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# DECOMPOSITION OF COEQUALITY RELATION ON THE CARTESIAN PRODUCT OF SETS WITH APARTNESSES

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ABSTRACT. A coequality relation on a set with an apartness is defined by standard way as a consistent, symmetric and cotransitive relation. The coequality relation on the product $\prod X_i$ of sets with apartnesses is called decomposible if it is determined by its special projections on $X_i$ respectively. This paper contains some theorems about characterization of decomposible coequality on the Cartesian product $\prod X_i$ of sets $X_i$ with apartnesses which are generalization of result of the main theorem in a paper by the author. As application of these theorems, we give the exact description of coideals of the commutative rings $\prod_{i=1}^{n} X_i$ and $\prod_{i=1}^{\infty} X_i$.

## Introduction

This paper continues the program of [7,8,9,10,12] to develop of *coequality* relations from the constructive mathematics ([1],[6],[14],[15]). The author is introduced the notion of coequality relation on sets with *apartnesses* in his papers [7],[8] and describes their basic properties in his papers [8] and [12]. Coequality relations on the Cartesian product of sets with apartnesses plays a central role in the developments [9] and [11].

At the beginning of the seventies, there appeared a number of papers dealing with decomposible congruences on the direct product of algebras, see e.g. papers [3] and [16]. In them it is used the well-known concept of diagonal operation ([2],[4]). The notion of compatibility of relation and the operation on the set given in the classical book [5]. If $C$ is a coequality relation on the set $(X, =, \neq)$, and if $w$ is an internal binary operation on $X$, then we say ([8],[10],[11]) that they are *compatible* if and only if

$$(\forall x, x', y, y' \in X)((w(x,y), w(x',y')) \in C \Rightarrow (x,x') \in C \vee (y,y') \in C.$$

In this paper we give the necessary and sufficient conditions for decomposibility relation on the Cartesian product $\prod_{i=1}^{n} X_i$ and $\prod_{i=1}^{\infty} X_i$ which are generalizations of the main theorem in the paper [13].

A coequality relation $C$ on a commutative ring $(R, =, \neq, +, \cdot)$ ([6],[10],[14]) with an apartness is a *cocongruence* on $R$ ([10]) if it is a coequality relation compatible with the operations in $R$ and if holds

$$(\forall x, x' \in R)((xx', 0) \in C \Rightarrow (x, 0) \in C \land (y, 0) \in C).$$

We get, as applications of the main theorems that every cocongruences on the commutative rings $\prod_{i=1}^{n} X_i$ and $\prod_{i=1}^{\infty} X_i$ with apartnesses are decomposible. If $C$ is a cocongruence on the commutative ring $R$, then ([10]) the set $S = \{x \in R : (x, 0) \in C\}$ is a *coideal* ([10],[14],[15]) of the ring $R$. It is a *strongly extensional* subset ([14],[15]) of $R$ such that

$$\neg(0 \in S), x \in S \Rightarrow -x \in S, x + y \in S \Rightarrow x \in S \lor y \in S, xy \in S \Rightarrow x \in S \land y \in S.$$

As the last we give a description of coideals of the commutative rings $\prod_{i=1}^{n} X_i$ and $\prod_{i=1}^{\infty} X_i$ using the coideals of $X_i$.

## Results I

**Theorem 1.** *Let $C$ be a coequality relation on the Cartesian product $\prod_{i=1}^{n} X_i$ of sets $X_i, (i = 1, \dots, n)$ with apartnesses. Then relations $q_i$ on $X_i, (i = 1, \dots, n)$, defined by $(x_i, x_i') \in q_i \Leftrightarrow$*

$$(\forall j = \{1, \dots, n\} - \{i\})(((x_1, \dots, x_i, \dots, x_n), (x_1, \dots, x_i', \dots, x_n)) \in C)$$

*is a coequality relation on $X_i, (i = 1, \dots, n)$.*

*Proof.* We give the proof for $q_1$. For $q_2, \dots, q_n$ the proofs are analogous.

$$\begin{aligned}(x_1, x_1') \in q_1 &\Leftrightarrow (\exists x_2 \in X_2) \dots (\exists x_n \in X_n)(((x_1, \dots, x_n), (x_1', x_2, \dots, x_n)) \in C) \\ &\Rightarrow ((x_1, x_2, \dots, x_n), (x_1', x_2, \dots, x_n)) \neq ((a, x_2, \dots, x_n), (a, x_2, \dots, x_n)) \\ &\Leftrightarrow (x_1, x_2, \dots, x_n) \neq (a, x_2, \dots, x_n) \lor (x_1', x_2, \dots, x_n) \neq (a, x_2, \dots, x_n) \\ &\Rightarrow x_1 \neq a \lor x_1' \neq a \\ &\Leftrightarrow (x_1, x_1') \neq (a, a);\end{aligned}$$

$$\begin{aligned}(x_1, x_1') \in q_1 &\Leftrightarrow (\exists x_2 \in X_2) \dots (\exists x_n \in X_n)(((x_1, \dots, x_n), (x_1', x_2, \dots, x_n)) \in C) \\ &\Leftrightarrow (\exists x_2 \in X_2) \dots (\exists x_n \in X_n)(((x_1', x_2, \dots, x_n), (x_1, x_2, \dots, x_n)) \in C) \\ &\Leftrightarrow (x_1', x_1) \in q_1.\end{aligned}$$

$$(x_1, x_1'') \in q_1 \Leftrightarrow (\exists x_2 \in X_2) \dots (\exists x_n \in X_n)(((x_1, x_2, \dots, x_n), (x_1'', x_2, \dots, x_n)) \in C)$$
$$\Rightarrow ((x_1, x_2, \dots, x_n), (x_1', x_2, \dots, x_n)) \in C \lor ((x_1', x_2, \dots, x_n), (x_1'', x_2, \dots, x_n)) \in C$$
$$\Leftrightarrow (x_1, x_1') \in q_1 \lor (x_1', x_1'') \in q_1. \quad \square$$

Using the strongly extensional and embedding bijection

$$f : \left(\prod_{i=1}^{n} X_i\right)^2 \ni ((x_1, \dots, x_n), (y_1, \dots, y_n)) \longmapsto ((x_1, y_1), \dots, (x_n, y_n)) \in \prod_{i=1}^{n} X_i^2,$$

we have the following

**Corollary 1.1.** *Let $C \subseteq (\prod_{i=1}^n X_i)^2$ be a coequality relation. Then*

$$f(C) \subseteq \bigcup_{i=1}^n \left( \prod_{j=1}^{i-1} X_j^2 \times q_i \times \prod_{j=i+1}^{n} X_j^2 \right).$$

*Proof.* $((x_1, y_1), \ldots, (x_n, y_n)) \in f(C) \iff$

$$\begin{aligned}
&((x_1, \ldots, x_n), (y_1, \ldots, y_n)) \in C \Rightarrow ((x_1, x_2, \ldots, x_n), (y_1, y_2, \ldots, y_n)) \in C \\
&\vee ((y_1, x_2, \ldots, x_n), (y_1, y_2, x_3, \ldots, x_n)) \in C \vee \ldots \\
&\vee ((y_1, \ldots, y_{n-1}, x_n), (y_1, y_2, \ldots, y_n)) \in C \Rightarrow \\
&(x_1, y_1) \in q_1 \vee (x_2, y_2) \in q_2 \vee \cdots \vee (x_n, y_n) \in q_n \Rightarrow \\
&((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)) \in (q_1 \times {X_2}^2 \times \ldots {X_n}^2) \\
&\bigcup ({X_1}^2 \times q_2 \times {X_3}^2 \times \cdots \times {X_n}^2) \bigcup \cdots \bigcup ({X_1}^2 \times \cdots \times {X_{n-1}}^2 \times q_n) \Rightarrow \\
&((x_1, y_1), \ldots, (x_n, y_n)) \in \bigcup_{i=1}^n \left( \prod_{j=1}^{i-1} X_j^2 \times q_i \times \prod_{j=i+1}^{n} X_j^2 \right). \quad \square
\end{aligned}$$

**Definition 1.** A coequality relation $C$ on the Cartesian product $\prod_{i=1}^n X_i$ is *decomposible* if and only if

$$f(C) = \bigcup_{i=1}^n \left( \prod_{j=1}^{i-1} X_j^2 \times q_i \times \prod_{j=i+1}^{n} X_j^2 \right).$$

**Theorem 2.** *Let $C$ be a coequality relation on the product $\prod_{i=1}^n X_i$ of sets with apartnesses. Then $C$ is decomposible if and only if $C$ is compatible with the diagonal operation $d$ defined by*

$$d : \left( \prod_{i=1}^n X_i \right)^n \ni (x^1, \ldots, x^n) \longmapsto (x_1^1, \ldots, x_n^n) \in \prod_{i=1}^n X_i.$$

*Proof.* (1) Let $C$ be a decomposible on the Cartesian product $\prod_{i=1}^n X_i$. We have

$$\begin{aligned}
&(d(x^1, x^2, \ldots, x^n), d(y^1, y^2, \ldots, y^n)) \in C \\
&\Leftrightarrow ((x_1^1, x_2^2, \ldots, x_n^n), (y_1^1, y_2^2, \ldots, y_n^n)) \in C \\
&\Leftrightarrow ((x_1^1, y_1^1), (x_2^2, y_2^2), \ldots, (x_n^n, y_n^n)) \in f(C) \\
&\Leftrightarrow (\exists i = 1, \ldots, n)((x_i^i, y_i^i) \in q_i) \\
&\Rightarrow (\exists i = 1, \ldots, n)(((x_1^i, y_1^i), \ldots, (x_n^i, y_n^i)) \in f(C)) \\
&\Leftrightarrow (\exists i = 1, \ldots, n)(((x_1^i, x_2^i, \ldots, x_n^i), (y_1^i, y_2^i, \ldots, y_n^i)) \in C) \\
&\Leftrightarrow (\exists i = 1, \ldots, n)((x^i, y^i) \in C).
\end{aligned}$$

(2) Let $C$ be a coequality relation on $\prod_{i=1}^n X_i$ compatible with the diagonal operation $d$. Let $((a_1, b_1), \ldots, (a_n, b_n))$ be an arbitrary element of

$$\bigcup_{i=1}^n \left( \prod_{j=1}^{i-1} X_j^2 \times q_i \times \prod_{j=i+1}^n X_j^2 \right).$$

Then there exists $i = 1, \ldots, n$ such that $(a_i, b_i) \in q_i$, i.e there exists $i = 1, \ldots, n$ and there exists $x_1 \in X_1, \ldots, x_{i-1} \in X_{i-1}, x_{i+1} \in X_{i+1}, \ldots, x_n \in X_n$ such that $((x_1, \ldots, x_{i-1}, a_i, x_{i+1}, \ldots, x_n), (x_1, \ldots, x_{i-1}, b_i, x_{i+1}, \ldots, x_n)) \in C$. Therefore

$$((a_1, \ldots, a_n), (b_1, \ldots, b_n)) \in C. \quad \square$$

**Theorem 3.** *Let $X_i$ be commutative rings with apartnesses and let $S$ be a coideal of the ring $\prod_{i=1}^n X_i$. Then there exists coideal s $S_i$ of $X_i (i = 1, \ldots, n)$ such that*

$$S = \bigcup_{i=1}^n \left( \prod_{j=1}^{i-1} X_j \times S_i \times \prod_{j=i+1}^n X_j \right).$$

*Proof.* Let $S$ be a coideal of the ring $\prod_{i=1}^n X_i$ . Then there exists cocongruemce $C$ on $\prod_{i=1}^n X_i$ such thet

$$(x, y) \in C \Leftrightarrow x - y \in S.$$

The cocongruence $C$ is compatible with the diagonal operation $d$ because the diagonal operation $d$ can be expressed as follows

$$\begin{aligned}
d(x^1, \ldots, x^n) &= (x_1^1, \ldots, x_n^n) \\
&= (x_1^1, 0, \ldots, 0) + \cdots + (0, \ldots, 0, x_n^n) \\
&= (x_1^1, x_2^1, \ldots, x_n^1)(1, 0, \ldots, 0) + \cdots + (x_1^n, x_2^n, \ldots, x_n^n)(0, 0, \ldots, 1).
\end{aligned}$$

Therefore, if we put $(1,0,\dots,0)=e^1,\dots,(0,\dots,0,1)=e^n$,

$$\begin{aligned}&(d(x^1,\dots,x^n),(y^1,\dots,y^n))\in C\\ &\Leftrightarrow\left(\sum_{i=1}^{n}x^ie^i,\sum_{i=1}^{n}y^ie^i\right)\in C\Rightarrow\bigvee_{i=1}^{n}((x^ie^i,y^ie^i)\in C)\\ &\Rightarrow\bigvee_{i=1}^{n}((x^i,y^i)\in C)(\text{because }(\forall i=1,\dots,n)\neg((e^i,e^i)\in C)).\end{aligned}$$

By Theorem 2, the cocongruence $C$ is decomposible such that

$$f(C)=\bigcup_{i=1}^{n}\left(\prod_{j=1}^{i-1}X_j^2\times q_i\times\prod_{j=i+1}^{n}X_j^2\right).$$

It is easy to prove that $q_i$ is a cocongruence on $X_i,(i=1,\dots,n)$. Thus, by Proposition 2.5 in [10], there exists the coideal $S_i$ of $X_i,(i=1,\dots,n)$ such that

$$(x_i,x_i')\in q_i\iff x_i-x_i'\in S_i.$$

Now, we have

$$\begin{aligned}&x\in S\Leftrightarrow(x,0)\in C\\ &\Leftrightarrow((x_1,\dots,x_n),(0,\dots,0))\in f(C)=\bigcup_{i=1}^{n}\left(\prod_{j=1}^{i-1}X_j^2\times q_i\times\prod_{j=i+1}^{n}X_j^2\right)\\ &\Leftrightarrow(\exists i,1\le i\le n)((x_i,0)\in q_i)\Leftrightarrow(\exists i,1\le i\le n)(x_i\in S_i)\\ &\Leftrightarrow x\in\prod_{j=1}^{i-1}X_j\times S_i\times\prod_{j=i+1}^{n}.\quad\Box\end{aligned}$$

## Results II

**Theorem 4.** *Let $C$ be a coequality relation on the Cartesian product $\prod_{i=1}^{\infty}X_i$. Then the relation $q_i$ on $X_i(i\in\mathbb{N})$, defined by*

$$\begin{aligned}&(x,y)\in q_i\Leftrightarrow(\exists a,b\in\textstyle\prod_{j=1}^{\infty}X_j)\\ &\qquad(a(i)=x\wedge b(i)=y\wedge(\forall k\in\mathbb{N}-\{i\})(a(k)=b(k)\wedge(a,b)\in C),\end{aligned}$$

*is a coequality relation on $X_i(i\in\mathbb{N})$.*

*Proof.* (1) Let $x,y$ be elements of $X_i$ such that $(x,y)\in q_i$ and let $u$ be an arbitrary element of $X_i$. Then there exist $a,b,c\in\prod_{i=1}^{\infty}X_i$ such that

$a(i) = x \wedge b(i) = y \wedge (\forall k \in \mathbb{N} - \{i\})(a(k) = b(k) \wedge (a,b) \in C)$, and $c(i) = u \wedge (\forall k \in \mathbb{N} - \{i\})(c(k) = a(k))$. From here, we have

$$\begin{aligned}(a,b) \in C &\Rightarrow (a,c) \in C \vee (c,b) \in C \Rightarrow a \neq c \vee c \neq b \\ &\Rightarrow x \neq u \vee u \neq y \Leftrightarrow (x,y) \neq (u,u).\end{aligned}$$

(2) $(x,y) \in q_i \Leftrightarrow (\exists a,b \in \prod_{j=1}^{\infty} X_j)(a(i) = x \wedge b(j) = y \wedge (\forall k \in N - \{i\})(a(k) = b(k)) \wedge (a,b) \in C) \Leftrightarrow (y,x) \in q_i$.

(3) Let $x,y,z$ be elements of $X_i$ such that $(x,z) \in q_i$. Then there exists $a,b,c \in \prod_{j=1}^{\infty} X_j$ such that $a(i) = x \wedge c(i) = z$ and $(\forall k \in N - \{i\})(a(k) = b(k)) \wedge (a,c) \in C$ and

$$b(i) = y \wedge (\forall k \in N - \{i\})(b(k) = a(k) = c(k)).$$

Therefore

$$\begin{aligned}(a,c) \in C &\Rightarrow (a,b) \in C \vee (b,c) \in C \\ &\Rightarrow (x,y) \in q_i \vee (y,z) \in q_i. \qquad \square\end{aligned}$$

Using the strongly extensional and embedding bijection

$$f : \left(\prod_{j=1}^{\infty} X_j\right)^2 \ni (a,b) \longmapsto \{(a(i), b(i)) : i \in N\} \in \prod_{j=1}^{\infty} X_j^2,$$

we have the following

**Corollary 4.1.** *Let $C \subset (\prod_{j=1}^{\infty} X_j)^2$ be a coequality relation. Then*

$$f(C) \subseteq \bigcup_{i=0}^{\infty} \left(\prod_{j=1}^{i} X_i^2 \times q_{i+1} \times \prod_{j=i+2}^{\infty} X_j^2\right).$$

*Proof.* Let $a,b$ be elements of $\prod_{j=1}^{\infty} X_j$ such that $(a,b) \in C$. If we put

$$a^t \in \prod_{j=1}^{\infty} X_j (t \in \{0\} \cup N \cup \{\infty\})\ a^0 = a, a^{\infty} = b,$$

$$(\forall j \in N)(j \leq t \Rightarrow a^t(j) = a(j) \wedge t > j \Rightarrow a^t(j) = b(j)),$$

we have

$$\begin{aligned}(a,b)\in C &\Rightarrow \bigvee_{i=0}^{\infty}((a^i,a^{i+1})\in C)\\ &\Rightarrow \bigvee_{i=0}^{\infty}((a(i),a(i+1)\in q_{i+1})\\ &\Rightarrow \bigvee_{i=0}^{\infty}\left((a,b)\in \prod_{j=1}^{i}X_j^2\times q_{i+1}\times \prod_{j=i+2}^{\infty}X_j^2\right)\\ &\Leftrightarrow (a,b)\in \bigcup_{i=0}^{\infty}\left(\prod_{j=1}^{i}X_j^2\times q_{i+1}\times \prod_{j=i+2}^{\infty}X_j^2\right).\end{aligned}$$

Therefore

$$f(C)\subseteq \bigcup_{i=0}^{\infty}\left(\prod_{j=1}^{i}X_j^2\times q_{i+1}\times \prod_{j=i+2}^{\infty}X_j^2\right).\quad \square$$

**Definition 2.** A coequality relation $C$ on the Cartesian product $\prod_{i=1}^{\infty}X_i$ is *decomposible* if only if

$$f(C)=\bigcup_{i=0}^{\infty}\left(\prod_{j=1}^{i}X_j^2\times q_{i+1}\times \prod_{j=i+2}^{\infty}X_j^2\right).$$

**Theorem 5.** *Let $C\subset(\prod_{i=1}^{\infty}X_i)^2$ be a coequality relation on the Cartesian product $\prod_{i=1}^{\infty}X_i$ of sets with aparthnesses. Then $C$ is decomposible if and only if $C$ is compatible with the diagonal operation $d$ on $\prod_{i=1}^{\infty}X_i$ defined by*

$$d:\left(\prod_{i=1}^{\infty}X_i\right)^{\mathbb{N}}\ni F\longmapsto\{F_n(n)\in X_n:n\in\mathbb{N}\}\in\prod_{i=1}^{\infty}X_i.$$

*Proof.* (1)Let $C$ be a decomposible relation on the Cartesian product $\prod_{i=1}^{\infty}X_i$ of sets and let $F\equiv\{\{F_n(j)\in X_j:j\in\mathbb{N}\}:n\in N\}$ and $G\equiv\{\{G_n(j)\in X_j:j\in\mathbb{N}\}:n\in\mathbb{N}\}$ be arbitrary elements of $(\prod_{i=1}^{\infty}X_i)^{\mathbb{N}}$. Then

$$\begin{aligned}&(d(F),d(G))\in C\\ &\Leftrightarrow(\{F_n(n)\in X_n:n\in\mathbb{N}\},\{G_n(n)\in X_n:n\in\mathbb{N}\})\in C\\ &\Leftrightarrow\{(F_n(n),G_n(n))\in X_n^2:n\in\mathbb{N}\}\in f(C)=\bigcup_{i=0}^{\infty}\left(\prod_{j=1}^{i}X_j^2\times q_{i+1}\times\prod_{j=i+2}^{\infty}X_j^2\right)\\ &\Leftrightarrow(\exists n\in\mathbb{N})((F_n(n),G_n(n))\in q_n)\\ &\Rightarrow(\exists n\in\mathbb{N})(\{(F_n(i),G_n(i))\in X_i^2:i\in\mathbb{N}\}\in f(C))\\ &\Leftrightarrow(\exists n\in\mathbb{N})((F_n,G_n)\in C.\end{aligned}$$

(2) Let $C$ be a coequality relation on $\prod_{i=1}^{\infty} X_i$ compatible with the diagonal operation $d$ and let $\{(a_i, b_i) : i \in \mathrm{N}\}$ be an arbitrary element of

$$\bigcup_{i=0}^{\infty}\left(\prod_{j=1}^{i} X_j^2 \times q_{i+1} \times \prod_{j=i+2}^{\infty} X_j^2\right).$$

Then there exists $n$ in N such that $(a_n, b_n) \in q_n$, i.e. there exists $n \in \mathrm{N}$ and there exist $x, y \in \prod_{j=1}^{\infty} X_j$ such that

$$(x, y) \in C \wedge x(n) = a_n \wedge y(n) = b_n \wedge (\forall k \in \mathrm{N} - \{n\})(x(k) = y(k) = x_k \in X_k).$$

Let we define $x^i, y^i \in \prod_{j=1}^{\infty} X_j (i \in \mathrm{N})$ such that

$$x^n = a \wedge y^n = b \wedge (\forall i \in \mathrm{N} - \{n\})(\forall j \in \mathrm{N})(x^i(j) = x_i = y^i(j)).$$

Then, by compatibility of $d$, we have

$$\begin{aligned}
&(x, y) \in C \Leftrightarrow \bigvee_{j=1}^{\infty}((x^i, y^i) \in C) \\
&\Rightarrow (x^n, y^n) \in C (\text{because } (\forall j \in \mathrm{N} - \{n\})\neg((x^j, y^j) \in C)) \\
&\Leftrightarrow (a, b) \in C \Leftrightarrow \{(a(i), b(i)) \in X_i^2 : i \in \mathrm{N}\} \in f(C). \quad \square
\end{aligned}$$

**Theorem 6.** *Let $X_i (i \in N)$ be commutative ring with an apartness and let $S$ be a coideal of the ring $\prod_{i=1}^{\infty} X_i$. Then there exists the coideal $S_i$ of $X_i (i \in N)$ such that*

$$S = \bigcup_{i=0}^{\infty}\left(\prod_{j=1}^{i} X_j \times S_{i+1} \times \prod_{j=i+2}^{\infty} X_j\right).$$

*Proof.* Let $S$ be a coideal of the ring $\prod_{i=1}^{\infty} X_i$. Then there exists the cocongruence $C$ on $\prod_{i=1}^{\infty} X_i$ such that $(x, y) \in C \Leftrightarrow x - y \in S$. The cocongrunce $C$ is compatible with the diagonal operation $d$ because the diagonal operation $d$ can be expressed as follows,if we put $e^i = (0, \ldots, 0, 1, 0, \ldots)(i \in N)$

$$d(F) = \{F_n(n) \in X_n : n \in N\} = \sum_{n=1}^{\infty} F_n e^n.$$

Therefore

$$(d(F), d(G)) \in C \Leftrightarrow \left(\sum_{n=1}^{\infty} F_n e^n, \sum_{n=1}^{\infty} G_n e^n\right) \in C$$
$$\Rightarrow \bigvee_{n=1}^{\infty} ((F_n e^n, G_n e^n) \in C)$$
$$\Rightarrow \bigvee_{n=1}^{\infty} ((F_n, G_n) \in C).$$

By Theorem 5, the cocongruence $C$ is decomposible such that

$$f(C) = \bigcup_{i=0}^{\infty} \left(\prod_{j=1}^{i} X_j^2 \times q_{i+1} \times \prod_{j=i+2}^{\infty} X_j^2\right).$$

It is easy to prove that $q_i$ is a cocongruence on $X_i (i \in N)$. Then, by Proposition 2.5 in [10], there exists the coideal $S_i$ of $X_i (i \in N)$, such that $(x, y) \in q_i \Leftrightarrow x - y \in S_i (i \in N)$. Now, we have

$$a \in S \Leftrightarrow (a, 0) \in C$$
$$\Leftrightarrow \{(a(i), 0) \in X_i^2 : i \in N\} \in f(C) = \bigcup_{i=0}^{\infty} \left(\prod_{j=1}^{i} X_j^2 \times q_{i+1} \times \prod_{j=i+2}^{\infty} X_j^2\right)$$
$$\Leftrightarrow (\exists n \in N)((a(n+1), 0) \in q_{n+1})$$
$$\Leftrightarrow (\exists n \in N)(a(n+1) \in S_{n+1})$$
$$a \in \bigcup_{n=0}^{\infty} \left(\prod_{j=1}^{n} X_j \times S_{n+1} \times \prod_{j=n+2}^{\infty} X_j\right). \quad \square$$

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