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THE COMPLETE LIST OF $F(2)$ TYPE STRUCTURES IN THE COMPLEX FINSLER SPACE

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ABSTRACT. *The complex Finsler space E' is formed in such a way, that its tangent space $T(E')$ is equal to $T(F_1) \oplus iT(F_2)$, where F_1 and F_2 are two 2n-dimensional Finsler spaces. Using the nonlinear connections N and $\overline{N}$ of F_1 and F_2 respectively, the adapted basis B' of $T(E')$ is formed. There is given the complete list of $F(2)$ type structures. Some of them for different values of parameters are almost complex, almost product or tangent structures.*

1. Complex Finsler spaces

Let us consider two n-dimensional Finsler spaces $F_1(x, \dot{x})$ and $F_2(y, \dot{y})$. The allowable coordinate transformations in F_1 and F_2 are given by

$$
(1.1)\qquad
\begin{aligned}
x^{a'} &= x^{a'}(x) & \qquad y^{i'} &= y^{i'}(y)\\
\dot{x}^{a'} &= A^{a'}_a(x)\,\dot{x}^a & \qquad \dot{y}^{i'} &= B^{i'}_i(y)\dot{y}^i\\
A^{a'}_a &= \tfrac{\partial x^{a'}}{\partial x^a} & \qquad B^{i'}_i &= \tfrac{\partial y^{i'}}{\partial y^i},
\end{aligned}
$$

where

$$rank[A^{a'}_a] = n, \quad rank[B^{i'}_i] = n,$$

so the inverse transformations exist.

The adapted basis of $T(F_1)$ is $B_1 = \{\frac{\delta}{\delta x^a}, \frac{\partial}{\partial \dot{x}^a}\}$ and the adapted basis of $T(F_2)$ is $B_2 = \{\frac{\delta}{\delta y^i}, \frac{\partial}{\partial \dot{y}^i}\}$, where

$$\frac{\delta}{\delta x^a} = \frac{\partial}{\partial x^a} - N^b_a(x, \dot{x})\frac{\partial}{\partial \dot{x}^b}, \quad \frac{\delta}{\delta y^i} = \frac{\partial}{\partial y^i} - \overline{N}^j_i(y, \dot{y})\frac{\partial}{\partial \dot{y}^j}.$$

$N^b_a(x, \dot{x})$ and $\overline{N}^j_i(y, \dot{y})$ are coefficients of the non-linear connections, which satisfy the usual transformation law with respect to (1).

The complex Finsler space $E'(x, \dot{x}, y, \dot{y})$ is formed in such a way that B', the adapted basis of $T(E')$, is given by $B' = B_1 \cup iB_2$.

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For the further exploration we shall use five kinds of indices

$a, b, c, d, e, f, g = 1, 2, \dots, n,$ $\quad i, j, h, k, l, m, p, q = n+1, \dots, 2n$
$A, B, C, D, E, F, G = 2n+1, .., 3n,$ $\quad I, J, H, K, L, M, P, Q = 3n+1, .., 4n$
$\alpha, \beta, \gamma, \delta, \kappa, \nu, \mu = 1, 2, \dots, 4n.$

The following equalities are valid

$$(1.2) \qquad \begin{aligned} a = i = A = I(mod\ n), \qquad & b = j = B = J(mod\ n) \\ c = h = C = H(mod\ n). & \end{aligned}$$

Using these indices, B' and its dual B'^* can be written in the form

$$(1.3) \qquad \begin{aligned} &\text{(a)} \quad B' = \{\partial_\alpha\} = \{\frac{\delta}{\delta x^a}, i\frac{\delta}{\delta y^i}, \frac{\partial}{\partial \dot{x}^A}, i\frac{\partial}{\partial \dot{y}^I}\} \\ &\text{(b)} \quad B'^* = \{d^\alpha\} = \{dx^b, -idy^j, \delta\dot{x}^B, -i\delta\dot{y}^J\}, \end{aligned}$$

where

$$\delta\dot{x}^B = d\dot{x}^B + N^B_c(x, \dot{x})dx^c, \quad \delta\dot{y}^J = d\dot{y}^J + N^J_i(y, \dot{y})dy^i.$$

If we introduce the notations

$$(1.4) \qquad \text{(a)} \quad R = [\frac{\delta}{\delta x^a} \; i\frac{\delta}{\delta y^i} \; \frac{\partial}{\partial \dot{x}^A} \; i\frac{\partial}{\partial \dot{y}^I}]$$

$$\text{(b)} \quad D = \begin{bmatrix} A^a_{a'}(x') & 0 & 0 & 0 \\ 0 & B^i_{i'}(y') & 0 & 0 \\ 0 & 0 & A^A_{A'}(x') & 0 \\ 0 & 0 & 0 & B^I_{I'}(y') \end{bmatrix}$$

$$\text{(c)} \quad K' = \begin{bmatrix} dx^{a'} \\ -idy^{i'} \\ \delta\dot{x}^{A'} \\ -i\delta\dot{y}^{I'} \end{bmatrix},$$

then the following relations are valid.

$$(1.5) \qquad R' = RD \qquad K = DK'.$$

R' is obtained from R if indices a, i, A and I are substituted by a', i', A' and I' respectively, similarly K is obtained from K' if in K' the sign "$'$" over all indices is dropped. D is regular matrix, so exists D^{-1}. From (4a) we have

$$(1.6) \qquad R = R'D^{-1} \qquad K' = D^{-1}K,$$

where

$$D^{-1} = \begin{bmatrix} A_b^{b'}(x) & 0 & 0 & 0 \\ 0 & B_j^{j'}(y) & 0 & 0 \\ 0 & 0 & A_B^{B'}(x) & 0 \\ 0 & 0 & 0 & B_J^{J'}(y) \end{bmatrix}.$$

2. The $F(2)$ type structures defined on E'

Definition 2.1. *The tensor field F of type (1) defined on E' is the structure of $F(k)$ type if in the basis B' its matrix can be decomposed on 4×4 blocks of format $n \times n$, such that in each row and each column are k scalar matrices and $4 - k$ zero blocks.*

Notation. Every one of the scalar fields a, b, c, d, e, f, g, h denotes the corresponding real or complex scalar matrix of type $n \times n$ (for example $a = a(x, \dot{x}, y, \dot{y})I$).

Theorem 2.1. *There exist 90 $F(2)$ type structures on E'. They are:*

$$\begin{bmatrix} a & 0 & e & 0 \\ b & 0 & f & 0 \\ 0 & c & 0 & g \\ 0 & d & 0 & h \end{bmatrix} \begin{bmatrix} a & 0 & e & 0 \\ b & 0 & 0 & g \\ 0 & c & f & 0 \\ 0 & d & 0 & h \end{bmatrix} \begin{bmatrix} a & 0 & e & 0 \\ b & 0 & 0 & g \\ 0 & c & 0 & h \\ 0 & d & f & 0 \end{bmatrix} \begin{bmatrix} a & 0 & 0 & g \\ b & 0 & e & 0 \\ 0 & c & f & 0 \\ 0 & d & 0 & h \end{bmatrix}$$

$$\begin{bmatrix} a & 0 & 0 & g \\ b & 0 & e & 0 \\ 0 & c & 0 & h \\ 0 & d & f & 0 \end{bmatrix} \begin{bmatrix} a & 0 & 0 & g \\ b & 0 & 0 & h \\ 0 & c & e & 0 \\ 0 & d & f & 0 \end{bmatrix} \begin{bmatrix} a & 0 & e & 0 \\ 0 & c & f & 0 \\ b & 0 & 0 & g \\ 0 & d & 0 & h \end{bmatrix} \begin{bmatrix} a & 0 & e & 0 \\ 0 & c & 0 & g \\ b & 0 & f & 0 \\ 0 & d & 0 & h \end{bmatrix}^{*} \quad (1)$$

$$\begin{bmatrix} a & 0 & e & 0 \\ 0 & c & 0 & g \\ b & 0 & 0 & h \\ 0 & d & f & 0 \end{bmatrix} \begin{bmatrix} a & 0 & 0 & g \\ 0 & c & e & 0 \\ b & 0 & f & 0 \\ 0 & d & 0 & h \end{bmatrix} \begin{bmatrix} a & 0 & 0 & g \\ 0 & c & e & 0 \\ b & 0 & 0 & h \\ 0 & d & f & 0 \end{bmatrix} \begin{bmatrix} a & 0 & 0 & g \\ 0 & c & 0 & h \\ b & 0 & e & 0 \\ 0 & d & f & 0 \end{bmatrix}$$

$$\begin{bmatrix} a & 0 & e & 0 \\ 0 & c & f & 0 \\ 0 & d & 0 & g \\ b & 0 & 0 & h \end{bmatrix} \begin{bmatrix} a & 0 & e & 0 \\ 0 & c & 0 & g \\ 0 & d & f & 0 \\ b & 0 & 0 & h \end{bmatrix} \begin{bmatrix} a & 0 & e & 0 \\ 0 & c & 0 & g \\ 0 & d & 0 & h \\ b & 0 & f & 0 \end{bmatrix} \begin{bmatrix} a & 0 & 0 & g \\ 0 & c & e & 0 \\ 0 & d & f & 0 \\ b & 0 & 0 & h \end{bmatrix}^{*} \quad (2)$$

$$\begin{bmatrix} a & 0 & 0 & g \\ 0 & c & e & 0 \\ 0 & d & 0 & h \\ b & 0 & f & 0 \end{bmatrix} \begin{bmatrix} a & 0 & 0 & g \\ 0 & c & 0 & h \\ 0 & d & e & 0 \\ b & 0 & f & 0 \end{bmatrix} \begin{bmatrix} 0 & c & e & 0 \\ a & 0 & f & 0 \\ b & 0 & 0 & g \\ 0 & d & 0 & h \end{bmatrix} \begin{bmatrix} 0 & c & e & 0 \\ a & 0 & 0 & g \\ b & 0 & f & 0 \\ 0 & d & 0 & h \end{bmatrix}$$

$$\begin{bmatrix} 0 & c & e & 0 \\ a & 0 & 0 & g \\ b & 0 & 0 & h \\ 0 & d & f & 0 \end{bmatrix}^{*}_{(3)} \begin{bmatrix} 0 & c & 0 & g \\ a & 0 & e & 0 \\ b & 0 & f & 0 \\ 0 & d & 0 & h \end{bmatrix} \begin{bmatrix} 0 & c & 0 & g \\ a & 0 & e & 0 \\ b & 0 & 0 & h \\ 0 & d & f & 0 \end{bmatrix} \begin{bmatrix} 0 & c & 0 & g \\ a & 0 & 0 & h \\ b & 0 & e & 0 \\ 0 & d & f & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & c & e & 0 \\ a & 0 & f & 0 \\ 0 & d & 0 & g \\ b & 0 & 0 & h \end{bmatrix} \begin{bmatrix} 0 & c & e & 0 \\ a & 0 & 0 & g \\ 0 & d & f & 0 \\ b & 0 & 0 & h \end{bmatrix} \begin{bmatrix} 0 & c & e & 0 \\ a & 0 & 0 & g \\ 0 & d & 0 & h \\ b & 0 & f & 0 \end{bmatrix} \begin{bmatrix} 0 & c & 0 & g \\ a & 0 & e & 0 \\ 0 & d & f & 0 \\ b & 0 & 0 & h \end{bmatrix}$$

$$\begin{bmatrix} 0 & c & 0 & g \\ a & 0 & e & 0 \\ 0 & d & 0 & h \\ b & 0 & f & 0 \end{bmatrix}^{*}_{(4)} \begin{bmatrix} 0 & c & 0 & g \\ a & 0 & 0 & h \\ 0 & d & e & 0 \\ b & 0 & f & 0 \end{bmatrix} \begin{bmatrix} 0 & c & e & 0 \\ 0 & d & f & 0 \\ a & 0 & 0 & g \\ b & 0 & 0 & h \end{bmatrix} \begin{bmatrix} 0 & c & e & 0 \\ 0 & d & 0 & g \\ a & 0 & f & 0 \\ b & 0 & 0 & h \end{bmatrix}$$

$$\begin{bmatrix} 0 & c & e & 0 \\ 0 & d & 0 & g \\ a & 0 & 0 & h \\ b & 0 & f & 0 \end{bmatrix} \begin{bmatrix} 0 & c & 0 & g \\ 0 & d & e & 0 \\ a & 0 & f & 0 \\ b & 0 & 0 & h \end{bmatrix} \begin{bmatrix} 0 & c & 0 & g \\ 0 & d & e & 0 \\ a & 0 & 0 & h \\ b & 0 & f & 0 \end{bmatrix} \begin{bmatrix} 0 & c & 0 & g \\ 0 & d & 0 & h \\ a & 0 & e & 0 \\ b & 0 & f & 0 \end{bmatrix}$$

$$\begin{bmatrix} a & c & 0 & 0 \\ b & 0 & e & 0 \\ 0 & d & 0 & g \\ 0 & 0 & f & h \end{bmatrix} \begin{bmatrix} a & c & 0 & 0 \\ b & 0 & 0 & g \\ 0 & d & e & 0 \\ 0 & 0 & f & h \end{bmatrix} \begin{bmatrix} a & c & 0 & 0 \\ b & 0 & e & 0 \\ 0 & 0 & f & g \\ 0 & d & 0 & h \end{bmatrix} \begin{bmatrix} a & c & 0 & 0 \\ b & 0 & 0 & g \\ 0 & 0 & e & h \\ 0 & d & f & 0 \end{bmatrix}$$

$$\begin{bmatrix} a & c & 0 & 0 \\ 0 & d & e & 0 \\ b & 0 & 0 & g \\ 0 & 0 & f & h \end{bmatrix} \begin{bmatrix} a & c & 0 & 0 \\ 0 & d & 0 & g \\ b & 0 & e & 0 \\ 0 & 0 & f & h \end{bmatrix} \begin{bmatrix} a & c & 0 & 0 \\ 0 & 0 & e & g \\ b & 0 & f & 0 \\ 0 & d & 0 & h \end{bmatrix} \begin{bmatrix} a & c & 0 & 0 \\ 0 & 0 & e & g \\ b & 0 & 0 & h \\ 0 & d & f & 0 \end{bmatrix}$$

$$\begin{bmatrix} a & c & 0 & 0 \\ 0 & d & e & 0 \\ 0 & 0 & f & g \\ b & 0 & 0 & h \end{bmatrix} \begin{bmatrix} a & c & 0 & 0 \\ 0 & d & 0 & g \\ 0 & 0 & e & h \\ b & 0 & f & 0 \end{bmatrix} \begin{bmatrix} a & c & 0 & 0 \\ 0 & 0 & e & g \\ 0 & d & f & 0 \\ b & 0 & 0 & h \end{bmatrix} \begin{bmatrix} a & c & 0 & 0 \\ 0 & 0 & e & g \\ 0 & d & 0 & h \\ b & 0 & f & 0 \end{bmatrix}$$

$$\begin{bmatrix} b & 0 & e & 0 \\ a & c & 0 & 0 \\ 0 & d & 0 & g \\ 0 & 0 & f & h \end{bmatrix} \begin{bmatrix} b & 0 & 0 & g \\ a & c & 0 & 0 \\ 0 & d & e & 0 \\ 0 & 0 & f & h \end{bmatrix} \begin{bmatrix} b & 0 & e & 0 \\ a & c & 0 & 0 \\ 0 & 0 & f & g \\ 0 & d & 0 & h \end{bmatrix} \begin{bmatrix} b & 0 & 0 & g \\ a & c & 0 & 0 \\ 0 & 0 & e & h \\ 0 & d & f & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & d & e & 0 \\ a & c & 0 & 0 \\ b & 0 & 0 & g \\ 0 & 0 & f & h \end{bmatrix} \begin{bmatrix} 0 & d & 0 & g \\ a & c & 0 & 0 \\ b & 0 & e & 0 \\ 0 & 0 & f & h \end{bmatrix} \begin{bmatrix} 0 & 0 & e & g \\ a & c & 0 & 0 \\ b & 0 & f & 0 \\ 0 & d & 0 & h \end{bmatrix} \begin{bmatrix} 0 & 0 & e & g \\ a & c & 0 & 0 \\ b & 0 & 0 & h \\ 0 & d & f & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & d & e & 0 \\ a & c & 0 & 0 \\ 0 & 0 & f & g \\ b & 0 & 0 & h \end{bmatrix} \begin{bmatrix} 0 & d & 0 & g \\ a & c & 0 & 0 \\ 0 & 0 & e & h \\ b & 0 & f & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & e & g \\ a & c & 0 & 0 \\ 0 & d & f & 0 \\ b & 0 & 0 & h \end{bmatrix} \begin{bmatrix} 0 & 0 & e & g \\ a & c & 0 & 0 \\ 0 & d & 0 & h \\ b & 0 & f & 0 \end{bmatrix}$$

$$\begin{bmatrix} b & 0 & e & 0 \\ 0 & d & 0 & g \\ a & c & 0 & 0 \\ 0 & 0 & f & h \end{bmatrix} \begin{bmatrix} b & 0 & 0 & g \\ 0 & d & e & 0 \\ a & c & 0 & 0 \\ 0 & 0 & f & h \end{bmatrix} \begin{bmatrix} b & 0 & e & 0 \\ 0 & 0 & f & g \\ a & c & 0 & 0 \\ 0 & d & 0 & h \end{bmatrix} \begin{bmatrix} b & 0 & 0 & g \\ 0 & 0 & e & h \\ a & c & 0 & 0 \\ 0 & d & f & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & d & e & 0 \\ b & 0 & 0 & g \\ a & c & 0 & 0 \\ 0 & 0 & f & h \end{bmatrix} \begin{bmatrix} 0 & d & 0 & g \\ b & 0 & e & 0 \\ a & c & 0 & 0 \\ 0 & 0 & f & h \end{bmatrix} \begin{bmatrix} 0 & 0 & e & g \\ b & 0 & f & 0 \\ a & c & 0 & 0 \\ 0 & d & 0 & h \end{bmatrix} \begin{bmatrix} 0 & 0 & e & g \\ b & 0 & 0 & h \\ a & c & 0 & 0 \\ 0 & d & f & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & d & e & 0 \\ 0 & 0 & f & g \\ a & c & 0 & 0 \\ b & 0 & 0 & h \end{bmatrix} \begin{bmatrix} 0 & d & 0 & g \\ 0 & 0 & e & h \\ a & c & 0 & 0 \\ b & 0 & f & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & e & g \\ 0 & d & f & 0 \\ a & c & 0 & 0 \\ b & 0 & 0 & h \end{bmatrix} \begin{bmatrix} 0 & 0 & e & g \\ 0 & d & 0 & h \\ a & c & 0 & 0 \\ b & 0 & f & 0 \end{bmatrix}$$

$$\begin{bmatrix} b & 0 & e & 0 \\ 0 & d & 0 & g \\ 0 & 0 & f & h \\ a & c & 0 & 0 \end{bmatrix} \begin{bmatrix} b & 0 & 0 & g \\ 0 & d & e & 0 \\ 0 & 0 & f & h \\ a & c & 0 & 0 \end{bmatrix} \begin{bmatrix} b & 0 & e & 0 \\ 0 & 0 & f & g \\ 0 & d & 0 & h \\ a & c & 0 & 0 \end{bmatrix} \begin{bmatrix} b & 0 & 0 & g \\ 0 & 0 & e & h \\ 0 & d & f & 0 \\ a & c & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & d & e & 0 \\ b & 0 & 0 & g \\ 0 & 0 & f & h \\ a & c & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & d & 0 & g \\ b & 0 & e & 0 \\ 0 & 0 & f & h \\ a & c & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & e & g \\ b & 0 & f & 0 \\ 0 & d & 0 & h \\ a & c & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & e & g \\ b & 0 & 0 & h \\ 0 & d & f & 0 \\ a & c & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & d & e & 0 \\ 0 & 0 & f & g \\ b & 0 & 0 & h \\ a & c & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & d & 0 & g \\ 0 & 0 & e & h \\ b & 0 & f & 0 \\ a & c & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & e & g \\ 0 & d & f & 0 \\ b & 0 & 0 & h \\ a & c & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & e & g \\ 0 & d & 0 & h \\ b & 0 & f & 0 \\ a & c & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} a & c & 0 & 0 \\ b & d & 0 & 0 \\ 0 & 0 & e & g \\ 0 & 0 & f & h \end{bmatrix}^{*}_{(5)} \begin{bmatrix} a & c & 0 & 0 \\ 0 & 0 & e & g \\ b & d & 0 & 0 \\ 0 & 0 & f & h \end{bmatrix} \begin{bmatrix} a & c & 0 & 0 \\ 0 & 0 & e & g \\ 0 & 0 & f & h \\ b & d & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & e & g \\ a & c & 0 & 0 \\ b & d & 0 & 0 \\ 0 & 0 & f & h \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & e & g \\ a & c & 0 & 0 \\ 0 & 0 & f & h \\ b & d & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & e & g \\ 0 & 0 & f & h \\ a & c & 0 & 0 \\ b & d & 0 & 0 \end{bmatrix}^{*}_{(6)} .$$

The first 36 matrices are formed in such a way that in the first two columns the chosen elements are always in different rows; in the next 48 matrices the first two columns have once two elements in the same row (ac) and two elements in different rows; in the last 6 matrices the first and second columns have two times, two elements in the same row.

Definition 2.2. *The tensor field F of type (1,1) defined on E' is almost complex structure (a.c.s.) iff $F^2 = -I$, almost product structure (a.p.s.) iff $F^2 = I$, or tangent structure (t.s.) iff $F^2 = 0$.*

Theorem 2.2. *The $F(2)$ type structure, which in the former list do not have the sign "*" can not be a.c.s., or o.p.s., or t.s.*

Proof. Some $F_i (i = 1, \dots, 90)$ from the above list of $F(2)$ type structures can be a.c.s., or a.p.s., or t.s. if F_i^2 has the property, that all elements, which are not on the main diagonal are equal to zero. All F_i's, which do not have the sign "*" (there are 84) are such, that F_i^2 has at least on one place, which is not on the main diagonal, product of two elements. This product is zero if at least one of the factor is equal to zero, but in this case F_i is not $F(2)$ type structure.

Theorem 2.3. *There are only six $F(2)$ type structures defined on E', which for some special values of parameters can be a.c.s., or a.p.s., or t.s. They are denoted by "*" in the above list of $F(2)$ type structures.*

Proof. For special values of parameters we have

$$F_1 = \begin{bmatrix} a & 0 & b & 0 \\ 0 & c & 0 & d \\ e & 0 & -a & 0 \\ 0 & g & 0 & -c \end{bmatrix} \quad F_2 = \begin{bmatrix} a & 0 & 0 & b \\ 0 & c & d & 0 \\ 0 & e & -c & 0 \\ f & 0 & 0 & -a \end{bmatrix}$$

$$F_3 = \begin{bmatrix} 0 & a & b & 0 \\ -ce & 0 & 0 & cd \\ cd & 0 & 0 & -ca \\ 0 & d & e & 0 \end{bmatrix} \quad F_4 = \begin{bmatrix} 0 & a & 0 & -b \\ ce & 0 & bc & 0 \\ 0 & d & 0 & e \\ -cd & 0 & ac & 0 \end{bmatrix}$$

$$F_5 = \begin{bmatrix} a & b & 0 & 0 \\ c & -a & 0 & 0 \\ 0 & 0 & e & f \\ 0 & 0 & d & -e \end{bmatrix} \quad F_6 = \begin{bmatrix} 0 & 0 & ae & -ab \\ 0 & 0 & -ac & d \\ a^{-1}b & b & 0 & 0 \\ c & e & 0 & 0 \end{bmatrix}.$$

By direct calculation we obtain

$$\begin{aligned} F_1^2 &= diag[a^2+be, c^2+dg, a^2+be, c^2+dg] \\ F_2^2 &= diag[a^2+bf, c^2+de, c^2+de, a^2+bf] \\ F_3^2 &= c(bd-ae)I \\ F_4^2 &= c(bd+ae)I \\ F_5^2 &= diag[a^2+bc, a^2+bc, e^2+df, e^2+df] \\ F_6^2 &= (de-abc)I. \end{aligned}$$

From Theorem 2.3 follows

Theorem 2.4. *The $F(2)$ type structures $F_1 - F_6$ are a.c.s. if*

$$\begin{aligned} &in\ F_1 \quad a^2+be = c^2+dg = -1, \\ &in\ F_2 \quad a^2+bf = c^2+de = -1, \\ &in\ F_3 \quad c(bd-ae) = -1, \\ &in\ F_4 \quad c(bd+ae) = -1, \\ &in\ F_5 \quad a^2+bc = e^2+df = -1, \\ &in\ F_6 \quad de-abc = -1. \end{aligned}$$

If in the above equations -1 is everywhere replaced by 1, the structures $F_1 - F_6$ become a.p.s.; if -1 is everywhere replaced by 0, the structures $F_1 - F_6$ become t.s.

3. The tensor character of $F(2)$ type structures

Theorem 3.1. *All 90 $F(2)$ type structures from the list in paragraph 2 determine tensor fields of type $(1,1)$ in the basis B', with respect to the coordinate transformation (1).*

Proof. As the proof is the same for all structures, we shall give it for F_1. The structure F_1 in the basis B' determines the following transformation:

$$\begin{aligned}
F_1(\tfrac{\delta}{\delta x^a}) &= a\tfrac{\delta}{\delta x^a} + e\tfrac{\partial}{\partial \dot{x}^A}\\
F_1(i\tfrac{\delta}{\delta y^i}) &= c(i\tfrac{\delta}{\delta y^i}) + g(i\tfrac{\partial}{\partial \dot{y}^I})\\
F_1(\tfrac{\partial}{\partial \dot{x}^A}) &= b\tfrac{\delta}{\delta x^a} - a\tfrac{\partial}{\partial \dot{x}^A}\\
F_1(i\tfrac{\partial}{\partial \dot{y}^I}) &= d(i\tfrac{\delta}{\delta y^i}) - c(i\tfrac{\partial}{\partial \dot{y}^I}).
\end{aligned}$$

The precise form of F_1 is the matrix

$$F_1 = \begin{bmatrix} a\delta^b_a & 0 & b\delta^B_a & 0 \\ 0 & c\delta^j_i & 0 & d\delta^J_i \\ e\delta^b_A & 0 & -a\delta^B_A & 0 \\ 0 & g\delta^j_I & 0 & -c\delta^J_I \end{bmatrix}.$$

The tensor F_1, which is determined by the matrix F_1 can be written in the following way:

$$F_1 = RF_1 \otimes K.$$

In the basis R' and K' F_1 has the form (see (4)):

$$F_1 = R'D^{-1}F_1D \otimes K' = R'F'_1 \otimes K',$$

where

$$F'_1 = D^{-1}F_1D = \begin{bmatrix} a\delta^b_a A^a_{a'} A^{b'}_b & 0 & b\delta^B_a A^a_{a'} A^{B'}_B & 0 \\ 0 & c\delta^j_i B^i_{i'} B^{j'}_j & 0 & d\delta^J_i B^i_{i'} B^{J'}_J \\ e\delta^b_A A^{b'}_b A^A_{A'} & 0 & -a\delta^B_A A^A_{A'} A^{B'}_B & 0 \\ 0 & g\delta^j_I B^I_{I'} B^{j'}_j & 0 & -c\delta^J_I B^I_{I'} B^{J'}_J \end{bmatrix}.$$

For F'_1 we have

$$F'^2_1 = (D^{-1}F_1D)(D^{-1}F_1D) = D^{-1}F^2_1D.$$

From the above relation follows:

$$\begin{aligned}
&\text{if } F^2_1 = -I \Rightarrow F'^2_1 = -I,\\
&\text{if } F^2_1 = I \Rightarrow F'^2_1 = I,\\
&\text{if } F^2_1 = 0 \Rightarrow F'^2_1 = 0.
\end{aligned}$$

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