and $m_3 = 9(210k^2 - 42k + 2)$.*
- (10^0) *G is a strongly regular graph of order $n = 21(10k + 1)^2$ and degree $r = 3(210k^2 + 42k + 2)$ with $\tau = 189k^2 + 77k + 5$ and $\theta = 21k(9k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 63k + 6$ and $\lambda_3 = -(7k + 1)$ with $m_2 = 210k^2 + 42k + 2$ and $m_3 = 9(210k^2 + 42k + 2)$.*
- $(\bar{10}^0)$ *G is a strongly regular graph of order $n = 21(10k + 1)^2$ and degree $r = 7(210k^2 + 42k + 2)$ with $\tau = 7(147k^2 + 27k + 1)$ and $\theta = 7(7k + 1)(21k + 2)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k$ and $\lambda_3 = -(63k + 7)$ with $m_2 = 9(210k^2 + 42k + 2)$ and $m_3 = 210k^2 + 42k + 2$.*

Theorem 2.9 ([6]). *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 10m_3$ or $m_3 = 10m_2$. Then, G is one of the following strongly regular graphs.*

- (1^0) *G is the complete bipartite graph $K_{6,6}$ of order $n = 12$ and degree $r = 6$ with $\tau = 0$ and $\theta = 6$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -6$ with $m_2 = 10$ and $m_3 = 1$.*
- (2^0) *G is the strongly regular graph $\overline{5K_9}$ of order $n = 45$ and degree $r = 36$ with $\tau = 27$ and $\theta = 36$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -9$ with $m_2 = 40$ and $m_3 = 4$.*
- (3^0) *G is the strongly regular graph $\overline{10K_{10}}$ of order $n = 100$ and degree $r = 90$ with $\tau = 80$ and $\theta = 90$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -10$ with $m_2 = 90$ and $m_3 = 9$.*
- (4^0) *G is a strongly regular graph of order $n = (11k - 1)^2$ and degree $r = k(11k - 2)$ with $\tau = k^2 + 8k - 1$ and $\theta = k(k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 10k - 1$ and $\lambda_3 = -k$ with $m_2 = k(11k - 2)$ and $m_3 = 10k(11k - 2)$.*
- $(\bar{4}^0)$ *G is a strongly regular graph of order $n = (11k - 1)^2$ and degree $r = 10k(11k - 2)$ with $\tau = 100k^2 - 19k - 1$ and $\theta = 10k(10k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = k - 1$ and $\lambda_3 = -10k$ with $m_2 = 10k(11k - 2)$ and $m_3 = k(11k - 2)$.*
- (5^0) *G is a strongly regular graph of order $n = (11k + 1)^2$ and degree $r = k(11k + 2)$ with $\tau = k^2 - 8k - 1$ and $\theta = k(k + 1)$, where $k \geq 9$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(10k + 1)$ with $m_2 = 10k(11k + 2)$ and $m_3 = k(11k + 2)$.*

- ($\bar{5}^0$) G is a strongly regular graph of order $n = (11k+1)^2$ and degree $r = 10k(11k+2)$ with $\tau = 100k^2 + 19k - 1$ and $\theta = 10k(10k+1)$, where $k \geq 9$. Its eigenvalues are $\lambda_2 = 10k$ and $\lambda_3 = -(k+1)$ with $m_2 = k(11k+2)$ and $m_3 = 10k(11k+2)$.
- (6^0) G is a strongly regular graph of order $n = 3(11k-2)^2$ and degree $r = 5(33k^2 - 12k + 1)$ with $\tau = 75k^2 - 42k + 4$ and $\theta = 15k(5k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 3k-1$ and $\lambda_3 = -(30k-5)$ with $m_2 = 10(33k^2 - 12k + 1)$ and $m_3 = 33k^2 - 12k + 1$.
- ($\bar{6}^0$) G is a strongly regular graph of order $n = 3(11k-2)^2$ and degree $r = 6(33k^2 - 12k + 1)$ with $\tau = 27k(4k-1)$ and $\theta = 6(3k-1)(6k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 30k-6$ and $\lambda_3 = -3k$ with $m_2 = 33k^2 - 12k + 1$ and $m_3 = 10(33k^2 - 12k + 1)$.
- (7^0) G is a strongly regular graph of order $n = 3(11k+2)^2$ and degree $r = 5(33k^2 + 12k + 1)$ with $\tau = 75k^2 + 42k + 4$ and $\theta = 15k(5k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 30k+5$ and $\lambda_3 = -(3k+1)$ with $m_2 = 33k^2 + 12k + 1$ and $m_3 = 10(33k^2 + 12k + 1)$.
- ($\bar{7}^0$) G is a strongly regular graph of order $n = 3(11k+2)^2$ and degree $r = 6(33k^2 + 12k + 1)$ with $\tau = 27k(4k+1)$ and $\theta = 6(3k+1)(6k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 3k$ and $\lambda_3 = -(30k+6)$ with $m_2 = 10(33k^2 + 12k + 1)$ and $m_3 = 33k^2 + 12k + 1$.
- (8^0) G is a strongly regular graph of order $n = 5(11k-3)^2$ and degree $r = 2(55k^2 - 30k + 4)$ with $\tau = 20k^2 - 33k + 7$ and $\theta = 2k(10k-3)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 3k-1$ and $\lambda_3 = -(30k-8)$ with $m_2 = 10(55k^2 - 30k + 4)$ and $m_3 = 55k^2 - 30k + 4$.
- ($\bar{8}^0$) G is a strongly regular graph of order $n = 5(11k-3)^2$ and degree $r = 9(55k^2 - 30k + 4)$ with $\tau = 27(3k-1)(5k-1)$ and $\theta = 9(3k-1)(15k-4)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 30k-9$ and $\lambda_3 = -3k$ with $m_2 = 55k^2 - 30k + 4$ and $m_3 = 10(55k^2 - 30k + 4)$.
- (9^0) G is a strongly regular graph of order $n = 5(11k+3)^2$ and degree $r = 2(55k^2 + 30k + 4)$ with $\tau = 20k^2 + 33k + 7$ and $\theta = 2k(10k+3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 30k+8$ and $\lambda_3 = -(3k+1)$ with $m_2 = 55k^2 + 30k + 4$ and $m_3 = 10(55k^2 + 30k + 4)$.
- ($\bar{9}^0$) G is a strongly regular graph of order $n = 5(11k+3)^2$ and degree $r = 9(55k^2 + 30k + 4)$ with $\tau = 27(3k+1)(5k+1)$ and $\theta = 9(3k+1)(15k+4)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 3k$ and $\lambda_3 = -(30k+9)$ with $m_2 = 10(55k^2 + 30k + 4)$ and $m_3 = 55k^2 + 30k + 4$.
- (10^0) G is a strongly regular graph of order $n = 15(11k-5)^2$ and degree $r = 3(165k^2 - 150k + 34)$ with $\tau = 3(45k^2 - 54k + 15)$ and $\theta = 3(3k-1)(15k-7)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 6k-3$ and $\lambda_3 = -(60k-27)$ with $m_2 = 10(165k^2 - 150k + 34)$ and $m_3 = 165k^2 - 150k + 34$.
- ($\bar{10}^0$) G is a strongly regular graph of order $n = 15(11k-5)^2$ and degree $r = 8(165k^2 - 150k + 34)$ with $\tau = 2(480k^2 - 429k + 95)$ and $\theta = 24(2k-1)(20k-9)$,

- where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 60k - 28$ and $\lambda_3 = -(6k - 2)$ with $m_2 = 165k^2 - 150k + 34$ and $m_3 = 10(165k^2 - 150k + 34)$.
- (11⁰) G is a strongly regular graph of order $n = 15(11k + 5)^2$ and degree $r = 3(165k^2 + 150k + 34)$ with $\tau = 3(45k^2 + 54k + 15)$ and $\theta = 3(3k + 1)(15k + 7)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 60k + 27$ and $\lambda_3 = -(6k + 3)$ with $m_2 = 165k^2 + 150k + 34$ and $m_3 = 10(165k^2 + 150k + 34)$.
- (11⁰) G is a strongly regular graph of order $n = 15(11k + 5)^2$ and degree $r = 8(165k^2 + 150k + 34)$ with $\tau = 2(480k^2 + 429k + 95)$ and $\theta = 24(2k + 1)(20k + 9)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 6k + 2$ and $\lambda_3 = -(60k + 28)$ with $m_2 = 10(165k^2 + 150k + 34)$ and $m_3 = 165k^2 + 150k + 34$.
- (12⁰) G is a strongly regular graph of order $n = 70(11k - 5)^2$ and degree $r = 4(770k^2 - 700k + 159)$ with $\tau = 2(560k^2 - 469k + 97)$ and $\theta = 28(2k - 1)(20k - 9)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 140k - 64$ and $\lambda_3 = -(14k - 6)$ with $m_2 = 770k^2 - 700k + 159$ and $m_3 = 10(770k^2 - 700k + 159)$.
- (12⁰) G is a strongly regular graph of order $n = 70(11k - 5)^2$ and degree $r = 7(770k^2 - 700k + 159)$ with $\tau = 14(245k^2 - 226k + 52)$ and $\theta = 14(7k - 3)(35k - 16)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 14k - 7$ and $\lambda_3 = -(140k - 63)$ with $m_2 = 10(770k^2 - 700k + 159)$ and $m_3 = 770k^2 - 700k + 159$.
- (13⁰) G is a strongly regular graph of order $n = 70(11k + 5)^2$ and degree $r = 4(770k^2 + 700k + 159)$ with $\tau = 2(560k^2 + 469k + 97)$ and $\theta = 28(2k + 1)(20k + 9)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 14k + 6$ and $\lambda_3 = -(140k + 64)$ with $m_2 = 10(770k^2 + 700k + 159)$ and $m_3 = 770k^2 + 700k + 159$.
- (13⁰) G is a strongly regular graph of order $n = 70(11k + 5)^2$ and degree $r = 7(770k^2 + 700k + 159)$ with $\tau = 14(245k^2 + 226k + 52)$ and $\theta = 14(7k + 3)(35k + 16)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 140k + 63$ and $\lambda_3 = -(14k + 7)$ with $m_2 = 770k^2 + 700k + 159$ and $m_3 = 10(770k^2 + 700k + 159)$.

Theorem 2.10 ([7]). *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 11m_3$ or $m_3 = 11m_2$. Then, G is one of the following strongly regular graphs.*

- (1⁰) G is the strongly regular graph $\overline{11K_{11}}$ of order $n = 121$ and degree $r = 110$ with $\tau = 99$ and $\theta = 110$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -11$ with $m_2 = 110$ and $m_3 = 10$.
- (2⁰) G is a strongly regular graph of order $n = (12k - 1)^2$ and degree $r = 2k(6k - 1)$ with $\tau = k^2 + 9k - 1$ and $\theta = k(k - 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 11k - 1$ and $\lambda_3 = -k$ with $m_2 = 2k(6k - 1)$ and $m_3 = 22k(6k - 1)$.
- (2⁰) G is a strongly regular graph of order $n = (12k - 1)^2$ and degree $r = 22k(6k - 1)$ with $\tau = 121k^2 - 21k - 1$ and $\theta = 11k(11k - 1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k - 1$ and $\lambda_3 = -11k$ with $m_2 = 22k(6k - 1)$ and $m_3 = 2k(6k - 1)$.
- (3⁰) G is a strongly regular graph of order $n = (12k + 1)^2$ and degree $r = 2k(6k + 1)$ with $\tau = k^2 - 9k - 1$ and $\theta = k(k + 1)$, where $k \geq 10$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(11k + 1)$ with $m_2 = 22k(6k + 1)$ and $m_3 = 2k(6k + 1)$.

- ($\bar{3}^0$) G is a strongly regular graph of order $n = (12k+1)^2$ and degree $r = 22k(6k+1)$ with $\tau = 121k^2 + 21k - 1$ and $\theta = 11k(11k+1)$, where $k \geq 10$. Its eigenvalues are $\lambda_2 = 11k$ and $\lambda_3 = -(k+1)$ with $m_2 = 2k(6k+1)$ and $m_3 = 22k(6k+1)$.
- (4^0) G is a strongly regular graph of order $n = 385(12k-5)^2$ and degree $r = 5(4620k^2 - 3850k + 802)$ with $\tau = 5(1925k^2 - 1645k + 351)$ and $\theta = 35(5k-2)(55k-23)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 35k-15$ and $\lambda_3 = -(385k-160)$ with $m_2 = 11(4620k^2 - 3850k + 802)$ and $m_3 = 4620k^2 - 3850k + 802$.
- ($\bar{4}^0$) G is a strongly regular graph of order $n = 385(12k-5)^2$ and degree $r = 7(4620k^2 - 3850k + 802)$ with $\tau = 7(2695k^2 - 2225k + 459)$ and $\theta = 35(7k-3)(77k-32)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 385k - 161$ and $\lambda_3 = -(35k-14)$ with $m_2 = 4620k^2 - 3850k + 802$ and $m_3 = 11(4620k^2 - 3850k + 802)$.
- (5^0) G is a strongly regular graph of order $n = 385(12k+5)^2$ and degree $r = 5(4620k^2 + 3850k + 802)$ with $\tau = 5(1925k^2 + 1645k + 351)$ and $\theta = 35(5k+2)(55k+23)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 385k + 160$ and $\lambda_3 = -(35k+15)$ with $m_2 = 4620k^2 + 3850k + 802$ and $m_3 = 11(4620k^2 + 3850k + 802)$.
- ($\bar{5}^0$) G is a strongly regular graph of order $n = 385(12k+5)^2$ and degree $r = 7(4620k^2 + 3850k + 802)$ with $\tau = 7(2695k^2 + 2225k + 459)$ and $\theta = 35(7k+3)(77k+32)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 35k+14$ and $\lambda_3 = -(385k+161)$ with $m_2 = 11(4620k^2 + 3850k + 802)$ and $m_3 = 4620k^2 + 3850k + 802$.

Theorem 2.11 ([7]). *Let G be a connected strongly regular graph of order n and degree r with $m_2 = 12m_3$ or $m_3 = 12m_2$. Then, G is one of the following strongly regular graphs.*

- (1^0) G is the complete bipartite graph $K_{7,7}$ of order $n = 14$ and degree $r = 7$ with $\tau = 0$ and $\theta = 7$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -7$ with $m_2 = 12$ and $m_3 = 1$.
- (2^0) G is the strongly regular graph $\overline{3K_9}$ of order $n = 27$ and degree $r = 18$ with $\tau = 9$ and $\theta = 18$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -9$ with $m_2 = 24$ and $m_3 = 2$.
- (3^0) G is the strongly regular graph $\overline{4K_{10}}$ of order $n = 40$ and degree $r = 30$ with $\tau = 20$ and $\theta = 30$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -10$ with $m_2 = 36$ and $m_3 = 3$.
- (4^0) G is the strongly regular graph $\overline{6K_{11}}$ of order $n = 66$ and degree $r = 55$ with $\tau = 44$ and $\theta = 55$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -11$ with $m_2 = 60$ and $m_3 = 5$.
- (5^0) G is the strongly regular graph $\overline{12K_{12}}$ of order $n = 144$ and degree $r = 132$ with $\tau = 120$ and $\theta = 132$. Its eigenvalues are $\lambda_2 = 0$ and $\lambda_3 = -12$ with $m_2 = 132$ and $m_3 = 11$.
- (6^0) G is a strongly regular graph of order $n = (13k-1)^2$ and degree $r = k(13k-2)$ with $\tau = k^2 + 10k - 1$ and $\theta = k(k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = 12k-1$ and $\lambda_3 = -k$ with $m_2 = k(13k-2)$ and $m_3 = 12k(13k-2)$.

- ($\bar{6}^0$) G is a strongly regular graph of order $n = (13k-1)^2$ and degree $r = 12k(13k-2)$ with $\tau = 144k^2 - 23k - 1$ and $\theta = 12k(12k-1)$, where $k \geq 2$. Its eigenvalues are $\lambda_2 = k-1$ and $\lambda_3 = -12k$ with $m_2 = 12k(13k-2)$ and $m_3 = k(13k-2)$.
- (7^0) G is a strongly regular graph of order $n = (13k+1)^2$ and degree $r = k(13k+2)$ with $\tau = k^2 - 10k - 1$ and $\theta = k(k+1)$, where $k \geq 11$. Its eigenvalues are $\lambda_2 = k$ and $\lambda_3 = -(12k+1)$ with $m_2 = 12k(13k+2)$ and $m_3 = k(13k+2)$.
- ($\bar{7}^0$) G is a strongly regular graph of order $n = (13k+1)^2$ and degree $r = 12k(13k+2)$ with $\tau = 144k^2 + 23k - 1$ and $\theta = 12k(12k+1)$, where $k \geq 11$. Its eigenvalues are $\lambda_2 = 12k$ and $\lambda_3 = -(k+1)$ with $m_2 = k(13k+2)$ and $m_3 = 12k(13k+2)$.
- (8^0) G is a strongly regular graph of order $n = 3(13k-3)^2$ and degree $r = 4(39k^2 - 18k + 2)$ with $\tau = 48k^2 - 45k + 7$ and $\theta = 12k(4k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 3k-1$ and $\lambda_3 = -(36k-8)$ with $m_2 = 12(39k^2 - 18k + 2)$ and $m_3 = 39k^2 - 18k + 2$.
- ($\bar{8}^0$) G is a strongly regular graph of order $n = 3(13k-3)^2$ and degree $r = 9(39k^2 - 18k + 2)$ with $\tau = 3(81k^2 - 34k + 3)$ and $\theta = 9(3k-1)(9k-2)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 36k-9$ and $\lambda_3 = -3k$ with $m_2 = 39k^2 - 18k + 2$ and $m_3 = 12(39k^2 - 18k + 2)$.
- (9^0) G is a strongly regular graph of order $n = 3(13k+3)^2$ and degree $r = 4(39k^2 + 18k + 2)$ with $\tau = 48k^2 + 45k + 7$ and $\theta = 12k(4k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 36k+8$ and $\lambda_3 = -(3k+1)$ with $m_2 = 39k^2 + 18k + 2$ and $m_3 = 12(39k^2 + 18k + 2)$.
- ($\bar{9}^0$) G is a strongly regular graph of order $n = 3(13k+3)^2$ and degree $r = 9(39k^2 + 18k + 2)$ with $\tau = 3(81k^2 + 34k + 3)$ and $\theta = 9(3k+1)(9k+2)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 3k$ and $\lambda_3 = -(36k+9)$ with $m_2 = 12(39k^2 + 18k + 2)$ and $m_3 = 39k^2 + 18k + 2$.
- (10^0) G is a strongly regular graph of order $n = 10(13k-2)^2$ and degree $r = 3(130k^2 - 40k + 3)$ with $\tau = 90k^2 - 70k + 8$ and $\theta = 15k(6k-1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k-1$ and $\lambda_3 = -(60k-9)$ with $m_2 = 12(130k^2 - 40k + 3)$ and $m_3 = 130k^2 - 40k + 3$.
- ($\bar{10}^0$) G is a strongly regular graph of order $n = 10(13k-2)^2$ and degree $r = 10(130k^2 - 40k + 3)$ with $\tau = 5(200k^2 - 59k + 4)$ and $\theta = 10(5k-1)(20k-3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 60k-10$ and $\lambda_3 = -5k$ with $m_2 = 130k^2 - 40k + 3$ and $m_3 = 12(130k^2 - 40k + 3)$.
- (11^0) G is a strongly regular graph of order $n = 10(13k+2)^2$ and degree $r = 3(130k^2 + 40k + 3)$ with $\tau = 90k^2 + 70k + 8$ and $\theta = 15k(6k+1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 60k+9$ and $\lambda_3 = -(5k+1)$ with $m_2 = 130k^2 + 40k + 3$ and $m_3 = 12(130k^2 + 40k + 3)$.
- ($\bar{11}^0$) G is a strongly regular graph of order $n = 10(13k+2)^2$ and degree $r = 10(130k^2 + 40k + 3)$ with $\tau = 5(200k^2 + 59k + 4)$ and $\theta = 10(5k+1)(20k+3)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 5k$ and $\lambda_3 = -(60k+10)$ with $m_2 = 12(130k^2 + 40k + 3)$ and $m_3 = 130k^2 + 40k + 3$.

- (12⁰) G is a strongly regular graph of order $n = 14(13k-1)^2$ and degree $r = 6(182k^2 - 28k + 1)$ with $\tau = 504k^2 - 119k + 5$ and $\theta = 42k(12k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k - 1$ and $\lambda_3 = -(84k - 6)$ with $m_2 = 12(182k^2 - 28k + 1)$ and $m_3 = 182k^2 - 28k + 1$.
- (12⁰) G is a strongly regular graph of order $n = 14(13k - 1)^2$ and degree $r = 7(182k^2 - 28k + 1)$ with $\tau = 14k(49k - 5)$ and $\theta = 7(7k - 1)(14k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 84k - 7$ and $\lambda_3 = -7k$ with $m_2 = 182k^2 - 28k + 1$ and $m_3 = 12(182k^2 - 28k + 1)$.
- (13⁰) G is a strongly regular graph of order $n = 14(13k + 1)^2$ and degree $r = 6(182k^2 + 28k + 1)$ with $\tau = 504k^2 + 119k + 5$ and $\theta = 42k(12k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 84k + 6$ and $\lambda_3 = -(7k + 1)$ with $m_2 = 182k^2 + 28k + 1$ and $m_3 = 12(182k^2 + 28k + 1)$.
- (13⁰) G is a strongly regular graph of order $n = 14(13k + 1)^2$ and degree $r = 7(182k^2 + 28k + 1)$ with $\tau = 14k(49k + 5)$ and $\theta = 7(7k + 1)(14k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 7k$ and $\lambda_3 = -(84k + 7)$ with $m_2 = 12(182k^2 + 28k + 1)$ and $m_3 = 182k^2 + 28k + 1$.
- (14⁰) G is a strongly regular graph of order $n = 30(13k - 6)^2$ and degree $r = 5(390k^2 - 360k + 83)$ with $\tau = 10(75k^2 - 76k + 19)$ and $\theta = 10(5k - 2)(15k - 7)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 10k - 5$ and $\lambda_3 = -(120k - 55)$ with $m_2 = 12(390k^2 - 360k + 83)$ and $m_3 = 390k^2 - 360k + 83$.
- (14⁰) G is a strongly regular graph of order $n = 30(13k - 6)^2$ and degree $r = 8(390k^2 - 360k + 83)$ with $\tau = 2(960k^2 - 865k + 194)$ and $\theta = 40(2k - 1)(24k - 11)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 120k - 56$ and $\lambda_3 = -(10k - 4)$ with $m_2 = 390k^2 - 360k + 83$ and $m_3 = 12(390k^2 - 360k + 83)$.
- (15⁰) G is a strongly regular graph of order $n = 30(13k + 6)^2$ and degree $r = 5(390k^2 + 360k + 83)$ with $\tau = 10(75k^2 + 76k + 19)$ and $\theta = 10(5k + 2)(15k + 7)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 120k + 55$ and $\lambda_3 = -(10k + 5)$ with $m_2 = 390k^2 + 360k + 83$ and $m_3 = 12(390k^2 + 360k + 83)$.
- (15⁰) G is a strongly regular graph of order $n = 30(13k + 6)^2$ and degree $r = 8(390k^2 + 360k + 83)$ with $\tau = 2(960k^2 + 865k + 194)$ and $\theta = 40(2k + 1)(24k + 11)$, where $k \geq 0$. Its eigenvalues are $\lambda_2 = 10k + 4$ and $\lambda_3 = -(120k + 56)$ with $m_2 = 12(390k^2 + 360k + 83)$ and $m_3 = 390k^2 + 360k + 83$.
- (16⁰) G is a strongly regular graph of order $n = 66(13k - 1)^2$ and degree $r = 2(858k^2 - 132k + 5)$ with $\tau = 264k^2 - 143k + 9$ and $\theta = 22k(12k - 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 11k - 1$ and $\lambda_3 = -(132k - 10)$ with $m_2 = 12(858k^2 - 132k + 5)$ and $m_3 = 858k^2 - 132k + 5$.
- (16⁰) G is a strongly regular graph of order $n = 66(13k - 1)^2$ and degree $r = 11(858k^2 - 132k + 5)$ with $\tau = 22(11k - 1)(33k - 2)$ and $\theta = 11(11k - 1)(66k - 5)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 132k - 11$ and $\lambda_3 = -11k$ with $m_2 = 858k^2 - 132k + 5$ and $m_3 = 12(858k^2 - 132k + 5)$.

- (17⁰) G is a strongly regular graph of order $n = 66(13k + 1)^2$ and degree $r = 2(858k^2 + 132k + 5)$ with $\tau = 264k^2 + 143k + 9$ and $\theta = 22k(12k + 1)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 132k + 10$ and $\lambda_3 = -(11k + 1)$ with $m_2 = 858k^2 + 132k + 5$ and $m_3 = 12(858k^2 + 132k + 5)$.
- (17⁰) G is a strongly regular graph of order $n = 66(13k + 1)^2$ and degree $r = 11(858k^2 + 132k + 5)$ with $\tau = 22(11k + 1)(33k + 2)$ and $\theta = 11(11k + 1)(66k + 5)$, where $k \in \mathbb{N}$. Its eigenvalues are $\lambda_2 = 11k$ and $\lambda_3 = -(132k + 11)$ with $m_2 = 12(858k^2 + 132k + 5)$ and $m_3 = 858k^2 + 132k + 5$.

3. CONCLUDING REMARKS

Using equations (1.3) and (1.4), and applying the same procedure as in articles [4–7], we can establish the parameters n , r , τ and θ for strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for any fixed value $q \in \mathbb{N}$, by using only one parameter k . All results presented in this work has been verified using a computer program `srgpar.exe`, which was written by the author in the programming language `Borland C++ Builder 5.5`. using only one parameter k .

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REFERENCES

- [1] D. Cvetković, M. Doob and H. Sachs, *Spectra of Graphs - Theory and Applications*, 3rd Edition, J.A. Barth Verlag, Heidelberg, Leipzig, 1995.
- [2] R. J. Elzinga, *Strongly regular graphs: values of λ and μ for which there are only finitely many feasible (v, k, λ, μ)* , Electron. J. Linear Algebra **10** (2003), 232–239.
- [3] C. Godsil and G. Royle, *Algebraic Graph Theory*, Springer-Verlag, New York, 2001.
- [4] M. Lepović, *On strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$* , Serdica Math. J. **37** (2011), 353–364.
- [5] M. Lepović, *On strongly regular graphs with $m_2 = qm_3$ and $m_3 = qm_2$ for $q = 5, 6, 7, 8$* , Sarajevo J. Math. **15**(28) (2019), 209–225. <https://doi.org/10.5644/SJM.15.02.06>
- [6] M. Lepović, *On strongly regular graphs with $m_2 = qm_3$ and $n_3 = qm_2$ for $q = 9, 10$* , Bull. Int. Math. Virtual Inst. **13**(2) (2023), 219–232. <https://doi.org/10.7251/BIMVI2302219L>
- [7] M. Lepović, *On strongly regular graphs with $m_2 = qm_3$ and $n_3 = qm_2$ for $q = 11, 12$* , Scientific Publications of the State University of Novi Pazar, Series A: Applied Mathematics, Informatics & Mechanics **15** (2023), 21–35.

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