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RANDOM INTEGRAL CONTRACTOR OF STOCHASTIC INTEGRODIFFERENTIAL EQUATION OF ITO TYPE

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Abstract. *The object of this paper is to study a random integral contractor for a class of stochastic integrodifferential equation of Ito type. We consider existence and uniqueness theorem of solution of this equation by using a principle of random integral contractor.*

1. Preliminaries

In this paper, following the basic ideas of Altman ([1]), Kuo ([3]) and immediately of Murge and Pachpatte ([4]), we study the existence and uniqueness of solution of one type of stochastic integrodifferential equation. This equation is considered in the paper [2], in which the proof of existence and uniqueness of solution is, accordingly, based on the Picard-Lindelöf method of sucessive approximations.

We consider the following stochastic integrodifferential equation (shorter SIDE)

(1)

$$x(t) = c + \int_0^t \left[f_1(s, x(s)) + \int_0^s f_2(s, r, x(r))dr + \int_0^s f_3(s, r, x(r))dW(r) \right] ds$$
$$+ \int_0^t \left[\sigma_1(s, x(s)) + \int_0^s \sigma_2(s, r, x(r))dr + \int_0^s \sigma_3(s, r, x(r))dW(r) \right] dW(s),$$

where $W = \{(W_t, \mathcal{F}_t), t \geq 0\}$ is a given standard Wiener process and c is a constant almost surely, independent of W. The random functions $f_1(t,x)$ and $\sigma_1(t,x)$ are defined and Borel measurable on $[0,T] \times R \times \Omega$ and nonanticipating in t for each x, $f_i(t,s,x)$ and $\sigma_i(t,s,x)$, $i = 2,3$ are defined and Borel merasurable on $J \times R \times \Omega$, $J = \{(t,s) : (t,s) \in [0,T] \times [0,T], s \leq t\}$, where $f_2(t,s,x)$ and $\sigma_i(t,s,x)$ are nonanticipating in t for each (s,x), while $f_3(t,s,x)$ and $\sigma_3(t,s,x)$ are to be nonanticipating in s for each (t,x).

Let C be the collection of one-dimensional real-valued stochastic processes defined on an interval $[0,T]$, nonanticipating with respect to the family of σ-fields $(\mathcal{F}, t \geq 0)$, which trajectories are continuons almost surely.

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For arbitrary $x, y \in C$ let as consider the linear operator $Ax : C \to C$ given by

$$
\begin{aligned}
((Ax)y)(t) &= y(t) \\
&+ \int_0^t \left[\Gamma_1(s, x(s))y(s) + \int_0^s \Gamma_2(s, r, x(r))y(r)dr \right] ds \\
&+ \int_0^t \int_0^s \Gamma_3(s, r, x(r))y(r)dW(r)ds \\
&+ \int_0^t \left[\Phi_1(s, x(s))y(s) + \int_0^s \Phi_2(s, r, x(r))y(r)dr \right] dW(s) \\
&+ \int_0^t \int_0^s \Phi_3(s, r, x(r))y(r)dW(r)dW(s),
\end{aligned} \tag{2}
$$

where $0 \le r \le s \le t \le T$ and $\Gamma_1 : [0, T] \times R \to R$, $\Phi_1 : [0, T] \times R \to R$, $\Gamma_i : J \times R \to R, \Phi_i : J \times R \to R$, $i = 2, 3$ are bounded Borel functions.

Definition 1.1. *Suppose let there exists a positive constant K, such that for all $x, y \in C$ and $t \in [0, T]$, the following inequalities hold almost surely:*

(3)

$$
\begin{aligned}
&|f_1(t, x(t) + ((Ax)y)(t)) - f_1(t, x(t)) - \Gamma_1(t, x(t))y(t)| \le K|y(t)| \\
&|f_i(t, s, x(s) + ((Ax)y)(s)) - f_i(t, s, x(s)) - \Gamma_i(t, s, x(s))y(s)| \le K|y(s)|, \ i = 2, 3 \\
&|\sigma_1(t, x(t) + ((Ax)y)(t)) - \sigma_1(t, x(t)) - \Phi_1(t, x(t))y(t)| \le K|y(t)| \\
&|\sigma_i(t, s, x(s) + ((Ax)y)(s)) - \sigma_i(t, s, x(s)) - \Phi_i(t, s, x(s))y(s)| \le K|y(s)|, \ i = 2, 3
\end{aligned}
$$

Then we say that the functions f_i and σ_i, $i = 1, 2, 3$ have a bounded random integral contractor, denoted by

$$
\begin{aligned}
\{I &+ \int_0^t \Gamma_1 ds + \int_0^t \int_0^s \Gamma_2 dr ds + \int_0^t \int_0^s \Gamma_3 dW(r) ds \\
&+ \int_0^t \Phi_1 dW(s) + \int_0^t \int_0^s \Phi_2 dr dW(s) + \int_0^t \cdot \int_0^s \Phi_3 dW(r) dW(s)\}
\end{aligned} \tag{4}
$$

Definition 1.2. *We say that a bounded random integral contractor is regular if the SIDE (2) has a solution y in C for anu x and z in C.*

Definition 1.3. *For any x_n and x in C such that $x_n \to x$ and $V(\cdot, x_n(\cdot)) \to y$ in $L^2([0, T] \times \Omega)$, a function $V : [0, T] \times R \to R$ is said to be stochastically closed if $y(t) = V(t, x(t))$ holds for every $t \in [0, T]$.*

Note that the analogue definition applies for a function $U : J \times R \to R$.

Remark 1.1. It is easy to prove that if the functions f_i, and σ_i, $i = 1, 2, 3$, satisfy the uniform Lipschitz condition in J, then they are stochastically closed and have a regular bounded random integral contractor (4) with $\Gamma_i = 0, \Phi_i = 0,\ i = 1, 2, 3$.

2. Main result

Theorem 2.1. *Suppose that the functions f_i, and σ_i , $i = 1, 2, 3$, have a bounded random integral contractor and they are stochastically closed. Suppose further that*

$$\int_0^T (f_1(t,c))^2 dt < \infty, \quad \int_0^T (\sigma_1(t,c))^2 dt < \infty,$$

$$\int_0^T \int_0^t (f_i(t,s,c))^2 ds dt < \infty, \quad \int_0^T \int_0^t (\sigma_i(t,s,c))^2 ds dt < \infty, \ i = 2, 3,$$

almost surely hold in J. Then there exists a solution x in C of the SIDE (1). Moreover, if the bounded random integral contractor is regular, then the solution x in C is unique.

Proof of existence: In our discussion we shall use the sequences $\{x_n\}$ and $\{y_n\}$ in C, such that

$$x_0(t) \equiv c,$$

and for $n \geq 0$

$$\begin{aligned} x_{n+1}(t) &= x_n(t) - ((Ax_n)y_n)(t) = x_n(t) - y_n(t) \\ &- \int_0^t \left[\Gamma_1(s, x_n(s)) y_n(s) + \int_0^s \Gamma_2(s, r, x_n(r)) y_n(r) dr \right] ds \\ &- \int_0^t \int_0^s \Gamma_3(s, r, x_n(r)) y_n(r) dW(r) ds \\ &- \int_0^t \left[\Phi_1(s, x_n(s)) y_n(s) + \int_0^s \Phi_2(s, r, x_n(r)) y_n(r) dr \right] dW(s) \\ &- \int_0^t \int_0^s \Phi_3(s, r, x_n(r)) y_n(r) dW(r) dW(s), \end{aligned} \tag{5}$$

$$y_n(t) = x_n(t) - c - \int_0^t \left[f_1(s, x_n(s)) + \int_0^s f_2(s, r, x_n(r)) dr \right] ds \tag{6}$$

$$-\int_0^t\int_0^s f_3(s,r,x_n(r))dW(r)ds$$

$$-\int_0^t\left[\sigma_1(s,x_n(s))+\int_0^s\sigma_2(s,r,x_n(r))dr\right]dW(s)$$

$$-\int_0^t\int_0^s\sigma_3(s,r,x_n(r))dW(r)dW(s).$$

Using (5) and (6), we find

(7)

$$y_{n+1}(t)=\int_0^t\left[f_1(s,x_n(s))-\Gamma_1(s,x_n(s))y_n(s)-f_1(s,x_{n+1}(s))\right]ds$$

$$+\int_0^t\int_0^s\left[f_2(s,r,x_n(r))-\Gamma_2(s,r,x_n(r))y_n(r)-f_2(s,r,x_{n+1}(r))\right]drds$$

$$+\int_0^t\int_0^s\left[f_3(s,r,x_n(r))-\Gamma_3(s,r,x_n(r))y_n(r)-f_3(s,r,x_{n+1}(r))\right]dW(r)ds$$

$$+\int_0^t\left[\sigma_1(s,x_n(s))-\Phi_1(s,x_n(s))y_n(s)-\sigma_1(s,x_{n+1}(s))\right]dW(s)$$

$$+\int_0^t\int_0^s\left[\sigma_2(s,r,x_n(r))-\Phi_2(s,r,x_n(r))y_n(r)-\sigma_2(s,r,x_{n+1}(r))\right]drdW(s)$$

$$+\int_0^t\int_0^s\left[\sigma_3(s,r,x_n(r))-\Phi_3(s,r,x_n(r))y_n(r)-\sigma_3(s,r,x_{n+1}(r))\right]dW(r)dW(s).$$

By using (7), $(a+b+c+d+e+f)^2\le 6(a^2+b^2+c^2+d^2+e^2+f^2)$, Schwarz inequality and some well-known properties of the Lesbegue and Ito integrals, we get:

$$Ey_{n+1}(t)^2\le 6K^2A\int_0^t E|y_n(s)|^2ds,$$

where $A=(T+1)\left(\frac{T^2}{2}+T+1\right)$.

By succesive iteration, we observe that,

$$Ey_n(t)^2\le\frac{(6K^2A)^n}{(n-1)!}\int_0^t(t-s)^{n-1}E|y_0(s)|^2ds.$$

From (6), we have $Ey_0(t)^2 \le a$, for all $t \in [0,T]$, where

$$a = 6\left\{T\int_0^T E(f_1(t,c))^2 dt + T^2 \int_0^T\int_0^t E(f_2(t,s,c))^2 dsdt\right\}$$
$$+ 6\left\{T\int_0^T\int_0^t E(f_3(t,s,c))^2 dsdt + T\int_0^T E(\sigma_1(t,c))^2 dt\right\}$$
$$+ 6\left\{T\int_0^T\int_0^t E(\sigma_2(t,s,c))^2 dsdt + \int_0^T\int_0^t E(\sigma_2(t,s,c))^2 dsdt\right\}.$$

Therefore,

$$Ey_n(t)^2 \le \frac{(6K^2A)^n}{n!} at^n, \quad t \in [0,T], \tag{8}$$

and it will be used to estimate $P\left\{\sup_{0\le t\le T} |y_n(t)| > 6^{-n}\right\}$ by induction.

Denote by

$$\begin{aligned}
\xi_1(t) &= f_1(t, x_n(t)) - \Gamma_1(t, x_n(t))y_n(t) - f_1(t, x_{n+1}(t))\\
\xi_i(t,s) &= f_i(t,s,x_n(s)) - \Gamma_i(t,s,x_n(s))y_n(s) - f_i(t,s,x_{n+1}(s)) \ , i = 2,3\\
\eta_1(t) &= \sigma_1(t, x_n(t)) - \Phi_1(t, x_n(t))y_n(t) - \sigma_1(t, x_{n+1}(t))\\
\eta_i(t,s) &= \sigma_i(t,s,x_n(s)) - \Phi_i(t,s,x_n(s))y_n(s) - \sigma_i(t,s,x_{n+1}(s)) \ , i = 2,3.
\end{aligned}$$

We can rewrite (7) as

$$y_{n+1}(t) = \int_0^t \left[\xi_1(s) + \int_0^s \xi_2(s,r)dr + \int_0^s \xi_3(s,r)dW(r)\right] ds$$
$$+ \int_0^t \left[\eta_1(s) + \int_0^s \eta_2(s,r)dr + \int_0^s \eta_3(s,r)dW(r)\right] dW(s).$$

Obviously,

$$\begin{aligned}
\sup_{0\le t\le T} |y_{n+1}(t)| \le& \int_0^T |\xi_1(s)|ds + \int_0^T \left|\int_0^s \xi_2(s,r)dr\right| ds + \int_0^T \left|\int_0^s \xi_3(s,r)dW(r)\right| ds\\
&+ \sup_{0\le t\le T}\left|\int_0^t \eta_1(s)dW(s)\right| + \sup_{0\le t\le T}\left|\int_0^t\int_0^s \eta_2(s,r)drdW(s)\right|\\
&+ \sup_{0\le t\le T}\left|\int_0^t\int_0^s \eta_3(s,r)dW(r)dW(s)\right|
\end{aligned} \tag{9}$$

But, using Chebysev's inequality and well-known Doob's martingale inequality, we observe that,

(10)

$$P\left\{\int_0^T |\xi_1(s)|ds > 6^{-n-2}\right\} \le 6^{2n+4}K^2\frac{(6K^2A)^n}{(n+1)!}aT^{n+2}$$

$$P\left\{\int_0^T \left|\int_0^s \xi_2(s,r)dr\right| ds > 6^{-n-2}\right\} \le 6^{2n+4}K^2\frac{(6K^2A)^n}{(n+1)!(n+3)}aT^{n+4}$$

$$P\left\{\int_0^T \left|\int_0^s \xi_3(s,r)dW(r)\right| ds > 6^{-n-2}\right\} \le 6^{2n+4}K^2\frac{(6K^2A)^n}{(n+2)!}aT^{n+3}$$

$$P\left\{\sup_{0\le t\le T}\left|\int_0^t |\eta_1(s)dW(s)\right| > 6^{-n-2}\right\} \le 4\cdot 6^{2n+4}K^2\frac{(6K^2A)^n}{(n+1)!}aT^{n+1}$$

$$P\left\{\sup_{0\le t\le T}\left|\int_0^t\int_0^s \eta_2(s,r)drdW(s)\right| > 6^{-n-2}\right\} \le 4\cdot 6^{2n+4}K^2\frac{(6K^2A)^n}{(n+1)!(n+3)}aT^{n+3}$$

$$P\left\{\sup_{0\le t\le T}\left|\int_0^t\int_0^s \eta_3(s,r)dW(r)dW(s)\right| > 6^{-n-2}\right\} \le 4\cdot 6^{2n+4}K^2\frac{(6K^2A)^n}{(n+2)!}aT^{n+2}.$$

Thus, from (9) and (10), we obtain

(11)

$$P\{\sup_{0\le t\le T}|y_{n+1}(t)| > 6^{-n-1}\}$$
$$\le 6^{2n+2}a36(6A)^n(K^2T)^{n+1}\left(\frac{T+4}{(n+1)!}+\frac{T(T+4)}{(n+2)!}+\frac{T^2(T+4)}{(n+1)!(n+3)}\right).$$

This will be used in the next estimation.

From (5), we have

(12)

$$\sup_{0\le t\le T}|x_{n+1}(t)-x_n(t)| \le \sup_{0\le t\le T}|y_n(t)| + \int_0^T |\Gamma_1(s,x_n(s))y_n(s)|\,ds$$
$$+\int_0^T\left|\int_0^s \Gamma_2(s,r,x_n(r))y_n(r)dr\right|ds + \int_0^T\left|\int_0^s \Gamma_3(s,r,x_n(r))y_n(r)dW(r)\right|ds$$
$$+\sup_{0\le t\le T}\left|\int_0^t \Phi_1(s,x_n(s))y_n(s)dW(s)\right|$$

$$+\sup_{0\le t\le T}\left|\int_0^t\int_0^s \Phi_2(s,r,x_n(r))y_n(r)drdW(s)\right|$$

$$+\sup_{0\le t\le T}\left|\int_0^t\int_0^s \Phi_3(s,r,x_n(r))y_n(r)dW(r)dW(s)\right|.$$

Since the functions Γ_i and Φ_i, $i=1,2,3$, are bounded, denote by

$$\begin{aligned}
&\alpha_1=\sup_{0\le t\le T}\{|\Gamma_1(t,x(t))|,x\in R\} && \beta_1=\sup_{0\le t\le T}\{|\Phi_1(t,x(t))|,x\in R\}\\
(13)\quad &\alpha_1=\sup_{(t,s)\in J}\{|\Gamma_2(t,s,x(s))|,x\in R\} && \beta_1=\sup_{(t,s)\in J}\{|\Phi_2(t,s,x(s))|,x\in R\}\\
&\alpha_3=\sup_{(t,s)\in J}\{|\Gamma_3(t,s,x(s))|,x\in R\} && \beta_3=\sup_{(t,s)\in J}\{|\Phi_3(t,s,x(s))|,x\in R\}.
\end{aligned}$$

By using Chebyshev's inequality, Doob's martingale inequality and (13), we get

$$P\left\{\int_0^T|\Gamma_1(s,x_n(s))y_n(s)|ds>6^{-n}\right\}\le\alpha_1^2\frac{(6K^2A)^n}{(n+1)!}aT^{n+2}6^{2n}$$

$$P\left\{\int_0^T\left|\int_0^s\Gamma_2(s,r,x_n(r))y_n(r)dr\right|ds>6^{-n}\right\}\le\alpha_2^2\frac{(6K^2A)^n}{(n+1)!(n+3)}aT^{n+4}6^{2n}$$

$$P\left\{\int_0^T\left|\int_0^s\Gamma_3(s,r,x_n(r))y_n(r)dW(r)\right|ds>6^{-n}\right\}\le\alpha_3^2\frac{(6K^2A)^n}{(n+2)!}aT^{n+3}6^{2n}$$

$$(14)\quad P\left\{\sup_{0\le t\le T}\left|\int_0^t|\Phi_1(s,x_n(s))y_n(s)dW(s)\right|>6^{-n}\right\}\le4\beta_1^2\frac{(6K^2A)^n}{(n+1)!}aT^{n+1}6^{2n}$$

$$P\left\{\sup_{0\le t\le T}\left|\int_0^t\int_0^s\Phi_2(s,r,x_n(r))y_n(r)drdW(s)\right|>6^{-n}\right\}$$

$$\le4\beta_2^2\frac{(6K^2A)^n}{(n+1)!(n+3)}aT^{n+3}6^{2n}$$

$$P\left\{\sup_{0\le t\le T}\left|\int_0^t\int_0^s\Phi_3(s,r,x_n(r))y_n(r)dW(r)dW(s)\right|>6^{-n}\right\}$$

$$\le4\beta_3^2\frac{(6K^2A)^n}{(n+2)!}aT^{n+2}6^{2n}.$$

Now from (12), (14) and (11), we have

$$P\left\{\sup_{0\le t\le T}|x_{n+1}(t)-x_n(t)|>\frac{7}{6^n}\right\}$$

$$\leq 6 \cdot 6^{2n} a(6K^2T)^n A^{n-1} \left(\frac{T+4}{n!} + \frac{T(T+4)}{(n+1)!} + \frac{T^2(T+4)}{n!(n+2)} \right)$$
$$+ 6^{2n} a(6K^2AT)^n T \left(\frac{T\alpha_1^2 + 4\beta_1^2}{(n+1)!} + \frac{T^2(T\alpha_2^2 + 4\beta_2^2)}{(n+1)!(n+3)} + \frac{T(T\alpha_3^2 + 4\beta_3^2)}{(n+2)!} \right).$$

Thus by Borel-Cantelli's lemma, it follows

$$P \left\{ \lim_{n\to\infty} \left(\sup_{0\leq t\leq T} |x_{n+1}(t) - x_n(t)| > \frac{7}{6^n} \right) \right\} = 0,$$

i.e., for largen n, $\sup\limits_{0\leq t\leq T} |x_{n+1}(t) - x_n(t)| \leq \dfrac{7}{6^n}$ almost surely. It means that the series $\sum\limits_{n=1}^{\infty} \sup |x_{n+1}(t) - x_n(t)|$ is convergent almost surely.

Therefore, the sequence $\{x_n(t)\}$ converges almost surely, uniformly in $[0,T]$ to the stochastic process $x^*(t)$. In accordance with the definition of the sequences $\{x_n(t)\}$ and $\{y_n(t)\}$, it follows that they are continuons and nonanticipating with respect to the given Wiener process. So, for each t, $t \in [0,T]$, $x^*(t)$ is continuons almost surely and nonanticipating. Hence, $x^*(t)$ is in C. In order to, prove that $x^*(t)$ is indeed a solution of the SIDE, we shall show that $x_n(t) \to x^*(t)$ as $n \to \infty$ in L^2 since.

Indeed, from (5) we have

$$E \int_0^T (x_{n+1}(t) - x_n(t))^2 \, dt \leq 7a(6K^2A)^n T^{n+1} \times$$
$$\left[\frac{1}{(n+1)!} + \frac{\alpha_1^2 T^2}{(n+1)!(n+3)} + \frac{\alpha_2^2 T^4}{(n+1)!(n+3)(n+5)} \right]$$
$$+ 7a(6K^2A)^n T^{n+1} \times$$
$$\left[\frac{\alpha_3^2 T^3}{(n+2)!(n+4)} + \frac{\beta_1^2 T}{(n+2)!} + \frac{\beta_2^2 T^3}{(n+1)!(n+3)(n+4)} + \frac{\beta_3^2 T^2}{(n+3)!} \right] \to 0, \ n \to \infty.$$

From the fact that the functions f_i, and σ_i, $i = 1,2,3$, are stochastically closed, it is not difficult to prove that

$$\int_0^t f_1(s, x_n(s))ds \to \int_0^t f_1(s, x^*(s))ds,$$
$$\int_0^t \sigma_1(s, x_n(s))dW(s) \to \int_0^t \sigma_1(s, x^*(s))dW(s),$$

in $L^2([0,T] \times \Omega)$ sence, and

$$\int_0^t \int_0^s f_2(s, r, x_n(r))drds \to \int_0^t \int_0^s f_2(s, r, x^*(r))drds,$$

$$\int_0^t \int_0^s \sigma_2(s,r,x_n(r))drdW(s) \to \int_0^t \int_0^s \sigma_2(s,r,x^*(r))drdW(s),$$

$$\int_0^t \int_0^s f_3(s,r,x_n(r))dW(r)ds \to \int_0^t \int_0^s f_3(s,r,x^*(r))dW(r)ds,$$

$$\int_0^t \int_0^s \sigma_3(s,r,x_n(r))dW(r)dW(s) \to \int_0^t \int_0^s \sigma_3(s,r,x^*(r))dW(r)dW(s),$$

in $L^2(J \times \Omega)$ sence.

By taking the L^2-limits in (5), we have that for each $t \in [0,T]$,
(15)

$$x^*(t) = c + \int_0^t \left[f_1(s,x^*(s)) + \int_0^s f_2(s,r,x^*(r))dr + \int_0^s f_3(s,r,x^*(r))dW(r) \right] ds$$
$$+ \int_0^t \left[\sigma_1(s,x^*(s)) + \int_0^s \sigma_2(s,r,x^*(r))dr + \int_0^s \sigma_3(s,r,x^*(r))dW(r) \right] dW(s)$$

holds almost surely. Both sides of the equality (15) have continuous sample paths, hence (15) holds for all $t \in J$ almost surely. It means x^* is a solution of the equation (15).

Proof of uniqueness: In order to prove the uniqueness of the solution, we shall use an analoguos method with the paper [3], and precisely with [4].

Let $x_1(t)$ and $x_2(t)$ be two continuous solutions of the SIDE (1) and let bounded random integral contractor be regular. If we put $x(t) = x_1(t)$ and $((Ax)y)(t) = x_2(t) - x_1(t)$ in (2), then there exists $y(t)$ in C such that

(16)

$$y(t) + \int_0^t \left[\Gamma_1(s,x_1(s))y(s) + \int_0^s \Gamma_2(s,r,x_1(r))y(r)dr \right] ds$$
$$+ \int_0^t \int_0^s \Gamma_3(s,r,x_1(r))y(r)dW(r)ds$$
$$+ \int_0^t \left[\Phi_1(s,x_1(s))y(s) + \int_0^s \Phi_2(s,r,x_1(r))y(r)dr \right] dW(s)$$
$$+ \int_0^t \int_0^s \Phi_3(s,r,x_1(r))y(r)dW(r)dW(s)$$
$$= x_2(t) - x_1(t).$$

Since x_1 and x_2 are the solutions of the SIDE (1), then

$$
\begin{aligned}
y(t) = &\int_0^t [f_1(s,x_2(s)) - \Gamma_1(s,x_1(s))y(s) - f_1(s,x_1(s))]\,ds \\
&+ \int_0^t\int_0^s [f_2(s,r,x_2(r)) - \Gamma_2(s,r,x_1(r))y(r) - f_2(s,r,x_1(r))]\,drds \\
&+ \int_0^t\int_0^s [f_3(s,r,x_2(r)) - \Gamma_3(s,r,x_1(r))y(r) - f_3(s,r,x_1(r))]\,dW(r)ds \\
&+ \int_0^t [\sigma_1(s,x_2(s)) - \Phi_1(s,x_1(s))y(s) - \sigma_1(s,x_1(s))]\,dW(s) \\
&+ \int_0^t\int_0^s [\sigma_2(s,r,x_2(r)) - \Phi_2(s,r,x_1(r))y(r) - \sigma_2(s,r,x_1(r))]\,drdW(s) \\
&+ \int_0^t\int_0^s [\sigma_3(s,r,x_2(r)) - \Phi_3(s,r,x_1(r))y(r) - \sigma_3(s,r,x_1(r))]\,dW(r)dW(s).
\end{aligned}
\tag{17}
$$

Since $Ey(t)^2$ is not necesserily finite, for $n > 0$, and $t \in [0,T]$, denote by

$$
I_N(t) = \begin{cases} 1, & \text{if } |y(s)| \le N \text{ for } 0 \le s \le t \\ 0, & \text{otherwise} \end{cases}.
$$

Then $I_N(t)$ is nonanticipating stochastic process and

$$
I_N(t) = I_N(t)I_N(s)I_N(r) \text{ for } 0 \le r \le s \le t \le T.
$$

Therefore, from (17), we have

$$
\begin{aligned}
I_N(t)y(t) = I_N(t) &\int_0^t I_N(s)\,[f_1(s,x_2(s)) - \Gamma_1(s,x_1(s))y(s) - f_1(s,x_1(s))]\,ds \\
&+ I_N(t)\int_0^t I_N(s)\int_0^s I_N(r)[f_2(s,r,x_2(r)) - \Gamma_2(s,r,x_1(r))y(r) \\
&- f_2(s,r,x_1(r))]drds \\
&+ I_N(t)\int_0^t I_N(s)\int_0^s I_N(r)[f_3(s,r,x_2(r)) - \Gamma_3(s,r,x_1(r))y(r) \\
&- f_3(s,r,x_1(r))]dW(r)ds
\end{aligned}
\tag{18}
$$

$$+ I_N(t) \int_0^t I_N(s) \left[\sigma_1(s, x_2(s)) - \Phi_1(s, x_1(s))y(s) - \sigma_1(s, x_1(s))\right] dW(s)$$

$$+ I_N(t) \int_0^t I_N(s) \int_0^s I_N(r)[\sigma_2(s, r, x_2(r)) - \Phi_2(s, r, x_1(r))y(r)$$
$$- \sigma_2(s, r, x_1(r))]drdW(s)$$

$$+ I_N(t) \int_0^t I_N(s) \int_0^s I_N(r)[\sigma_3(s, r, x_2(r)) - \Phi_3(s, r, x_1(r))y(r)$$
$$- \sigma_3(s, r, x_1(r))]dW(r)dW(s).$$

By using the inequalities (3) after replacing $x(t)$ by $x_1(t)$, also (17),

$$(a + b + c + d + e + f)^2 \leq 6(a^2 + b^2 + c^2 + d^2 + e^2 + f^2),$$

Schwarcz inequality and (18), we get

$$EI_N(t)y(t)^2 \leq 6K^2(T + 1) \int_0^t EI_N(s)y(s)^2 ds$$
$$+ 6K^2(T + 1)^2 \int_0^t I_N(s) \int_0^s EI_N(r)y(r)^2 drds.$$

By applying the lemma Pachpatte ([5]) we obtain $EI_N(t)y(t)^2 = 0,\ t \in [0, T]$¿

Since $y(t)$ has continuous sample paths, we conclude that $\lim_{N \to \infty} I_N(t) = 1$ almost surely. Hence by Lebesque's monotone convergence theorem,

$$Ey(t)^2 = \lim_{N \to \infty} EI_N(t)y(t)^2 = 0.$$

Therefore, for each $t \in [0, T]$, $y(t) \equiv 0$ almost surely. By (16), $x_1(t) \equiv x_2(t)$ almost surely.

It could be very interesting to study the speed of convergence of the sequence (5) with respect to the sequence of sucessive approximations based on the Picard-Lindelöf method, ie. coefficients of the equation (1) satisfy the uniform Lipschitz condition. Also, it could be very interesting to study how the speed of convergence of the sequence $\{x_n\}$, $n \in N$, to the solution x^* depends on a choice of integral contractor. However, it will be a subject of a forthcoming paper. Note that, we could extend the results of this paper for SIDE-s involving stochastic integrals with respect to any continuons martingales and martingale measures.

REFERENCES

[1] ALTMAN M., *Contractors and Contractor Directions, Theory and Applications*, Marcel Dekker, New York, 1978.

[2] BERGER M., MIZEL V., *Volterra equations with Ito integrals-I*, Journal of Integral Equations, 2(1980), 187-245.

[3] KUO H.H., *On integral contractors*, Journal of Integral Equations, 1(1979), 35-46.

[4] MURGE M.G., PACHPATTE B.G., *Existence and uniqueness of solution of nonlinear Ito type stochastic integral equations*, Acta Mathematica Scientia, 7(1987), 2,207-216.

[5] PACHPATTE B.G., *On random nonlinear contractions and Ito-Doob type stochastic integral equations*, J. of M.A. C T, 12(1979),77-89.

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