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ON INFINITESIMAL DEFORMATIONS OF A TOROID ROTATIONAL SURFACE GENERATED BY A QUADRANGULAR MERIDIAN

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ABSTRACT. *In this paper we consider a toroid rotational surface with a quadrangular meridian and obtain a necessary and sufficient condition for infinitesimal deformations of such a surface (eq.(1.18)). It is determined the field of deformations too.*

0. Introduction

In the paper [1] K.M. Belov gave necessary and sufficient condition for infinitesimal deformations of a toroid surface of rotation generated by a special case of the meridian.

One puts question of considering infintensimal deformations, i.e. of the rigidity of a surface with any quadrangular meridian.

In the plane of the meridian which rotates around the u-axis let's introduce Descartes' orthogonal coordinate system $uO\rho$ and let $\rho = \rho(u)$ be the equation of the meridian. If $\bar{e}$ is unit vector of the axis of rotation, $\bar{a}(v)$ unit vector of the ρ-axis, where v is the angle between the plane of initial position of the meridian and $\bar{a}(v)$ then $\bar{a}'(v) \perp \bar{a}(v)$ and $\bar{a}'(v) \perp \bar{e}$ (see [2], page 90, or [3] page 253).

The equation of a surface of rotation, in the coordinate system with the base $\bar{e}, \bar{a}, \bar{a}'$ is

$$\bar{r}(u,v) = u\bar{e} + \rho(u)\bar{a}(v). \tag{0.1}$$

As it is known ([2],page 91.) for every $k \in \{2, 3, ...\}$ there is a field of infinitesimal deformations

$$\begin{aligned}\bar{z}(u,v) = {} & [\varphi_k(u)e^{ikv} + \tilde{\varphi}_k(u)e^{-ikv}]\bar{e} \\ & + [\psi_k(u)e^{ikv} + \tilde{\psi}_k(u)e^{-ikv}]\bar{a}(v) \\ & + [\chi_k(u)e^{ikv} + \tilde{\chi}_k(u)e^{-ikv}]\bar{a}'(v)\end{aligned} \tag{0.2}$$

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of a surface (0.1),where e.g. $\tilde{\varphi}_k(u)$ is the conjugated value for $\varphi_k(u)$. The functions $\psi_k(u)$, $\chi_k(u)$ satisfy differential equation in the form of

$$\rho(u)\lambda''(u) + (k^2 - 1)\rho''(u)\lambda(u) = 0, \tag{0.3}$$

where $\lambda(u)$ is unknown function,and also satisfy the system

$$\begin{gathered} \varphi_k'(u) + \rho'(u)\psi_k'(u) = 0, \quad \psi_k(u) + ik\chi_k(u) = 0 \\ ik\varphi(u) + \rho'(u)[ik\psi_k(u) - \chi_k(u)] + \rho(u)\chi_k'(u) = 0. \end{gathered} \tag{0.4}$$

In the vertexes $u = \sigma$ of the meridian, $\psi_k(u)$ satisfy the equation ([2],page 112)

$$\rho(\sigma)[\psi_k'(\sigma + 0) - \psi_k'(\sigma - 0)] + (k^2 - 1)\psi_k(\sigma)[\rho'(\sigma + 0) - \rho'(\sigma - 0)] = 0, \tag{0.5}$$

supposing the function $\varphi_k(u), \chi_k(u)$ to be continuous in this points. Analogously,the equation (0.5) is satisfied for $\chi_k(u)$, if $\varphi_k(u), \psi_k(u)$ are continuous.

1. Condition for the existence of infinitesimal deformations

Suppose that quadrangle $A_i(u_i, \rho_i)$ $(i = 1,2,3,4;\ \rho_i > 0)$ rotates around the u-axis. If $\rho_{(1)}$ is value of ρ on the A_1A_2, $\rho_{(2)}$ on A_2A_3 etc., we get the equations of the sides of the meridian

$$\begin{gathered} A_iA_{i+1} : \rho_{(i)} = \rho_i + \frac{\rho_{i+1} - \rho_i}{u_{i+1} - u_i}(u - u_i), \\ (\ i = 1,2,3,4; A_5 \equiv A_1, \quad \rho_5 \equiv \rho_1, \quad u_5 \equiv u_1\ ), \end{gathered} \tag{1.1}$$

from where

$$\rho_{(i)}' = k_i = \frac{\rho_{i+1} - \rho_i}{u_{i+1} - u_i}. \tag{1.1'}$$

Dropping index k, let's designate with $\psi_{(i)}$ $(i = 1,2,3,4)$ the values of the function ψ on the sides $A_1A_2, ..., A_4A_1$ respectively. If we replace $\lambda(u)$ with $\psi_{(i)}(u)$ at (0.3) according to (1.1), we can see that the functions $\psi_{(i)}$ are linear, i.e.

$$\psi_{(i)} = M_iu + N_i \quad (i = 1,2,3,4) \tag{1.2}$$

Supposing that the functions $\psi_{(i)}(u)$ are continuous at the points $u = \sigma$ of the meridian $\rho = \rho(u)$, where $\rho'(\sigma - 0) \neq \rho'(\sigma + 0)$, we get the system

$$\begin{gathered} \psi_{(1)}(u_1) = \psi_{(4)}(u_1) = \psi_{(41)}(u_1) \\ \psi_{(2)}(u_2) = \psi_{(1)}(u_2) = \psi_{(12)}(u_2) \\ \psi_{(3)}(u_3) = \psi_{(2)}(u_3) = \psi_{(23)}(u_3) \\ \psi_{(4)}(u_4) = \psi_{(3)}(u_4) = \psi_{(34)}(u_4) \end{gathered} \tag{1.3}$$

According to (1.2) we have the system

$$
\begin{aligned}
M_1u_1+N_1&=M_4u_1+N_4\\
M_2u_2+N_2&=M_1u_2+N_1\\
M_3u_3+N_3&=M_2u_3+N_2\\
M_4u_4+N_4&=M_3u_4+N_3
\end{aligned}
\tag{1.4}
$$

i.e., if we consider this system as a system on N_i:

$$
\begin{array}{ccccl}
N_1 & & & -N_4 & =-M_1u_1+M_4u_1\\
N_1 & -N_2 & & & =-M_1u_2+M_2u_2\\
 & N_2 & -N_3 & & =-M_2u_3+M_3u_3\\
 & & N_3 & -N_4 & =-M_3u_4+M_4u_4
\end{array}
\tag{1.5}
$$

At the apeces of the meridian the condition (0.5) gives the equations:

$$
\begin{aligned}
A_1:&\quad \rho_1(M_1-M_4)+(k^2-1)(M_1u_1+N_1)(k_1-k_4)=0\\
A_2:&\quad \rho_2(M_2-M_1)+(k^2-1)(M_2u_2+N_2)(k_2-k_1)=0\\
A_3:&\quad \rho_3(M_3-M_2)+(k^2-1)(M_3u_3+N_3)(k_3-k_2)=0\\
A_4:&\quad \rho_4(M_4-M_3)+(k^2-1)(M_4u_4+N_4)(k_4-k_3)=0
\end{aligned}
$$

or:

$$
\begin{aligned}
&[\rho_1+(k^2-1)u_1(k_1-k_4)]M_1-\rho_1M_4+(k^2-1)(k_1-k_4)N_1=0\\
-&\rho_2M_1+[\rho_2+(k^2-1)u_2(k_2-k_1)]M_2+(k^2-1)(k_2-k_1)N_2=0\\
-&\rho_3M_2+[\rho_3+(k^2-1)u_3(k_3-k_2)]M_3+(k^2-1)(k_3-k_2)N_3=0\\
-&\rho_4M_3+[\rho_4+(k^2-1)u_4(k_4-k_3)]M_4+(k^2-1)(k_4-k_3)N_4=0
\end{aligned}
\tag{1.6}
$$

Necessary and surfficient condition for the compatibility of system (1.5) is $rank\, M = rank\, P$, where M is the matrix of the system and P is extended matrix of the system.In order to explore the system, we are making elementary transformations of the matrices M and P.

According to (1.5) :

$$
P=\begin{array}{c}
\begin{array}{cccc} N_1 & N_2 & N_3 & N_4 \end{array}\\
\left[\begin{array}{cccccc}
1 & 0 & 0 & -1 & \vdots & -M_1u_1+M_4u_1\\
1 & -1 & 0 & 0 & \vdots & -M_1u_2+M_2u_2\\
0 & 1 & -1 & 0 & \vdots & -M_2u_3+M_3u_3\\
0 & 0 & 1 & -1 & \vdots & -M_3u_4+M_4u_4
\end{array}\right]
\end{array}
\tag{1.7}
$$

Applying Gauss'algorithm for matrix P, let's realize following elementary transformations successively:

1) $-I \to II$, 2) $II \to III$, 3) $III \to IV$,
which means: 1) the first row is transcribed, it's elements are multiplied with -1 and added to corresponding elements of the second row,

2) the elements of the second row obtained in 1) we add to the corresponding elements of the third row , etc.. Thus we obtain

$$(1.8)\qquad P\sim\begin{array}{c}\begin{array}{cccccc} N_1 & N_2 & N_3 & N_4 & & \\ \end{array}\\ \left[\begin{array}{cccccc} 1 & 0 & 0 & -1 & \vdots & m_1\\ 0 & -1 & 0 & 1 & \vdots & m_2\\ 0 & 0 & -1 & 1 & \vdots & m_3\\ 0 & 0 & 0 & 0 & \vdots & m_4 \end{array}\right]\end{array}$$

where

$$\begin{aligned}
m_1 &= -M_1u_1 + M_4u_1\\
m_2 &= M_1u_1 - M_4u_1 - M_1u_2 + M_2u_2\\
m_3 &= M_1u_1 - M_4u_1 - M_1u_2 + M_2u_2 - M_2u_3 + M_3u_3\\
m_4 &= M_1u_1 - M_4u_1 - M_1u_2 + M_2u_2 - M_2u_3 + M_3u_3 - M_3u_4 + M_4u_4
\end{aligned}$$

Hence, the system is compatible if

$$m_4 = M_1(u_1-u_2) + M_2(u_2-u_3) + M_3(u_3-u_4) + M_4(u_4-u_1) = 0.$$

When $u_i = u_{i+1}$ $(i = 1, 2, 3, 4, u_5 = u_1)$ the meridian contains a side which is orthogonal on the axis of rotation, generated surface contains a plane part and it is non rigid (see[4]). We omit this case in following consideration.

Suppose that $u_4 \neq u_1$. Then

$$(1.9)\qquad M_4 = \frac{1}{u_1-u_4}[(u_1-u_2)M_1 + (u_2-u_3)M_2 + (u_3-u_4)M_3].$$

Reduced system (according to (1.8)) is:

$$(1.10)\qquad \begin{aligned}
N_1 - N_4 &= -M_1u_1 + M_4u_1\\
-N_2 + N_4 &= M_1(u_1-u_2) + M_2u_2 - M_4u_1\\
-N_3 + N_4 &= M_1(u_1-u_2) + M_2(u_2-u_3) + M_3u_3 - M_4u_1
\end{aligned}$$

From (1.9,10) we have

$$(1.11)\qquad \begin{aligned}
N_1 &= N_4 + \frac{u_1(u_4-u_2)}{u_1-u_4}M_1 + \frac{u_1(u_2-u_3)}{u_1-u_4}M_2 + \frac{u_1(u_3-u_4)}{u_1-u_4}M_3\\
N_2 &= N_4 + \frac{u_4(u_1-u_2)}{u_1-u_4}M_1 + \frac{u_2u_4-u_1u_3}{u_1-u_4}M_2 + \frac{u_1(u_3-u_4)}{u_1-u_4}M_3\\
N_3 &= N_4 + \frac{u_4(u_1-u_2)}{u_1-u_4}M_1 + \frac{u_4(u_2-u_3)}{u_1-u_4}M_2 + \frac{u_4(u_3-u_1)}{u_1-u_4}M_3
\end{aligned}$$

By the equations (1.9,11) unknowns M_4, N_1, N_2, N_3 are expressed by M_1, M_2, M_3 and N_4. Substituting (1.9) and (1.11) at (1.6) and designating

$$\begin{aligned} u_i - u_j &= u_{ij} \\ k_i - k_j &= k_{ij} \end{aligned} \tag{1.12}$$

we get the system

$$\begin{aligned}
&[\rho_1 u_{24} + (k^2-1)k_{14}u_1u_{12}]M_1 + [(k^2-1)k_{14}u_1 - \rho_1]u_{23}M_2 + \\
&\qquad + [(k^2-1)k_{14}u_1 - \rho_1]u_{34}M_3 + (k^2-1)k_{14}u_{14}N_4 = 0 \\
&[\rho_2 u_{41} + (k^2-1)k_{21}u_{12}u_4]M_1 + [\rho_2 u_{14} + (k^2-1)k_{21}u_{23}u_1]M_2 + \\
&\qquad + (k^2-1)k_{21}u_{34}u_1M_3 + (k^2-1)k_{21}u_{14}N_4 = 0 \\
&(k^2-1)k_{32}u_{12}u_4M_1 + [\rho_3 u_{41} + (k^2-1)k_{32}u_{23}u_4]M_2 + \\
&\qquad + [\rho_3 u_{14} + (k^2-1)k_{32}u_{34}u_1]M_3 + (k^2-1)k_{32}u_{14}N_4 = 0 \\
&[\rho_4 u_{12} + (k^2-1)u_{12}k_{43}u_4]M_1 + [\rho_4 u_{23} + (k^2-1)u_{23}k_{43}u_4]M_2 + \\
&\qquad + [\rho_4 u_{31} + (k^2-1)u_{34}k_{43}u_4]M_3 + (k^2-1)u_{14}k_{43}N_4 = 0.
\end{aligned} \tag{1.13}$$

Necessary and sufficient condition for this system of linear homogeneous equations to have nontrivial solutions is the *rank* of matrix

$$H = \begin{array}{c} \begin{array}{cccc} N_4 & M_3 & M_2 & M_1 \end{array} \\ \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \end{array} \tag{1.14}$$

of the system to be less then 4. We have to find a condition under which it is valid. According to (1.13) we have

$$\begin{aligned}
A_{11} &= (k^2-1)k_{14}u_{14} & A_{12} &= \rho_1 u_{43} + (k^2-1)k_{14}u_{34}u_1 \\
A_{13} &= \rho_1 u_{32} + (k^2-1)k_{14}u_{23}u_1 & A_{14} &= \rho_1 u_{24} + (k^2-1)k_{14}u_{12}u_1 \\
A_{21} &= (k^2-1)k_{21}u_{14} & A_{22} &= (k^2-1)k_{21}u_{34}u_1 \\
A_{23} &= \rho_2 u_{14} + (k^2-1)k_{21}u_{23}u_1 & A_{24} &= \rho_2 u_{41} + (k^2-1)k_{21}u_{12}u_4 \\
A_{31} &= (k^2-1)k_{32}u_{14} & A_{32} &= \rho_3 u_{14} + (k^2-1)k_{32}u_{34}u_1 \\
A_{33} &= \rho_3 u_{41} + (k^2-1)k_{32}u_{23}u_4 & A_{34} &= (k^2-1)k_{32}u_{12}u_4 \\
A_{41} &= (k^2-1)k_{43}u_{14} & A_{42} &= \rho_4 u_{31} + (k^2-1)k_{43}u_{34}u_4 \\
A_{43} &= \rho_4 u_{23} + (k^2-1)k_{43}u_{23}u_4 & A_{44} &= \rho_4 u_{12} + (k^2-1)k_{43}u_{12}u_4
\end{aligned} \tag{1.14'}$$

Evidently,it is always $k_{ii+1} \neq 0$,as on contrary the meridian will not be quadrangular. Applying at the same time following operations to the matrix (1.14)

$$I\frac{k_{12}}{k_{14}} \to II, \qquad I\frac{k_{23}}{k_{14}} \to III, \qquad II\frac{k_{34}}{k_{21}} \to IV,$$

we obtain

$$H \sim [B_{ij}], \tag{1.15}$$

where

$$\begin{aligned}
B_{1j} =&A_{1j}, B_{21} = 0, B_{22} = \rho_1 u_{43}\frac{k_{12}}{k_{14}}, B_{23} = \rho_1 u_{32}\frac{k_{12}}{k_{14}} + \rho_2 u_{14}\\
B_{24} =&\rho_1 u_{24}\frac{k_{12}}{k_{14}} + \rho_2 u_{41} + (k^2-1)u_{12}u_{14}k_{12}, B_{31} = 0\\
B_{32} =&\rho_1 u_{43}\frac{k_{23}}{k_{14}} + \rho_3 u_{14}, B_{33} = \rho_1 u_{32}\frac{k_{23}}{k_{14}} + \rho_3 u_{41} + (k^2-1)u_{23}k_{23}u_{14},\\
B_{34} =&\rho_1 u_{24}\frac{k_{23}}{k_{14}} + (k^2-1)u_{12}k_{23}u_{14}, B_{41} = 0, B_{42} = \rho_4 u_{31} + (k^2-1)u_{34}u_{14}k_{34}\\
B_{43} =&\rho_2 u_{14}\frac{k_{34}}{k_{21}} + \rho_4 u_{23} + (k^2-1)u_{23}k_{34}u_{14}, B_{44} = \rho_2 u_{41}\frac{k_{34}}{k_{21}} + \rho_4 u_{12}.
\end{aligned}$$

Further, we apply the operations

$$\begin{aligned}
II\frac{k_{41}}{\rho_1 u_{43}k_{12}}(\rho_1 u_{43}\frac{k_{23}}{k_{14}} + \rho_3 u_{14}) \rightarrow III\\
II\frac{k_{41}}{\rho_1 u_{43}k_{12}}[\rho_4 u_{31} + (k^2-1)u_{34}u_{14}k_{34}] \rightarrow IV
\end{aligned}$$

and obtain

$$H \sim [C_{ij}], \tag{1.16}$$

where

$$\begin{aligned}
C_{1j} =&B_{1j} = A_{1j}, C_{2j} = B_{2j}, C_{31} = C_{32} = 0\\
C_{33} =&\rho_2 u_{14}\frac{k_{32}}{k_{12}} + \rho_3\frac{u_{14}u_{24}}{u_{43}} + \frac{\rho_2\rho_3(u_{14})^2k_{41}}{\rho_1 u_{43}k_{12}} + (k^2-1)u_{23}u_{14}k_{23},\\
C_{34} =&\rho_2 u_{41}\frac{k_{32}}{k_{12}} + \rho_3 u_{41}\frac{u_{24}}{u_{43}} + \frac{\rho_3(k^2-1)u_{12}(u_{14})^2k_{41}}{\rho_1 u_{43}} + \frac{\rho_2\rho_3(u_{14})^2k_{14}}{\rho_1 u_{43}k_{12}},\\
C_{41} =&C_{42} = 0
\end{aligned}$$

$$\begin{aligned}
C_{43} =&\rho_2 u_{14}\frac{k_{34}}{k_{12}} + \rho_4 u_{23}\frac{u_{41}}{u_{43}} + \frac{\rho_2\rho_4 u_{14}u_{31}k_{41}}{\rho_1 u_{43}k_{12}} + \frac{\rho_2(k^2-1)(u_{14})^2k_{14}k_{3}4}{\rho_1 k_{12}}\\
C_{44} =&\frac{\rho_4 u_{14}u_{23}}{u_{43}} + \frac{\rho_2 u_{41}k_{34}}{k_{21}} + \frac{\rho_2(k^2-1)(u_{14})^2k_{41}k_{34}}{\rho_1 k_{12}} +\\
&+\frac{\rho_4(k^2-1)u_{12}u_{14}u_{31}k_{41}}{\rho_1 u_{43}} + \frac{\rho_2\rho_4 u_{41}u_{31}k_{41}}{\rho_1 u_{43}k_{12}} +\\
&+\frac{(k^2-1)^2u_{12}(u_{14})^2k_{14}k_{34}}{\rho_1} + (k^2-1)u_{24}u_{14}k_{34}.
\end{aligned} \tag{1.16'}$$

By transformation $III\left(-\frac{C_{43}}{C_{33}}\right) \rightarrow IV$ the matrix (1.16) take a form

$$H \sim [D_{ij}], \tag{1.17}$$

where

$$\begin{aligned} &D_{1j} = C_{1j} = B_{1j} = A_{1j}, \quad D_{2j} = C_{2j} = B_{2j}, \quad D_{3j} = C_{3j} \\ &D_{41} = D_{42} = D_{43} = 0, \quad D_{44} = -\frac{C_{43}C_{34}}{C_{33}} + C_{44} = \frac{1}{C_{33}}(C_{33}C_{44} - C_{43}C_{34}), \end{aligned} \tag{1.17'}$$

and C_{ij} are given by (1.16'). The rank of the matrix N will be less then 4 for $D_{44} = 0$,i.e. $C_{33}C_{44} - C_{43}C_{34} = 0$,what gives

$$\begin{aligned} &[\rho_1\rho_2 u_{43}k_{32} + \rho_1\rho_3 u_{24}k_{12} + \rho_2\rho_3 u_{14}k_{41} + \rho_1(k^2-1)u_{23}u_{43}k_{12}k_{23}]\times \\ &[\rho_4 u_{12}u_{31}k_{41} + (k^2-1)u_{12}u_{43}u_{14}k_{14}k_{34} + \rho_1 u_{43}u_{24}k_{34}]- \\ &-(\rho_1 u_{23}u_{43}k_{23} + \rho_3 u_{12}u_{14}k_{41})\times \\ &[\rho_1\rho_2 u_{34}k_{34} + \rho_1\rho_4 u_{32}k_{12} + \rho_2\rho_4 u_{31}k_{41} + \rho_2(k^2-1)u_{14}u_{43}k_{14}k_{34}] = 0, \end{aligned} \tag{1.18}$$

where u_{ij}, k_{ij} are given by (1.12) and (1.1').
Thus, we have

Theorem. *Necessary and sufficient condition for infinitesimal deformations of a toroid rotational surface,which is generated by a quadrangular meridian with apeces* $A_i(u_i, \rho_i)$ $(\rho_i > 0,\ u_{i+1} \neq u_i,\ u_5 \equiv u_1,\ i = 1,2,3,4)$, *around the Ou axis is the relation (1.18) where* u_{ij}, k_{ij} *are given by (1.12) and (1.1').*

Remark.. *If we apply (1.18) to the quadrangle of Belov* $A_1(-1,b)$, $A_2(0,b+c_1)$, $A_3(1,b)$, $A_4(0,b-c_2)$ *we obtain the relation* $1/c_2 - 1/c_1 = k^2/b$, *which Belov obtained in other manner. So, the previous theorem is a generalization of the result of Belov.*

2. Determination of the field of infinitesimal deformation

Above applied method makes possible to determine the field of infinitesimal deformation.From (1.17) one obtains reduced system

$$\begin{aligned} &D_{11}N_4 + D_{12}M_3 + D_{13}M_2 + D_{14}M_1 = 0 \\ &D_{22}M_3 + D_{23}M_2 + D_{24}M_1 = 0 \\ &D_{33}M_2 + D_{34}M_1 = 0, \end{aligned} \tag{2.1}$$

from where

$$M_2 = -\frac{D_{34}}{D_{33}}M_1 \tag{2.2a}$$

$$M_3 = \left(\frac{D_{23}D_{34}}{D_{22}D_{33}} - \frac{D_{24}}{D_{22}}\right)M_1 \tag{2.2b}$$

$$N_4 = \left[-\frac{D_{12}}{D_{11}}\left(\frac{D_{23}D_{34}}{D_{22}D_{33}} - \frac{D_{24}}{D_{22}}\right) + \frac{D_{13}D_{34}}{D_{11}D_{33}} - \frac{D_{14}}{D_{11}}\right]M_1 \tag{2.2c}$$

Further, from (1.9) we have

$$M_4 = \frac{1}{u_{14}}\left[u_{12} - \frac{u_{23}D_{34}}{D_{33}} + u_{34}\left(\frac{D_{23}D_{34}}{D_{22}D_{33}} - \frac{D_{24}}{D_{22}}\right)\right]M_1, \tag{2.2d}$$

$$\begin{aligned}N_1 = \Bigg\{ \Bigg[-\frac{D_{12}}{D_{11}} \left(\frac{D_{23}D_{34}}{D_{22}D_{33}} - \frac{D_{24}}{D_{22}} \right) + \frac{D_{13}D_{34}}{D_{11}D_{33}} - \frac{D_{14}}{D_{11}} \Bigg] + \\ + \frac{u_1u_{34}}{u_{14}} \left(\frac{D_{23}D_{34}}{D_{22}D_{33}} - \frac{D_{24}}{D_{22}} \right) - \frac{u_1u_{23}D_{34}}{u_{14}D_{33}} + \frac{u_1u_{12}}{u_{14}} - u_1 \Bigg\} M_1,\end{aligned} \tag{2.2e}$$

$$\begin{aligned}N_2 = \Bigg\{ \Bigg[-\frac{D_{12}}{D_{11}} \left(\frac{D_{23}D_{34}}{D_{22}D_{33}} - \frac{D_{24}}{D_{22}} \right) + \frac{D_{13}D_{34}}{D_{11}D_{33}} - \frac{D_{14}}{D_{11}} \Bigg] - u_1 + u_2 + \\ + \frac{D_{34}u_2}{D_{33}} + \frac{u_1u_{12}}{u_{14}} - \frac{u_1u_{23}D_{34}}{u_{14}D_{33}} + \frac{u_1u_{34}}{u_{14}} \left(\frac{D_{23}D_{34}}{D_{22}D_{33}} - \frac{D_{24}}{D_{22}} \right) \Bigg\} M_1,\end{aligned} \tag{2.2f}$$

$$\begin{aligned}N_3 = \Bigg\{ \Bigg[-\frac{D_{12}}{D_{11}} \left(\frac{D_{23}D_{34}}{D_{22}D_{33}} - \frac{D_{24}}{D_{22}} \right) + \frac{D_{13}D_{34}}{D_{11}D_{33}} - \frac{D_{14}}{D_{11}} \Bigg] - u_1 + u_2 + \\ + \frac{D_{34}u_2}{D_{33}} - \frac{D_{34}u_3}{D_{33}} - \left(\frac{D_{23}D_{34}}{D_{22}D_{33}} - \frac{D_{24}}{D_{22}} \right) u_3 + \frac{u_1u_{12}}{u_{14}} - \\ - \frac{D_{34}u_1u_{23}}{D_{33}u_{14}} + \frac{u_1u_{34}}{u_{14}} \left(\frac{D_{23}D_{34}}{D_{22}D_{33}} - \frac{D_{24}}{D_{22}} \right) \Bigg\} M_1.\end{aligned} \tag{2.2g}$$

By the equations $(2.2.a-g)$ we expressed M_i, N_i $(i=1,2,3,4)$ by M_1 (indefinit const.). Further, we obtain $\psi_{(i)}(u)$ on the base of (1.2).In this manner, we get the field $\overline{z}(u,v)$ of infinitesimal deformations, given by (0.2).

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References

[1] К.М. Белов, *О бесконечно малых изгибаниях торообразной поверхности вращения*, Сиб. мат. журнал, т Н3 (IX)(1968), 490-494.

[2] С.Э. Кон-Фоссен, Некоторые вопросы дифференциальной геометрии в целом, Физматгиз, Москва, 1959.

[3] В.Ф. Каган, Основы теории поверхностей, Т.И, ОГИЗ, Москва-Ленниград, 1947.

[4] Н.В. Ефимов, *Качественные вопросы теории деформаций поверхностей*, УМН 3,2(1948), 47-158.

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