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Original Research article

Initial problem solving competencies of the first year students, prospective elementary school teachers, at the Pedagogical faculty in Skopje (2006/07-2009/10)

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Abstract

The development of problem-solving competencies of students is one of the focal points of contemporary education. Mathematics education at all levels if designed and conducted appropriately could provide multiple opportunities for building and continuous development of students' problem-solving competencies. Initial and periodic assessment of these competencies plays crucial role in optimal planning of mathematics instruction for their development.

At the beginning of the first semester during four consecutive academic years (2006/07-2009/10) diagnostic tests on a range of mathematics competencies of the first year students, prospective elementary school teachers, in Macedonian and in Turkish language of instruction at the Pedagogical faculty "St. Kliment Ohridski" in Skopje were administered. The diagnostic tests consisted of fifteen math problems covering areas from the first of two compulsory one-semester mathematics courses. We analyze the results of the students on two problems: a context[2] problem which can be solved by logical reasoning or by using a proportion and a context problem which can be solved by logical reasoning or by modeling it as one linear equation with one unknown or as a system of two linear equations with two unknowns.

The results showed that problem-solving competencies of the majority of students are of very low levels. Each year more than half of students didn't even attempt to solve the problems, and only a small percent of those who tried did it correctly.

These results together with the results from other studies lead us to conclude that the role of problem-solving has to be incorporated more prominently in the mathematics curriculum at all levels of education. As means for achieving this goal we point at contemporary mathematics education approaches like realistic mathematics education whose foundations are based primarily on problem-solving activities and context problems.

Key words and phrases: mathematics, elementary school, teacher education, diagnostic testing, problem-solving competencies, context problems, realistic mathematics education.

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1. INTRODUCTION

Development in each sphere of human society occurs in cycles – emerging practical problems introduce the necessity for new research; research results in the development and adoption of new theories; their practical application opens new problems or calls for different approaches to old

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[2] In this paper, context problems are distinguished from word (textual) problems in the sense that the second ones refer to a wider class of problems. In addition to context problems word problems also include problems like: The number 53 equals the sum of two whole numbers whose product is 612. Find those two whole numbers.

problems; evaluation of now old practices and addressing the new set of questions produces a new cycle of research, practical application, evaluation, and so on.

Even in today's modern world of globalization, the development of education in general and mathematics education, in particular, bears national characteristics – being restricted by the country's sets of laws and regulations. Based on past national practices and current research, it is carried by individuals and groups of professionals who work within the constraints posed by available material and human resources. Assessment of current levels of development and identification of areas to be researched is a matter of national importance as well a matter of interest of an individual researcher. In certain areas educational systems of nations with limited resources may benefit immensely from results of research conducted worldwide and from careful appropriation of successful practices of other educational systems.

Worldwide, many studies have been conducted on the development of mathematical proficiency (see for ex. Resnick, 1989, or Kilpatrick, Swafford & Findell, 2001, for a review and analysis). According to Resnick (1987, p.13):

> "There is substantial evidence that students' difficulty in learning school mathematics derives in large part from their failure to recognize and apply the relationship between the formal rules taught in school and their own independently developed intuitions."

Mathematics instruction based on context problems and learning by problem-solving activities provides one of the answers to this problem. Within the theory of realistic mathematics education (the successful Dutch answer to contemporary challenges of mathematic education), Streefland (1990, p.1) defines:

> "Learning mathematics … means *constructing* mathematics or – to say it more explicitly – proceeding from one's own informal mathematical constructions to what could be accepted as formal mathematics."

There are studies (Bodenhausen et al., 1991; Heuvel-Panhuizen et al., 1990) which point out that elementary school teachers as well as teachers of mathematics[3] need to be able to base their instruction on context problems and problem-solving activities, to build from students' intuitive/informal knowledge, to be able to motivate their students to construct their own problem-solving strategies and to accept multiple solution strategies. The problem solving standard as given in The Principles and Standards for School Mathematics (National Council of Teachers of Mathematics, 2000, p.52) states:

> "Instructional programs from pre-kindergarten through grade 12 should enable all students to –
> - Build new mathematical knowledge through problem solving;
> - Solve problems that arise in mathematics and in other contexts;
> - Apply and adapt a variety of appropriate strategies to solve problems;
> - Monitor and reflect on the process of mathematical problem solving."

Teachers' own mathematics proficiency and problem-solving competence together with a clear vision of the horizons of elementary school mathematics play central role to the effective achievement of the goal of teaching for lasting acquisition of mathematical proficiency.

The basis for this work is the analysis of a subset of the results from an initial assessment of mathematical competencies of the first year students, prospective elementary school teachers, at the Pedagogical faculty in Skopje.

2. BACKGROUND

In Macedonia, elementary school teachers' professional development begins with the first cycle of studies offered at several university departments, the oldest of which being the Pedagogical faculty

[3] In Macedonia, the first 4 years of eight-year elementary school education (5 years of nine-year elementary school education) all subjects are taught by a single teacher. Starting from 5th grade (6th grade, respectively) mathematics is taught by a mathematics teacher trained at a mathematics department.

"St. Kliment Ohridski" within the University "Ss Kiril and Metodij" in Skopje. Additional information on teacher education in Macedonia as well as eleven other countries from South-east Europe can be found in a publication which appeared as a result of a research project (2004-2006) organized and executed within the South – east European Education Co – operation Network (SEE ECN) (Zgaga, 2006).

The curriculum for elementary school teachers provides students with two compulsory one-semester mathematics courses: Mathematics 1 and Mathematics 2 for elementary school teachers, offered in the first year of studies.

Students who enroll in the first year of studies for elementary school teachers have various secondary school background. According to the information provided by the students in the accompanying questionnaire, the approximate percents by year are:

- General gymnasiums where mathematics courses are compulsory all four years (social-humanistic, linguistic – artistic, natural science & mathematics) – 35 %, 46 %, 49 %, 64 % in 2006/07, 2007/08, 2008/09 and 2009/10, respectively.
- Professional secondary schools where mathematics courses are compulsory all four years (economy, tourism, transportation, mechanics, electro-techniques, construction (architecture), graphics, forestry, agriculture, textile industry) – 48 %, 37 %, 31 %, 17 % in 2006/07, 2007/08, 2008/09 and 2009/10, respectively.
- Professional secondary schools where mathematics courses are compulsory the first two years (law, medical, veterinary, physical education/sports, personal services: cosmetic technician, hairstylist) – 15 %, 13 %, 20 %, 17 % in 2006/07, 2007/08, 2008/09 and 2009/10, respectively.
- Professional secondary schools with mathematics courses compulsory only the first year (music, visual arts, and so on) – 1 %, 2 %, 0 %, 2 % in 2006/07, 2007/08, 2008/09 and 2009/10, respectively.

(Information about their secondary school wasn't provided by 3 students in 2006/07 and 2 students in 2007/08.)

Comparison of the content matter and course objectives of the mathematics courses offered in the above listed variety of secondary schools has not been done.

Since 2004/05 academic year there are no university entrance examinations in Republic of Macedonia. Traditionally the entrance exams for prospective elementary school teachers at the Pedagogical faculty in Skopje included a test in mathematics. Since 2005/06 university entrance exams have been cancelled and a national matriculation examination (matura) was introduced in 2007/08 academic year after a lot of opposition from the secondary school students in the country. On the national matura, mathematics is an elective exam. Graduating secondary school students choose between foreign language and mathematics (basic or advanced level) and students who have passed either of them (mathematics at any level) qualify for entrance in the studies for elementary school teachers at the Pedagogical faculty in Skopje. In 2007/08 and in 2008/09 small number of students enrolled at the first year of studies had taken the mathematics matura exam (less than 30 % and less than 15 %, respectively), none of the students took the advanced level, and more than half of them received the lowest passing grade, 2.

The diagnostic test

Appropriate assessment of mathematics knowledge and skills students acquired in their previous education is necessary for optimal planning of instruction for effective achievement of course objectives of Mathematics 1 for elementary school teachers. Diagnostic testing was administered in the first week of the first semester of 2006/07, 2007/08, 2008/09, 2009/10 academic years.

In addition, the test provided students with an opportunity to self assess their own abilities in this field and presented them with a preview of what is expected of them to be able to do at the end of the course. Self-assessment of students' own knowledge and skills at the beginning of this particular course is of great importance since without it their motivation to engage in the required course work is quite low due to the fact that almost all of the content matter students have already met with in their primary school.

The diagnostic test consisted of 15 open ended mathematics problems chosen from most chapters in Mathematics 1 for elementary school teachers. A complete list of mathematics problems included in the test is given in a related paper by Jakimovik & Timovski (in press) presented at the International

Conference “Teacher in Balkan Cultures” in Vranje, November 2010, organized by the Balkan Association of Pedagogical and Related Faculties.

The number of students, prospective elementary school teachers in Macedonian language of instruction and in Turkish language of instruction who took the test each year is given in the following table.

Academic year	**2006/07**	**2007/08**	**2008/09**	**2009/10**
Total number of students who took the course Mathematics 1	215	194	134	107
Number of students who took the diagnostic test	172	122	89	58
Percent of students who took the diagnostic test	80 %	62,89 %	66,42%	54,21 %

Taking the diagnostic test is not compulsory, it carries extra points (5 % of the total number of points) and the motivation of students to take it is decreasing each year, which in itself is an indicator of students’ attitude towards studying mathematics and towards their studies in general.

In this work we aim to analyze the results of the students on two of the test problems[4] (the problems are named by the variety of solutions students presented in their work):

Problem 1. Solving a context problem which can be reduced to solving a proportion or by logical reasoning.

Problem 2. Modeling and solving a context problem by using logical reasoning or by using linear equations.

3. RESULTS FROM THE DIAGNOSTIC TESTS AND THEIR ANALYSIS

The results on solving a context problem which can be reduced to solving a proportion

Problem 1. It takes 33 workers to finish certain project in 32 days. How many workers are required to finish the same project in 22 days?

The standard textbook solution taught in elementary schools in Macedonia includes the following steps.

- Determine that the number of workers is inversely proportional to the number of days required to finish the project (more workers would finish the project in less time).
- Draw a diagram.

↑	↓
33 workers	32 days
x workers	22 days

- Set a proportion.

$$x:33=32:22$$

- Solve the proportion using the “Simple Triple Rule”.

$$x=\frac{33\times 32}{22}=48.$$

- Interpret the result.

It takes 48 workers to finish the project in 22 days.

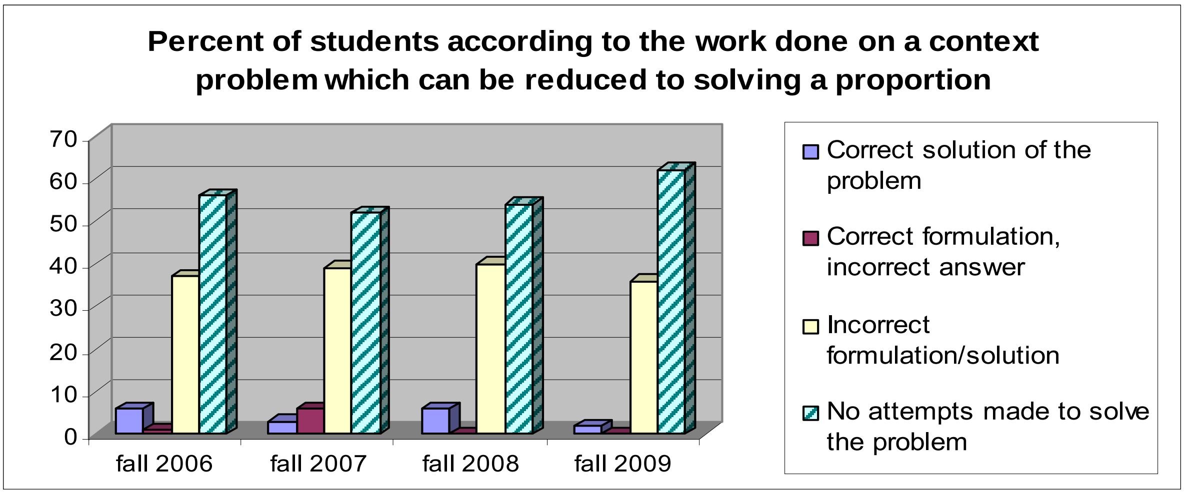
Another type of solution would be the following one.

The product, 33×32, which gives the number of days it takes one worker to finish the project, is divided by 22 and the result is the number of workers required to finish the project in 22 days.

The percent of students according to the work shown in presenting a solution and reaching the correct answer to Problem 1 is given in the following table and diagram.

[4] These two problems appear as Problem 6 and Problem 15, respectively, in the diagnostic test and their presentation in this paper is not based on the level of their difficulty.

PROBLEM 1	Academic year							
	2006/07		2007/08		2008/09		2009/10	
Number of students with:		**%**		**%**		**%**		**%**
Correct solution of the problem	10	**6**	4	**3**	5	**6**	1	**2**
Correct formulation, incorrect answer	2	**1**	7	**6**	0	**0**	0	**0**
Incorrect formulation or solution	64	**37**	47	**39**	36	**40**	21	**36**
No attempts made to solve the problem	96	**56**	64	**52**	48	**54**	36	**62**
Total number of students tested	**172**	**100**	**122**	**100**	**89**	**100**	**58**	**100**



A large number of the students each year didn't even attempt to solve Problem 1 (between 52 % and 62 %) and only a very small percent of all of the students reached a correct answer (between 2 % and 6 %).

At the beginning of 2006/07 academic year 44 % of the students attempted to solve the problem but only 1 student solved it correctly; 54 % of the students who tried to solve the problem recognized that the problem can be solved as a proportion but set the proportion incorrectly; and 30 % of the students who tried didn't even recognize the problem, but tried to solve it by adding or subtracting some of the given numbers or by inventing a linear equation. Only 2 students solved the problem without using the standard textbook solution.

In 2007/08 academic year 48 % of the students attempted to solve the problem, all but 6% of those students failed; 11 % of the students who tried recognized the problem but set the proportion incorrectly; and 65 % of the students who tried didn't even recognize the problem, but tried to solve it by adding or subtracting or by using a linear equation (twice as many as the previous year). Out of 4 students who solved the problem correctly, 3 students solved the problem without using the standard textbook solution, and additional 3 students correctly set up the problem (without using the standard textbook solution) but didn't reach the correct answer.

The following year (2008/09) 46 % of the students attempted to solve the problem, 12% of those students succeeded; 12 % of the students who tried recognized the problem but set the proportion incorrectly; and 76 % of the students who tried didn't even recognize the problem, but tried to solve it by adding or subtracting or by using a linear equation (2,5 times as many as the first year). Two students solved the problem without using the standard textbook solution.

The last year (2009/10) only 38 % of the students attempted to solve the problem, 5 % of those students succeeded (only 1 student); no other student recognized the problem and all the other students tried to solve the problem by adding or subtracting or by using a linear equation (95 % of those who tried). The only student who solved the problem correctly used the standard textbook solution.

The results show low levels of understanding of multiplicative structures and proportional thinking among the vast majority of the first year students, prospective elementary school teachers. The percent of students who attempted to solve the problem and didn't even recognize that the problem can be solved as a proportion grew rapidly each year (30 %, 65 %, 76 % and 95 %, respectively).

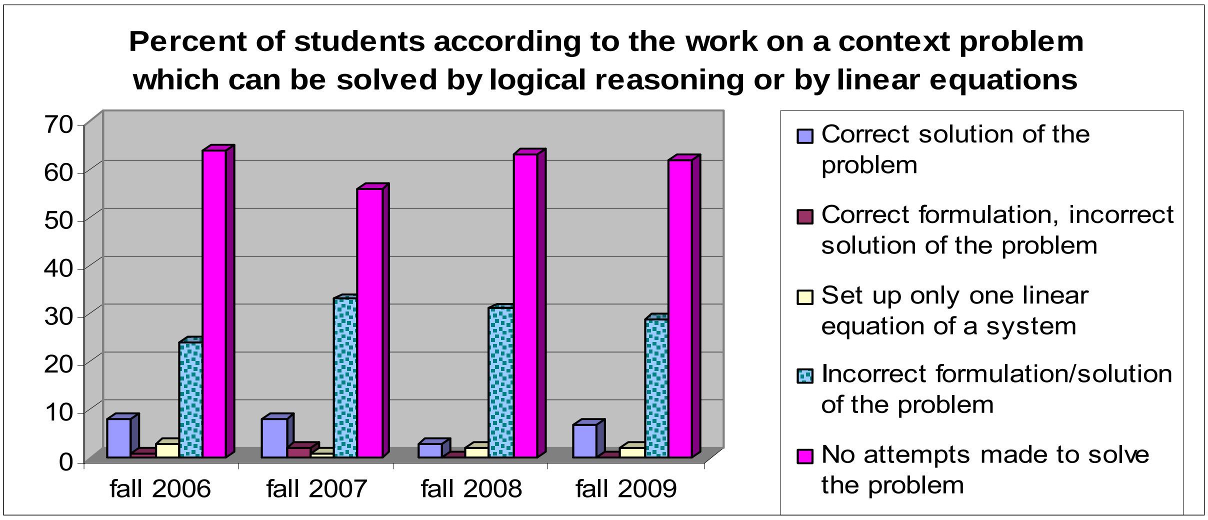
These results are not surprising having into consideration the results[5] from the international report Trends in International Mathematics and Science Study 2003 (TIMSS 2003) as only 1 % of the Macedonian eight-grade students reached the advanced international benchmark, 9 % reached the high international benchmark, 33 % reached the intermediate international benchmark and 66 % reached the low international benchmark. According to the issues that emerged from the benchmark description "…even at the intermediate benchmark, students demonstrate only limited achievement in problem solving beyond straightforward one-step problems…" (Mullis, Martin, Gonzalez and Chrostowski, 2004, p.101).

A context problem which can be solved by using logical reasoning or linear equations

Problem 2. There are 74 children in a school choir. The number of girls is 38 more than the number of boys. How many boys there are in the choir?

The percent of students according to the work shown in presenting a solution and reaching the correct answer to Problem 2 is given in the following table and diagram.

PROBLEM 2	Academic year							
	2006/07		2007/08		2008/09		2009/10	
Number of students with:		**%**		**%**		**%**		**%**
Correct solution of the problem	14	**8**	10	**8**	3	**3**	4	**7**
Correct formulation, incorrect solution	1	**1**	3	**2**	0	**0**	0	**0**
Set up only one linear equation of a system	5	**3**	1	**1**	2	**2**	1	**2**
Incorrect formulation/ incorrect solution	42	**24**	40	**33**	28	**31**	17	**29**
No attempts made to solve the problem	110	**64**	68	**56**	56	**63**	36	**62**
Total number of students who took the test	**172**	**100**	**122**	**100**	**89**	**1 0 0**	**58**	**1 0 0**



As with Problem 1 less than half of the students each year tried to solve Problem 2. The percent of students who made a correct mathematical model or gave a complete solution to the problem varies from about 9 % in 2006/07, about 10 % in 2007/08, only about 3 % in 2008/09 and approximately 7 % in 2009/10 academic year.

In 2006/07 academic year 64 % of the students didn't even try to solve the problem, and 68 % of those who tried – failed. Out of those who failed, 64 % made the typical error of modeling the problem as the difference between the given two numbers, 74 – 38, and interpreting the difference as the answer to the posed question. Among the students who solved the problem correctly, less than 15 % (2 students) did it by using logical reasoning (they halved the difference 74 – 38 and interpreted it as the number of boys in the choir). Half of the rest of the students who succeeded modeled the problem by

[5] The report from TIMSS 2003 is relevant for our consideration since the first year students who took the diagnostic test in 2008/09 academic year belong to the generation of eight-grade students whose sample was tested in this international study.

one linear equation with one unknown and the other half by modeling the problem by a system of linear equations with two unknowns.

In 2007/08 academic year 56 % of the students didn't try to solve the problem and 75% of those who tried did it incorrectly. One third out of those who failed made an error analogous to the typical error of modeling the problem as the difference between the given two numbers, 74 – 38, and interpreting the difference as the answer to the posed question. Among 10 students who solved the problem correctly, 9 students did it by using logical reasoning as described above. Only 1 student modeled the problem by a system of linear equations with two unknowns.

In 2008/09 academic year 63 % of the students didn't attempt to solve the problem at all, and 86 % of those who tried did it incorrectly. More than a half of those who failed made an error analogous to the typical error described above. Among the only 3 students who solved the problem correctly, 2 students did it by using logical reasoning and 1 student modeled the problem by a system of linear equations with two unknowns.

In 2009/10 academic year 62 % of the students didn't attempt to solve the problem at all, and 76% of those who tried did it incorrectly. Almost 60 % of those who failed made an error analogous to the typical error described above. All of the 4 students who solved the problem correctly did it by using logical reasoning.

It seems we cannot escape the conclusion that mathematics education, from the beginning of elementary school through the last years of secondary schools when mathematics courses were taken by students, made a very poor contribution to the development of their problem-solving competencies. Judging by the percents of students who solved either of the problems by logical reasoning without the use of formal strategies taught in elementary school, we may be tempted to ask ourselves whether our educational system have actually prevented students to use their common sense, i.e. their intuitive knowledge to find the solutions to each of the above problems.

4. CONCLUSIONS

The results from the diagnostic testing on these two context problems show serious lack of understanding of these types of problems and very low levels of strategic competence of the majority of the first year students, prospective elementary school teachers. Approximately 93 %, 91 %, 94 % and 98 %, each year respectively, earned 0 points on the first problem, and 88 %, 89 %, 94% and 91%, each year respectively, earned 0 points on the second problem. The reasons behind the high percent of students who earned 0 points on each problem are indeed complex and require a substantial in depth investigation. A list of some of the possible related factors, a set of goals of mathematics education for elementary school teachers and the necessary changes in planning and practicing mathematics instruction at teacher training departments was offered by Jakimovik & Timovski (in press). The quality of mathematics instruction in our schools, teachers' needs with respect to their pre-service and in-service training in mathematics, current mathematics curriculums at all levels of education in Macedonia are some of the major areas that need to be researched.

The realistic mathematics approach which is virtually unknown to the educational professionals in elementary schools and in teacher training departments in our country offers examples of successful practices in mathematics education and it is primarily based on the use of context problems and students' own free productions. More about the fundamental principles of this contemporary mathematics theory can be found in publications from the Freudenthal Institute (Streefland, 1991a; Heuvel-Panhuizen, Gravemeijer, & Streefland, 1990; Streefland, 1991b; Freudenthal, 1991) and other newer publications. Gravemeijer (1990, p.31) lists the main issues in the realistic approach instruction theory:

> - Applications, domain specific knowledge and strategies: Applicability is inherent to the curriculum by virtue of the stand on context problems.
> - (Mathematical) models: Rather than being offered right away, they arise from problem-solving activities. So they can function to bridge intuitive notions and formal mathematical objects.
> - Construction: With regard to the question whether students should construct their own knowledge or whether one should rely on explicit instruction and feedback, realist take the position that the construction process can and has to be guided by means of special assignments, free production and interaction.

- Social versus individual learning: The question may be answered by a reference Schoenfeld' idea of 'mathematical people'.
- Complexity of learning: As opposed to tasks analysis, the approach is rather holistic. As reflecting the complex reality, context problems are to get learning strands intertwined, which also serves the development of intelligible procedures and relational understanding.

It could prove beneficial to present this approach to practicing elementary school teachers and to introduce it in the curriculum for elementary school teacher studies. Contemporary theories on mathematics education like realistic mathematics education if carefully and appropriately adapted to fit the needs of our educational system could give a solid basis for improvement of our current practices. Similar suggestions with respect to their educational system (which has common roots with our educational system) are found in the works of Ibrahimpašić, Ibrahimpašić and Romano (2010), Romano (2010). As the development of problem-solving competencies of students is one of the ultimate goals of mathematics education and education in general, and at the same time it is probably the most complex one to achieve, it is important for all of us working in mathematics education from elementary school level to university level to make serious endeavors to both educate the wider public of the need for change for better mathematics education for all students and to offer ways to achieve this worthy goal.

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