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COLOR-SYMMETRY GROUPS OF BANDS

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Abstract. *Using color-symmetry characteristics of symmetry groups, all color-symmetry groups of bands* ${G_{321}}^P$, *are derived.*

1. Introduction

The concept of$P$-symmetry (permutation symmetry) introduced by A.M.Zamorzaev [1, 2] is defined as follows. If P is a subgroup of the symmetric permutation group of p indices, and G is a discrete symmetry group, every transformation $C = cS = Sc$, $c \in P$, $S \in G$ is a P-symmetry transformation. Every group G^P derived from G by subtituting its symmetries by the corresponding P-symmetries is a P-symmetry group. If the substitutions incl uded in G^P exhaust the group P, G^P is a complete P-symmetry group. Every complete P-symmetry group G^P can be derived from its generating group G by searching in G and P for normal subgroups H and Q for which the isomorphism $G/H \cong P/Q$ holds, by paired multiplication of the cosets corresponding in this isomorphism, and by the unification of the products obtained. The groups of complete P-symmetry fall, respectively, into the senior ($G = H$and $G^P = G \times P$), middle ($Q = P$, $Q = I$ and $I \subset Q \subset P$) and junior groups ($G/H \cong P$ and $G^P \cong G$). In this paper only the junior P-symmetry groups are considered.

There are few different criterions for the equality of junior P-symmetry groups. The most rafined ("strong") criterion is the following: let a color-permutation group P be decomposed in the product of different (irreducible) groups $P = P_1\alpha_1 ... P_i\alpha$, where $H_1, ..., H_\alpha$ ($\alpha = \alpha_1 + ... + \alpha_i$) are the subgroups of G such that $G/H_1 \cong P_1$, $G/H_2 \cong P_1$,...,$G/H\alpha_1 \cong P_1$,...,$G/H\alpha \cong P_n$, and H is their section ($G/H \cong P$). In this case every P-symmetry group can be uniquely defined as $G/(H_1, ..., H_\alpha)/H$ [2]. If subgroups which result in isomorphic quotient groups are taken as equivalent, or if only reduced symbols G/H are considered, two (sub)criterions (the "middle" and "weak"), are obtained. Knowing P-symmetry groups classified according to the "middle" criterion, they can be simply classified according to the "strong" criterion by permuting the equivalent subgroups.

If H is a normal subgroup of G, and P is the regular permutation group of the order $N(G/H \cong P)$, then $[G : H] = N$. In the case of an irregular permutation

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group P, instead of a group/subgroup symbol G/H [1, 2, 3, 4], a symbol $G/H_1/H$ will be used, where H_1 is the subgroup retaining invariant one index, and H is the symmetry subgroup of G^P [5]. Then, H_1 is not a normal s ubgroup of G, $[G:H_1]=N_1$, $[H_1:H]=N_2$, and $N=N_1N_2$.

Bohm symbols [6] are used to denote the corresponding categories of isometric symmetry groups. In a symbol $Gn...$ the first subscript n is the maximal dimension of space in which the transformations of the symmetry group act, while the following subscripts are the maximal dimensions of subspaces invariant with respect to the action of the symmetry group and properly included in each other. The corresponding categories of P-symmetry groups are denoted by additional P-superscripts.

The indices ascribed to the points of a figure with a P-symmetry group have an extrageometric sense with respect to the space in which the figure is considered. In additional dimensions such index permutations can be geometrically interpreted, making possible an investigation of multi-dimensional symmetry groups by means of P-symmetry groups [7]. A simple example illustrating this is the derivation of the symmetry groups of bands G_{321} from the symmetry groups of friezes G_{21}. The 31 symmetry group of bands G_{321} can be derived from the symmetry groups of friezes G_{21} by using antisymmetry (31 $G_{321} \simeq 7$ generating $G+7$ senior $(G\times \underline{1})+17$ junior G^1, $G\in G_{21}$). The same method can be used in order to derive the P-symmetry groups of bands from the P-symmetry groups of friezes [1, 2, 8].

If $P=C_2{}^l$, according to the "strong" criterion we have the simple $(l=1)$ and multiple $(l\geq 2)$ antisymmetry groups, and according to the "middle" criterion, so-called Mackay groups (or compound groups) [9, 10]. In the first case, all quotient groups C_2 are treated as non-equivalent, and in the second case as the equivalent ones. Hence, in the first case besides of their choice, there are considered permutations of the corresponding subgroups of the index 2.

Let a symmetry group G be given by the presentation:

$$\{S_1,S_2,...,S_r\} \qquad g_k(S_1,S_2,...,S_r)=I, \qquad k=1,2,...,s,$$

and let e_1, e_2, ..., e_l be the antiidentity transformations of the first, second,...,lth kind satisfying the relations:

$$e_i^2=I, \qquad e_ie_j=e_je_i, \qquad e_iS_q=S_qe_i,$$
$$i,j\in\{1,2,...,l\}, \quad q\in\{1,2,...,r\}.$$

Every transformation $S'=e'S$, $S\in G$, where e' is an antiidentity transformation or their product is called a (multiple) antisymmetry transformation. Every group G' derived from G, which contains at least one (multiple) antisymmetry transformation is called a (multiple) antisymmetry group, and the group G is called its generating group. A junior multiple antisymmetry group is called the M^m-type group if its corresponding antiidentity transformations are independent, this means, if they are not representable by the others. In the theory of simple and multiple antisymmetry [10] only the derivation of junior simple and multiple antisymmetry groups of the M^m-type is non-trivial. They can be derived very efficiently by using the antisymmetric characteristic method [11, 12].

In particular, for $l = 1$ we have the (simple) antisymmetry. Among (simple) antisymmetry groups we can distinguish the senior antisymmetry groups of the form $G \times \{e_1\}$, with the structure $G \times C_2$, where $\{e_1\}$ denotes the group generated by e_1, with the structure C_2, and the junior antisymmetry groups isomorphic to their generating group G. Since the antiidentity transformation e1 can be identified with the (hyper)reflection in the invariant $(n+1)$-plane, to every antisymmetry group of the category $G^1_{n\ldots}$ corresponds the symmetry group of the category $G_{(n+1)n\ldots}$. Namely, to every senior antisymmetry group $G \times \{e_1\}$ corresponds the symmetry group $G \times \{T_1\}$, where $\{T_1\}$ denotes the symmetry group with the structure D_1 ($D_1 \cong C_2$), generated by a plane reflection T_1, and to every junior antisymmetry group G^1 given by the presentation:

$$\{S_1{}^1, S_2{}^1, ..., S_r{}^1\} \quad g_k(S_1{}^1, S_2{}^1, ..., S_r{}^1) = I, \quad k = 1, 2, ..., s,$$

where the set $\{S_1{}^1, S_2{}^1, ..., S_r{}^1\}$ consists of the (anti)generators $S_q{}^1$ ($S_q{}^1 = S_q$ or $S_q{}^1 = e_1 S_q, \quad q = 1, 2, ..., r$), corresponds the $(n+1)$-dimensional symmetry group generated by the symmetries $S_q{}^1 = S_q$ or $S_q{}^1 = T_1 S_q$.

In the case of Belov (p)-symmetry [2] (or Cp-symmetry [4]), the group $P \cong Cp$ is generated by the permutation $c_1 = (12...p)$ satisfying the relations:

$$c_1^p = I, \quad c_1 S = S c_1, \quad S \in G.$$

In the case of Pawley (p')-symmetry [2] (or $D_{p(2p)}$-symmetry [4]), the group $P \cong D_{p(2p)}$ is the regular dihedral permutation group generated by the permutations c_1 and $e_1 = (11')$ satisfying the relations:

$$c_1^p = e_1{}^2 = (c_1 e_1)^2 = I \quad c_1 S = S c_1 \quad e_1 S = S e_1, \quad S \in G.$$

In the case of $(p2)$-symmetry [2] (or Dp-symmetry [4]), the group $P \cong D_p$ is the irregular dihedral permutation group generated by the permutations c_1 and $e_1 = (12)$ satisfying the same relations.

All remaining P-symmetries which the symmetry groups of friezes admit, are the combinations of $Cp-$, $Dp(2p)-$ and $Dp-$ symmetry with the simple and multiple antisymmetry.

With regard to their structure, 7 symmetry groups of friezes can be divided in 4 classes:

1) p111, p1a1 of the structure C_∞;
2) pm11, p112, pma2 of the structure D_∞;
3) p1m1 of the structure $C_\infty \times D_1$; and
4) pmm2 of the structure $D_\infty \times D_1$.

Having in mind the reducibility of the groups C_{4k-2} ($C_{4k-2} = C_{2k-1} \times C_2$) and D_{4k-2} ($D_{4k-2} = D_{2k-1} \times C_2$), from this we can conclude that P can be, respectively:

1) C_k;
2) D_k, $D_{k(2k)}$;
3) C_k, $C_k \times C_2$;

4) D_k, $D_{k(2k)}$, $D_k \times C_2$, $D_{k(2k)} \times C_2$, $D_{2k-1(4k-2)} \times {C_2}^2$.

Using the method of A.M.Zamorzaev and Kishinev school, all P-symmetry groups of friezes, are derived. From them, by applying antisymmetry, all P-symmetry groups of bands (classified according to the "strong" equality criterion), are derived.

2. Color-symmetry groups of bands

According to the relation of isomorphism, 31 symmetry group of bands can be distributed in 6 equivalency classes. Every such class consists of groups possessing the same structure:

1) {p111}, {p1a1, p11a, p2111} of the structure C_∞;

2) {pm11, p112, p121, pi}, {pma2, p12/a1, pm2a, p112/a, p2$_1$/m11, 2$_1$22} of the structure D_∞;

3) {p1m1, p11m, p211}, {p21ma, p21am, p2aa} of the structure $C_\infty \times D_1$;

4){pmm2, p12/m1, pm2m, p112/m, p2/m11, p222}, {pmma, pmam, 22a} of the structure $D_\infty \times D_1$;

5) {p2mm} of the structure $C_\infty \times {D_1}^2$;

6) {pmmm} of the structure $D_\infty \times {D_1}^2$.

From their structure, knowing that $C_{4k-2} = C_{2k-1} \times C_2$ and $D_{4k-2} = D_{2k-1} \times C_2$, we can directly conclude that the color-permutation group P, as a finite quotient group of the considered symmetry group can be, respectively:

1) C_k;

2) D_k, $D_{k(2k)}$;

3) C_k, $C_k \times C_2$;

4) D_k, $D_{k(2k)}$, $D_k \times C_2$, $D_{k(2k)} \times C_2$, $D_{2k-1(4k-2)} \times {C_2}^2$;

5) C_k, $C_k \times C_2$, $C_k \times {C_2}^2$;

6) D_k, $D_{k(2k)}$, $D_k \times C_2$, $D_{k(2k)} \times C_2$, $D_k \times {C_2}^2$, $D_{k(2k)} \times {C_2}^2$, $D_{2k-1(4k-2)} \times {C_2}^3$.

Hence, the color-symmetry groups of bands admit the following P-symmetries: $P \cong C_k, C_k \times C_2, C_k \times {C_2}^2, D_k, D_{k(2k)}, D_k \times C_2, D_{k(2k)} \times C_2, D_k \times {C_2}^2, D_{k(2k)} \times {C_2}^2, D_{2k-1(4k-2)} \times {C_2}^3$.

Definition 1. *Let the set of elements of a symmetry group G be divided in equivalency classes consisting of elements equivalent with regard to symmetry. The resulting system is called the color-symmetry characteristic $CC(G)$ of the group G.*

Theorem 1. *Two P-symmetry groups derived from the same generating symmetry group G are equal iff they possess equal color-symmetry characteristics.*

Theorem 2. *From two symmetry groups G and G_1 which possess isomorphic color-symmetry characteristics $CC(G) \cong CC(G_1)$ the same number of color-symmetry (P-symmetry) groups can be derived. All P-symmetry groups derived from G and G_1 are corresponding in this CC-isomorphism.*

With regard to the CC-isomorphism, the symmetry groups of bands belonging to the classes 1)-6) are distributed in 10 subclasses, denoted by . Consequently, the derivation of all P-symmetry groups of bands is reduced to the derivation of P-symmetry groups from 10 symmetry groups of bands, i.e. from the representatives of the subclasses mentioned: p111, p1a1, pm11, pma2, p1m1, p2$_1$ma, pmm2, pmma, p2mm and pmmm. By this means, all P-symmetry groups of bands classified according to the "middle" equality criterion (and resulting "strong" and "weak" criterion), are derived.

3. Catalogue of color-symmetry groups of bands

In Table 1 symmetry groups of bands G_{321} are denoted by the corresponding number 01—31 and classified according to their structure and CC-isomorphism. The symmetry groups of friezes G_{21} (G_{21} G_{321}) are denoted by $*$.

Table 1

C_∞		$C_\infty \times D_1$		$C_\infty \times D_1^2$	
01	$p111*$	15	$p1m1*$	30	$p2mm$
		16	$p11m$		
02	$p1a1*$	17	$p211$	$D_\infty \times D_1^2$	
03	$p11a$			31	$pmmm$
04	$p2_111$	18	$p2_1ma$		
		19	$p2_1am$		
D_∞		20	$p2aa$		
05	$pm11*$				
06	$p112*$	$D_\infty \times D_1$			
07	$p121$	21	$pmm2*$		
08	pi	22	$p12/m1$		
		23	$pm2m$		
09	$pma2*$	24	$p112/m$		
10	$p12/a1$	25	$p2/m11$		
11	$pm2a$	26	$p222$		
12	$p112/a$				
13	$p21/m11$	27	$pmma$		
14	$p2122$	28	$pmam$		
		29	$p22a$		

In Table 2.1- 2.4 Mackay groups (or compound groups [9]) of bands with $P \cong C_2^l$ $(1 \leq l \leq 4)$, derived from the groups-representatives (01, 02, 05, 09, 15, 18, 21, 27, 30, 31), are given. The multiple antisymmetry groups of bands [10] can be simply derived from them by permuting the equivalent subgroups of the index 2. The results according to the "strong" and "weak" criterion are given at the end of each table.

Table 2.1.

C_2			
01/01	15/01	27/21	30/15
		27/18	
02/01	18/15	27/14	31/31
	18/04	27/13	31/30

05/05	18/03	27/12	31/29
05/01		27/11	31/28
	21/21		31/27
09/06	21/15	30/30	31/26
09/05	21/09	30/20	31/25
09/02	21/06	30/19	31/24
	21/05	30/18	31/23
15/15		30/17	31/22
15/02	27/22	30/16	31/21

There are 117 ${G_{321}}^1$.

Table 2.2

${C_2}^2$		
05/(05,05)/01	27/(14,13)/04	31/(27,21)/21
05/(05,01)/01	27/(18,12)/03	31/(30,29)/20
	27/(18,11)/03	31/(29,29)/20
09/(06,05)/01	27/(12,11)/03	31/(30,28)/19
09/(06,02)/01		31/(28,28)/19
09/(05,02)/01	30/(30,20)/17	31/(30,27)/18
	30/(30,17)/17	31/(27,27)/18
15/(15,02)/01	30/(20,17)/17	31/(30,26)/17
15/(15,01)/01	30/(30,19)/16	31/(30,25)/17
15/(02,01)/01	30/(30,16)/16	31/(26,25)/17
	30/(19,16)/16	31/(30,24)/16
18/(15,04)/01	30/(30,18)/15	31/(30,23)/16
18/(15,03)/01	30/(30,15)/15	31/(24,23)/16
18/(04,03)/01	30/(18,15)/15	31/(30,22)/15
	30/(19,18)/04	31/(30,21)/15
21/(21,21)/15	30/(19,17)/04	31/(22,21)/15
21/(21,15)/15	30/(18,17)/04	31/(28,27)/14
21/(21,09)/06	30/(20,18)/03	31/(28,26)/14
21/(21,06)/06	30/(20,16)/03	31/(27,26)/14
21/(09,06)/06	30/(18,16)/03	31/(28,27)/13
21/(21,09)/05	30/(20,19)/02	31/(28,25)/13
21/(21,05)/05	30/(20,15)/02	31/(27,25)/13
21/(09,05)/05	30/(19,15)/02	31/(29,27)/12
21/(15,09)/02	30/(17,16)/01	31/(29,24)/12
21/(09,09)/02	30/(17,15)/01	31/(27,24)/12
21/(15,06)/01	30/(16,15)/01	31/(29,27)/11
21/(15,05)/01		31/(29,23)/11
21/(06,05)/01	31/(31,31)/30	31/(27,23)/11
	31/(31,30)/30	31/(29,28)/10
27/(22,21)/15	31/(31,29)/26	31/(29,22)/10

27/(22,18)/15	31/(31,26)/26	31/(28,22)/10
27/(21,18)/15	31/(29,26)/26	31/(29,28)/09
27/(22,13)/08	31/(31,29)/25	31/(29,21)/09
27/(22,12)/08	31/(31,25)/25	31/(28,21)/09
27/(13,12)/08	31/(29,25)/25	31/(25,24)/08
27/(22,14)/07	31/(31,28)/24	31/(25,22)/08
27/(22,11)/07	31/(31,24)/24	31/(24,22)/08
27/(14,11)/07	31/(28,24)/24	31/(26,23)/07
27/(21,14)/06	31/(31,28)/23	31/(26,22)/07
27/(21,12)/06	31/(31,23)/23	31/(23,22)/07
27/(14,12)/06	31/(28,23)/23	31/(26,24)/06
27/(21,13)/05	31/(31,27)/22	31/(26,21)/06
27/(21,11)/05	31/(31,22)/22	31/(24,21)/06
27/(13,11)/05	31/(27,22)/22	31/(25,23)/05
27/(18,14)/04	31/(31,27)/21	31/(25,21)/05
27/(18,13)/04	31/(31,21)/21	31/(23,21)/05

From 271 compound groups of the M^2-type can be derived $20 + 2 \times 251 = 522$ multiple antisymmetry groups of the M^2-type, and 97 $D_{2(4)}$-symmetry (or (2')-symmetry [2]) groups, according to the "weak" equality criterion (G/H).

For $l = 2, 3, 4$, the multiple antisymmetry groups can be obtained from Mackay groups given in Table 2.2- 2.4 by permuting the corresponding equivalent subgroups of the index 2. In non- exceptional cases, the number of multiple antisymmetry groups derived from a Mackay group will be $l!$ in the case that all subgroups mentioned are different, or $l!/2$ if two of them are identical. In exceptional cases (denoted by +) the section of two identical subgroups with a remaining subgroup results in distinct s ubgroups of the index 4, so these identical subgroups are not equivalent, and the number of multiple antisymmetry groups derived will be l!. If a Mackay group (Table 2.3-24) is not uniquely defined by its extended group/subgroup symbol, an additional symbol (given in parentheses () or ¡¿, indicating to Table 2.2 or 2.3), which points to the section of subgroups of the index 2, is given.

Table 2.3

C_2^3

21/(21,21,09)/01+	27/(22,18,11)/01	30/(30,20,16)/01
21/(21,21,06)/01	27/(22,14,13)/01	30/(30,20,15)/01
21/(21,21,05)/01	27/(22,14,12)/01	30/(30,19,18)/01
21/(21,15,09)(06)/01	27/(22,13,11)/01	30/(30,19,17)/01
21/(21,15,09)(05)/01	27/(22,12,11)/01	30/(30,19,15)/01
21/(21,15,06)/01	27/(22,18,14)/01	30/(30,18,17)/01
21/(21,15,05)/01	27/(21,18,13)/01	30/(30,18,16)/01
21/(21,09,09)/01+	27/(21,18,12)/01	30/(30,17,16)/01
21/(21,09,06)/01	27/(21,18,11)/01	30/(30,17,15)/01
21/(21,09,05)/01	27/(21,14,13)/01	30/(30,16,15)/01

21/(21, 06, 05)/01
21/(15, 09, 06)/01
21/(15, 09, 05)/01
21/(09, 09, 06)/01
21/(09, 09, 05)/01
21/(09, 06, 05)/01

27/(22, 21, 14)/01
27/(22, 21, 13)/01
27/(22, 21, 12)/01
27/(22, 21, 11)/01
27/(22, 18, 14)/01
27/(22, 18, 13)/01
27/(22, 18, 12)/01
30/(18, 17, 15)/01
30/(18, 16, 15)/01

31/(31, 31, 29)/17+
31/(31, 31, 28)/16+
31/(31, 31, 27)/15+
31/(31, 31, 26)/17
31/(31, 31, 25)/17
31/(31, 31, 24)/16
31/(31, 31, 23)/16
31/(31, 31, 22)/15
31/(31, 31, 21)/15
31/(31, 30, 29)(26)/17
31/(31, 30, 29)(25)/17
31/(31, 30, 28)(24)/16
31/(31, 30, 28)(23)/16
31/(31, 30, 27)(22)/15
31/(31, 30, 27)(21)/15
31/(31, 30, 26)/17
31/(31, 30, 25)/17
31/(31, 30, 24)/16
31/(31, 30, 23)/16
31/(31, 30, 22)/15
31/(31, 30, 21)/15
31/(31, 29, 29)/17+
31/(31, 29, 28)/08
31/(31, 29, 28)/07
31/(31, 29, 28)/06
31/(31, 29, 28)/05
31/(31, 29, 27)/08
31/(31, 29, 27)/07

27/(21, 14, 11)/01
27/(21, 13, 12)/01
27/(21, 12, 11)/01
27/(18, 14, 12)/01
27/(18, 14, 11)/01
27/(18, 13, 12)/01
27/(18, 13, 11)/01
27/(14, 13, 12)/01
27/(14, 13, 11)/01
27/(14, 12, 11)/01
27/(13, 12, 11)/01

30/(30, 20, 19)/01
30/(30, 20, 18)/01
31/(31, 27, 27)/15+
31/(31, 27, 26)/07
31/(31, 27, 26)/06
31/(31, 27, 25)/08
31/(31, 27, 25)/05
31/(31, 27, 24)/08
31/(31, 27, 24)/06
31/(31, 27, 23)/07
31/(31, 27, 23)/05
31/(31, 27, 22)/15
31/(31, 27, 21)/15
31/(31, 26, 25)/17
31/(31, 26, 24)/06
31/(31, 26, 23)/07
31/(31, 26, 22)/07
31/(31, 26, 21)/06
31/(31, 25, 24)/08
31/(31, 25, 23)/05
31/(31, 25, 22)/08
31/(31, 25, 21)/05
31/(31, 24, 23)/16
31/(31, 24, 22)/08
31/(31, 24, 21)/06
31/(31, 23, 22)/07
31/(31, 23, 21)/05
31/(31, 22, 21)/15
31/(30, 29, 28)(12)/03
31/(30, 29, 28)(11)/03
31/(30, 29, 28)(10)/02
31/(30, 29, 28)(09)/02
31/(30, 29, 26)/17

30/(20, 19, 18)/01
30/(20, 19, 17)/01
30/(20, 19, 16)/01
30/(20, 18, 17)/01
30/(20, 18, 15)/01
30/(20, 17, 16)/01
30/(20, 17, 15)/01
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30/(18, 17, 16)/01
31/(30, 24, 22)/01
31/(30, 24, 21)/01
31/(30, 23, 22)/01
31/(30, 23, 21)/01
31/(29, 29, 28)/02+
31/(29, 29, 27)/03+
31/(29, 29, 26)/04
31/(29, 29, 25)/04
31/(29, 29, 24)/03
31/(29, 29, 23)/03
31/(29, 29, 22)/02
31/(29, 29, 21)/02
31/(29, 28, 28)/02+
31/(29, 28, 27)/08
31/(29, 28, 27)/07
31/(29, 28, 27)/06
31/(29, 28, 27)/05
31/(29, 28, 26)/07
31/(29, 28, 26)/06
31/(29, 28, 25)/08
31/(29, 28, 25)/05
31/(29, 28, 24)/08
31/(29, 28, 24)/06
31/(29, 28, 23)/07
31/(29, 28, 23)/05
31/(29, 28, 22)/02
31/(29, 28, 21)/02
31/(29, 27, 27)/03+
31/(29, 27, 26)/07
31/(29, 27, 26)/06
31/(29, 27, 25)/08

31/(31, 29, 27)/06
31/(31, 29, 27)/05
31/(31, 29, 26)/17
31/(31, 29, 25)/17
31/(31, 29, 24)/08
31/(31, 29, 24)/06
31/(31, 29, 23)/07
31/(31, 29, 23)/05
31/(31, 29, 22)/08
31/(31, 29, 22)/07
31/(31, 29, 21)/06
31/(31, 29, 21)/05
31/(31, 28, 28)/16+
31/(31, 28, 27)/08
31/(31, 28, 27)/07
31/(31, 28, 27)/06
31/(31, 28, 27)/05
31/(31, 28, 26)/07
31/(31, 28, 26)/06
31/(31, 28, 25)/08
31/(31, 28, 25)/05
31/(31, 28, 24)/16
31/(31, 28, 23)/16
31/(31, 28, 22)/08
31/(31, 28, 22)/07
31/(31, 28, 21)/06
31/(31, 28, 21)/05
31/(28, 28, 22)/02
31/(28, 28, 21)/02
31/(28, 27, 27)/04+
31/(28, 27, 26)/04
31/(28, 27, 25)/04
31/(28, 27, 24)/08
31/(28, 27, 24)/06
31/(28, 27, 23)/07
31/(28, 27, 23)/05
31/(28, 27, 22)/08
31/(28, 27, 22)/07
31/(28, 27, 21)/06
31/(28, 27, 21)/05
31/(28, 26, 25)/04
31/(28, 26, 24)/06
31/(28, 26, 23)/07
31/(28, 26, 22)/07
31/(28, 26, 21)/06
31/(30, 29, 25)/17
31/(30, 29, 24)/03
31/(30, 29, 23)/03
31/(30, 29, 22)/02
31/(30, 29, 21)/02
31/(30, 28, 27)(13)/04
31/(30, 28, 27)(12)/04
31/(30, 28, 26)/04
31/(30, 28, 25)/04
31/(30, 28, 24)/16
31/(30, 28, 23)/16
31/(30, 28, 22)/02
31/(30, 28, 21)/02
31/(30, 27, 26)/04
31/(30, 27, 25)/04
31/(30, 27, 24)/03
31/(30, 27, 23)/03
31/(30, 27, 22)/15
31/(30, 27, 21)/15
31/(30, 26, 24)/01
31/(30, 26, 23)/01
31/(30, 26, 22)/01
31/(30, 26, 21)/01
31/(30, 25, 24)/01
31/(30, 25, 23)/01
31/(30, 25, 22)/01
31/(30, 25, 21)/01
31/(28, 24, 23)/16
31/(28, 24, 22)/08
31/(28, 24, 21)/06
31/(28, 23, 22)/07
31/(28, 23, 21)/05
31/(28, 22, 21)/02
31/(27, 27, 26)/04
31/(27, 27, 25)/04
31/(27, 27, 24)/03
31/(27, 27, 23)/03
31/(27, 27, 22)/02
31/(27, 27, 21)/02
31/(27, 26, 25)/04
31/(27, 26, 24)/06
31/(27, 26, 23)/07
31/(27, 26, 22)/07
31/(27, 26, 21)/06
31/(27, 25, 24)/08
31/(29, 27, 25)/05
31/(29, 27, 24)/03
31/(29, 27, 23)/03
31/(29, 27, 22)/08
31/(29, 27, 22)/07
31/(29, 27, 21)/06
31/(29, 27, 21)/05
31/(29, 26, 25)/17
31/(29, 26, 24)/06
31/(29, 26, 23)/07
31/(29, 26, 22)/07
31/(29, 26, 21)/06
31/(29, 25, 24)/08
31/(29, 25, 23)/05
31/(29, 25, 22)/08
31/(29, 25, 21)/05
31/(29, 24, 23)/03
31/(29, 24, 22)/08
31/(29, 24, 21)/06
31/(29, 23, 22)/07
31/(29, 23, 21)/05
31/(29, 22, 21)/02
31/(28, 28, 27)/04+
31/(28, 28, 26)/04
31/(28, 28, 25)/04
31/(28, 28, 24)/16
31/(28, 28, 23)/16
31/(27, 24, 22)/08
31/(27, 24, 21)/06
31/(27, 23, 22)/07
31/(27, 23, 21)/05
31/(27, 22, 21)/15
31/(26, 25, 24)/01
31/(26, 25, 23)/01
31/(26, 25, 22)/01
31/(26, 25, 21)/01
31/(26, 24, 23)/01
31/(26, 24, 22)/01
31/(26, 23, 21)/01
31/(26, 22, 21)/01
31/(25, 24, 23)/01
31/(25, 24, 21)/01
31/(25, 23, 22)/01
31/(25, 22, 21)/01
31/(24, 23, 22)/01

$31/(28,25,24)/08$
$31/(28,25,23)/05$
$31/(28,25,22)/08$
$31/(28,25,21)/05$
$31/(27,25,23)/05$
$31/(27,25,22)/08$
$31/(27,25,21)/05$
$31/(27,24,23)/03$
$31/(24,23,21)/01$
$31/(24,22,21)/01$
$31/(23,22,21)/01$

From 444 compound groups of the M^3-type (or 221-symmetry groups [2] can be derived $6 \times 396 + 3 \times 48 = 2520$ multiple antisymmetry groups of the M^3-type, and the only one group G/H.

Table 2.4

$\underline{C_2{}^4}$

$31/(31,31,29,28)(10)/01^+$
$31/(31,31,29,28)(09)/01^+$
$31/(31,31,29,27)(12)/01^+$
$31/(31,31,29,27)(11)/01^+$
$31/(31,31,29,24)/01^+$
$31/(31,31,29,23)/01^+$
$31/(31,31,29,22)/01^+$
$31/(31,31,29,21)/01^+$
$31/(31,31,28,27)(14)/01^+$
$31/(31,31,28,27)(13)/01^+$
$31/(31,31,28,26)/01^+$
$31/(31,31,28,25)/01^+$
$31/(31,31,28,22)/01^+$
$31/(31,31,28,21)/01^+$
$31/(31,31,27,26)/01^+$
$31/(31,31,27,25)/01^+$
$31/(31,31,27,24)/01^+$
$31/(31,31,27,23)/01^+$
$31/(31,31,26,24)/01$
$31/(31,31,26,23)/01$
$31/(31,31,26,22)/01$
$31/(31,31,26,21)/01$
$31/(31,31,25,24)/01$
$31/(31,31,25,23)/01$
$31/(31,31,25,22)/01$
$31/(31,30,28,22)(24)/01$
$31/(31,30,28,21)(24)/01$
$31/(31,30,28,27) < 06 > /01$
$31/(31,30,28,27) < 05 > /01$
$31/(31,30,28,26)(23)/01$
$31/(31,30,28,25)(23)/01$
$31/(31,30,28,22)(23)/01$
$31/(31,30,28,21)(23)/01$
$31/(31,31,25,21)/01$
$31/(31,31,24,22)/01$
$31/(31,31,24,21)/01$
$31/(31,31,23,22)/01$
$31/(31,31,23,21)/01$
$31/(31,30,29,28) < 08 > /01$
$31/(31,30,29,28) < 07 > /01$
$31/(31,30,29,27) < 08 > /01$
$31/(31,30,29,27) < 07 > /01$
$31/(31,30,29,24)(26)/01$
$31/(31,30,29,23)(26)/01$
$31/(31,30,29,22)(26)/01$
$31/(31,30,29,21)(26)/01$
$31/(31,30,29,27) < 06 > /01$
$31/(31,30,29,27) < 05 > /01$
$31/(31,30,29,27) < 06 > /01$
$31/(31,30,29,27) < 05 > /01$
$31/(31,30,29,24)(25)/01$
$31/(31,30,29,23)(25)/01$
$31/(31,30,29,22)(25)/01$
$31/(31,30,29,21)(25)/01$
$31/(31,30,28,27) < 08 > /01$
$31/(31,30,28,27) < 07 > /01$
$31/(31,30,28,26)(24)/01$
$31/(31,30,28,25)(24)/01$
$31/(31,29,27,21) < 08 > /01$
$31/(31,29,27,25) < 07 > /01$
$31/(31,29,27,24) < 07 > /01$
$31/(31,29,27,21) < 07 > /01$
$31/(31,29,27,25) < 06 > /01$
$31/(31,29,27,23) < 06 > /01$
$31/(31,29,27,22) < 06 > /01$
$31/(31,29,27,26) < 05 > /01$

31/(31, 30, 27, 26)(22)/01
31/(31, 30, 27, 25)(22)/01
31/(31, 30, 27, 24)(22)/01
31/(31, 30, 27, 23)(22)/01
31/(31, 30, 27, 26)(21)/01
31/(31, 30, 27, 25)(21)/01
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31/(31, 30, 23, 22)/01
31/(31, 30, 23, 21)/01
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31/(31, 29, 29, 28)(23)/01+
31/(31, 29, 29, 27)(22)/01+
31/(31, 29, 29, 27)(21)/01+
31/(31, 29, 29, 24)/01+
31/(31, 29, 29, 23)/01+
31/(31, 29, 29, 22)/01+
31/(31, 29, 29, 21)/01+
31/(31, 29, 28, 28)(26)/01+
31/(31, 29, 28, 28)(25)/01+
31/(31, 29, 28, 27) < 08 > /01
31/(31, 29, 28, 27) < 07 > /01
31/(31, 29, 28, 27) < 06 > /01
31/(31, 29, 28, 27) < 05 > /01
31/(31, 29, 28, 26) < 08 > /01
31/(31, 29, 28, 23) < 08 > /01
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31/(31, 29, 28, 25) < 07 > /01
31/(31, 29, 28, 24) < 07 > /01
31/(31, 29, 28, 21) < 07 > /01
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31/(31, 29, 28, 23) < 06 > /01
31/(31, 29, 28, 22) < 06 > /01
31/(31, 29, 28, 26) < 05 > /01
31/(31, 29, 28, 24) < 05 > /01
31/(31, 29, 27, 24) < 05 > /01
31/(31, 29, 27, 22) < 05 > /01
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31/(31, 29, 26, 22)/01
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31/(31, 29, 23, 22)/01
31/(31, 29, 23, 21)/01
31/(31, 29, 22, 21)(26)/01
31/(31, 29, 22, 21)(25)/01
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31/(31, 28, 28, 27)(21)/01+
31/(31, 28, 28, 26)/01+
31/(31, 28, 28, 25)/01+
31/(31, 28, 28, 22)/01+
31/(31, 28, 28, 21)/01+
31/(31, 28, 27, 27)(24)/01+
31/(31, 28, 27, 27)(23)/01+
31/(31, 28, 27, 26) < 08 > /01
31/(31, 28, 27, 23) < 08 > /01
31/(31, 28, 27, 21) < 08 > /01
31/(31, 28, 27, 25) < 07 > /01
31/(31, 28, 27, 24) < 07 > /01
31/(31, 28, 27, 21) < 07 > /01
31/(31, 28, 27, 25) < 06 > /01
31/(31, 28, 27, 23) < 06 > /01
31/(31, 28, 27, 22) < 06 > /01
31/(31, 28, 27, 26) < 05 > /01
31/(31, 28, 27, 24) < 05 > /01
31/(31, 28, 27, 22) < 05 > /01
31/(31, 28, 26, 25)(24)/01
31/(31, 28, 26, 25)(23)/01
31/(31, 28, 26, 24)/01
31/(31, 28, 26, 23)/01
31/(31, 28, 26, 22)/01
31/(31, 28, 26, 21)/01
31/(31, 28, 25, 24)/01

$31/(31,29,28,22) < 05 > /01$
$31/(31,29,27,27)(26)/016^{+}$
$31/(31,29,27,27)(25)/016^{+}$
$31/(31,29,27,26) < 08 > /01$
$31/(31,29,27,23) < 08 > /01$
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31/(31,28,23,21)/01
31/(31,28,22,21)(24)/01
31/(31,28,22,21)(23)/01
$31/(31,27,27,26)/01^{+}$
$31/(31,27,27,25)/01^{+}$
$31/(31,27,27,24)/01^{+}$
$31/(31,27,27,23)/01^{+}$
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31/(31,27,26,25)(21)/01
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31/(30,27,25,23)/01

$31/(30,29,28,27)<08>/01$
$31/(30,29,28,27)<07>/01$
$31/(30,29,28,27)<06>/01$
$31/(30,29,28,27)<05>/01$
$31/(30,29,28,26)(10)/01$
$31/(30,29,28,26)(09)/01$
$31/(30,29,28,25)(10)/01$
$31/(30,29,28,25)(09)/01$
$31/(30,29,28,24)(10)/01$
$31/(30,29,28,24)(09)/01$
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$31/(29,29,24,21)/01$
$31/(29,29,23,22)/01$
$31/(29,29,23,21)/01$
$31/(29,28,28,27)(12)/01^+$
$31/(29,28,28,27)(11)/01^+$
$31/(29,28,28,26)/01^+$
$31/(29,28,28,25)/01^+$
$31/(29,28,27,24)/01^+$
$31/(29,28,27,23)/01^+$
$31/(29,28,27,27)(10)/01^+$
$31/(29,28,27,27)(09)/01^+$
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$31/(29,28,27,21)<08>/01$
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$31/(29,28,27,24)<07>/01$
$31/(29,28,27,21)<07>/01$
$31/(29,28,27,25)<06>/01$
$31/(29,28,27,23)<06>/01$
$31/(29,28,27,22)<06>/01$
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$31/(29,29,28,27)(14)/01^+$
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$31/(29,24,22,21)/01$
$31/(29,23,22,21)/01$
$31/(28,28,27,24)/01^+$
$31/(28,28,27,23)/01^+$
$31/(28,28,27,22)/01^+$

31/(29,28,27,26) < 05 > /01
31/(29,28,27,24) < 05 > /01
31/(29,28,27,22) < 05 > /01
31/(29,28,26,25)(10)/01
31/(29,28,26,25)(09)/01
31/(29,28,26,24)/01
31/(29,28,26,23)/01
31/(29,28,26,22)/01
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31/(28,28,27,21)/01^{+}
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31/(27, 24, 22, 21)/01
31/(27, 23, 22, 21)/01

From 444 compound groups of the M^4-type can be derived $(24 \times 396 + 12 \times 48) = 10080$ multiple antisymmetry groups of the M^4-type, and the only one group according to the "weak" equality criterion (G/H).

The remaining groups $G_{321}{}^P$ are classified in the same way, according to P-symmetries (G/HP), using the "middle" equality criterion. In Table 3.1 — 3.10 every P-symmetry group is given by its (extended) group/subgroup symbol and by the number n. Each table is followed by data about the possible reducibility of the group P, the number of the P-symmetry groups for the given n, the number of crystallographic P-symmetry groups of bands [1, 2], by results corresponding to the "strong" equality criterion (which can be simply derived by permuting the equivalent subgroups of the index 2), and to the "weak" equality criterion (G/H). Owing to the reducibility of the groups C_{4k-2} and D_{4k-2}, $P \cong C_{4k-2} \times C_2^{l-1} = C_{2k-1} \times C_2^l$ and $P \cong D_{4k-2} \times C_2^{l-1} = D_{2k-1} times C_2^l$ $(l \leq 1)$. Therefore, the cases $P \cong C_{2k-1} \times C_2^l$, $D_{2k-1} \times C_2^l$ are not considered.

Table 3.1

$\underline{C_n}$

01/01	k	15/01	$4k-2$	30/30	k
				30/20	$2k$
02/02	$2k-1$	18/18	$2k-1$	30/19	$2k$
02/01	$2k$	18/15	$2k$	30/18	$2k$
		18/04	$4k-2$	30/17	$4k-2$
15/15	k	18/03	$4k-2$	30/16	$4k-2$
15/02	$2k$	18/02	$4k$	30/15	$4k-2$

$C_{4k-2} = C_{2k-1} \times C_2$

k	5	2	29
$2k-1$	6	3	11
$2k$	12	4	20
$4k$	3	6	29
$4k-2$	12		

Table 3.2

$D_{n(2n)}$					
05/01	k	21/01	$4k-2$	31/30	k
				31/20	$2k$
09/02	$2k-1$	27/18	$2k-1$	31/19	$2k$
09/01	$2k$	27/15	$2k$	31/18	$2k$
		27/04	$4k-2$	31/17	$4k-2$
21/15	k	27/03	$4k-2$	31/16	$4k-2$
21/02	$2k$	27/02	$4k$	31/15	$4k-2$

k	11	2	44
$2k-1$	9	3	20
$2k$	18	4	32
$4k$	3	6	44
$4k-2$	15		

Table 3.3

D_n $(n \geq 2)$					
05/05/01	k $(k \geq 3)$	27/27/18	$2k-1$	31/31/30	k $(k \geq 3)$
		27/22/15	$2k$	31/29/20	$2k$
09/09/02	$2k-1$	27/21/15	$2k$	31/28/19	$2k$
09/06/01	$2k$	27/14/04	$4k-2$	31/27/18	$2k$
09/05/01	$2k$	27/13/04	$4k-2$	31/26/17	$4k-2$
		27/12/03	$4k-2$	31/25/17	$4k-2$
21/21/15	k $(k \geq 3)$	27/11/03	$4k-2$	31/24/16	$4k-2$
21/09/02	$2k$	27/10/02	$4k$	31/23/16	$4k-2$
21/06/01	$4k-2$	27/09/02	$4k$	31/22/15	$4k-2$
21/05/01	$4k-2$			31/21/15	$4k-2$

$D_{4k-2} = D_{2k-1} \times C_2$

k	11	3	20
$2k-1$	9	4	44
$2k$	27	6	68
$4k$	6		
$4k-2$	30		

The transition from the "middle" (in this case coinciding to the "strong") equality criterion $(G/H_1/H)$ to the "weak" equality criterion (G/H), results in the transition from D_n-symmetry (or $p2$-symmetry [2]) to $D_{n(2n)}$-symmetry (or p'-symmetry [2]).

Table 3.4

$C_n \times C_2$					
15/(15,02)/01	$2k$	30/(20,17)/17	$2k$	30/(15,30)/15	$4k-2$
15/(15,01)/01	$2k$	30/(20,16)/03	$2k$	30/(17,20)/17	$4k-2$
15/(02,01)/01	$2k$	30/(20,15)/02	$2k$	30/(16,20)/03	$4k-2$
15/(02,02)/01	$4k$	30/(19,17)/04	$2k$	30/(15,20)/02	$4k-2$
15/(01,02)/01	$4k-2$	30/(19,16)/16	$2k$	30/(17,19)/04	$4k-2$
15/(01,15)/01	$4k-2$	30/(19,15)/02	$2k$	30/(16,19)/16	$4k-2$
15/(02,15)/01	$4k-2$	30/(18,17)/04	$2k$	30/(15,19)/02	$4k-2$
		30/(18,16)/03	$2k$	30/(17,18)/04	$4k-2$
18/(15,04)/01	$2k$	30/(18,15)/15	$2k$	30/(16,18)/03	$4k-2$
18/(15,03)/01	$2k$	30/(20,20)/17	$4k$	30/(15,18)/15	$4k-2$
18/(02,03)/01	$4k$	30/(20,19)/03	$4k$	30/(20,19)/02	$4k-2$
18/(02,04)/01	$4k$	30/(20,18)/02	$4k$	30/(19,20)/02	$4k-2$
18/(04,03)/01	$4k-2$	30/(19,20)/04	$4k$	30/(20,18)/03	$4k-2$
18/(03,04)/01	$4k-2$	30/(19,19)/16	$4k$	30/(18,20)/03	$4k-2$
18/(03,15)/01	$4k-2$	30/(19,18)/02	$4k$	30/(19,18)/04	$4k-2$
18/(04,15)/01	$4k-2$	30/(18,20)/04	$4k$	30/(18,19)/04	$4k-2$
		30/(18,19)/03	$4k$	30/(17,16)/01	$4k-2$
30/(30,20)/17	$2k$	30/(18,18)/15	$4k$	30/(16,17)/01	$4k-2$
30/(30,19)/16	$2k$	30/(20,30)/17	$4k-2$	30/(17,15)/01	$4k-2$
30/(30,18)/15	$2k$	30/(19,30)/16	$4k-2$	30/(15,17)/01	$4k-2$
30/(30,17)/17	$2k$	30/(18,30)/15	$4k-2$	30/(16,15)/01	$4k-2$
30/(30,16)/16	$2k$	30/(17,30)/17	$4k-2$	30/(15,16)/01	$4k-2$
30/(30,15)/15	$2k$	30/(16,30)/16	$4k-2$		

$C_{4k-2} = C_{2k-1} \times C_2$

$2k$	30	2	78
$4k$	14	4	48
$4k-2$	48	6	78

In transition from the "middle" (in this case coinciding to the "strong") equality criterion (G/(H1,H2)/H) to the "weak" equality criterion (G/H), we have the following results:

$2k$	12	2	13
$4k-2$	1	4	12
		6	13

Table 3.5

$D_{n(2n)} \times C_2$					
05/(01,05)/01	$2k-1$	31/(30,31)/30	$2k-1$	31/(18,24)/03	$2k$
		31/(30,29)/20	$2k-1$	31/(18,23)/03	$2k$

09/(02,06)/01	$2k-1$
09/(02,05)/01	$2k-1$
21/(15,21)/15	$2k-1$
21/(15,09)/02	$2k-1$
21/(15,06)/01	k
21/(15,05)/01	k
21/(15,09)/01	$2k$
21/(02,06)/01	$2k$
21/(02,05)/01	$2k$
21/(02,09)/01	$4k$
21/(02,21)/01	$4k-2$
21/(01,21)/01	$4k-2$
21/(01,09)/01	$4k-2$
27/(18,22)/15	$2k-1$
27/(18,21)/15	$2k-1$
27/(18,14)/04	$2k-1$
27/(18,13)/04	$2k-1$
27/(18,12)/03	$2k-1$
27/(18,11)/03	$2k-1$
27/(15,14)/01	$2k$
27/(15,13)/01	$2k$
27/(15,12)/01	$2k$
27/(15,11)/01	$2k$
27/(04,22)/01	$4k-2$
27/(04,21)/01	$4k-2$
27/(04,12)/01	$4k-2$
27/(04,11)/01	$4k-2$
27/(03,22)/01	$4k-2$
27/(03,21)/01	$4k-2$
27/(03,14)/01	$4k-2$
27/(03,13)/01	$4k-2$
27/(02,14)/01	$4k$
27/(02,13)/01	$4k$
27/(02,12)/01	$4k$
27/(02,11)/01	$4k$

31/(30,28)/19	$2k-1$
31/(30,27)/18	$2k-1$
31/(30,26)/17	k
31/(30,25)/17	k
31/(30,24)/16	k
31/(30,23)/16	k
31/(30,22)/15	k
31/(30,21)/15	k
31/(30,29)/17	$2k$
31/(30,28)/16	$2k$
31/(30,27)/15	$2k$
31/(20,26)/17	$2k$
31/(20,25)/17	$2k$
31/(20,24)/03	$2k$
31/(20,23)/03	$2k$
31/(20,22)/02	$2k$
31/(20,21)/02	$2k$
31/(20,31)/17	$4k-2$
31/(20,28)/02	$4k-2$
31/(20,27)/03	$4k-2$
31/(20,29)/17	$4k$
31/(20,28)/03	$4k$
31/(20,27)/02	$4k$
31/(19,26)/04	$2k$
31/(19,25)/04	$2k$
31/(19,24)/16	$2k$
31/(19,23)/16	$2k$
31/(19,22)/02	$2k$
31/(19,21)/02	$2k$
31/(19,31)/16	$4k-2$
31/(19,29)/02	$4k-2$
31/(19,27)/04	$4k-2$
31/(19,29)/04	$4k$
31/(19,28)/16	$4k$
31/(19,27)/02	$4k$
31/(18,26)/04	$2k$
31/(18,25)/04	$2k$

31/(18,22)/15	$2k$
31/(18,21)/15	$2k$
31/(18,31)/15	$4k-2$
31/(18,29)/03	$4k-2$
31/(18,28)/04	$4k-2$
31/(18,29)/04	$4k$
31/(18,28)/03	$4k$
31/(18,27)/15	$4k$
31/(17,31)/17	$4k-2$
31/(17,29)/17	$4k-2$
31/(17,28)/04	$4k-2$
31/(17,27)/04	$4k-2$
31/(17,24)/01	$4k-2$
31/(17,23)/01	$4k-2$
31/(17,22)/01	$4k-2$
31/(17,21)/01	$4k-2$
31/(16,31)/16	$4k-2$
31/(16,29)/03	$4k-2$
31/(16,28)/16	$4k-2$
31/(16,27)/03	$4k-2$
31/(16,26)/01	$4k-2$
31/(16,25)/01	$4k-2$
31/(16,22)/01	$4k-2$
31/(16,21)/01	$4k-2$
31/(15,31)/15	$4k-2$
31/(15,29)/02	$4k-2$
31/(15,28)/02	$4k-2$
31/(15,27)/15	$4k-2$
31/(16,26)/01	$4k-2$
31/(15,25)/01	$4k-2$
31/(15,24)/01	$4k-2$
31/(15,23)/01	$4k-2$

$2k-1$	50
k	18
$2k$	51
$4k-2$	75
$4k$	27

2	171
3	68
4	96
6	144

In the transition from the "middle" $(G/(H_1,H_2)/H)$ to the "weak" equality criterion (G/H), we have the following results:

$2k-1$	35	2	44
k	44	3	35
$2k$	6	4	50
$4k-2$	1	6	51

Table 3.6

$D_n \times C_2$			
21/(21/15, 09)/06/01	$2k$	31/(31/30, 26)/26/17	$2k$
21/(21/15, 09)/05/01	$2k$	31/(31/30, 25)/25/17	$2k$
21/(21/15, 06)/06/01	$2k$	31/(31/30, 24)/24/16	$2k$
21/(21/15, 05)/05/01	$2k$	31/(31/30, 23)/23/16	$2k$
21/(09/02, 06)/06/01	$2k$	31/(31/30, 22)/22/15	$2k$
21/(09/02, 05)/05/01	$2k$	31/(31/30, 21)/21/15	$2k$
21/(09/02, 09)/06/01	$4k$	31/(29/20, 26)/26/17	$2k$
21/(09/02, 09)/05/01	$4k$	31/(29/20, 25)/25/17	$2k$
21/(09/02, 21)/06/01	$4k-2$	31/(29/20, 24)/12/03	$2k$
21/(09/02, 21)/05/01	$4k-2$	31/(29/20, 23)/11/03	$2k$
21/(06/01, 21)/06/01	$4k-2$	31/(29/20, 22)/10/02	$2k$
21/(06/01, 09)/06/01	$4k-2$	31/(29/20, 21)/09/02	$2k$
21/(05/01, 21)/05/01	$4k-2$	31/(29/20, 29)/26/17	$4k$
21/(05/01, 09)/05/01	$4k-2$	31/(29/20, 29)/25/17	$4k$
		31/(29/20, 28)/12/03	$4k$
27/(22/15, 14)/08/01	$2k$	31/(29/20, 28)/11/03	$4k$
27/(22/15, 13)/07/01	$2k$	31/(29/20, 27)/10/02	$4k$
27/(22/15, 12)/08/01	$2k$	31/(29/20, 27)/09/02	$4k$
27/(22/15, 11)/07/01	$2k$	31/(29/20, 31)/26/17	$4k-2$
27/(21/15, 14)/06/01	$2k$	31/(29/20, 31)/25/17	$4k-2$
27/(21/15, 13)/05/01	$2k$	31/(29/20, 28)/10/02	$4k-2$
27/(21/15, 12)/06/01	$2k$	31/(29/20, 28)/09/02	$4k-2$
27/(21/15, 11)/05/01	$2k$	31/(29/20, 27)/12/03	$4k-2$
27/(10/02, 14)/07/01	$4k$	31/(29/20, 27)/11/03	$4k-2$
27/(10/02, 13)/08/01	$4k$	31/(28/19, 26)/14/04	$2k$
27/(10/02, 12)/08/01	$4k$	31/(28/19, 25)/13/04	$2k$
27/(10/02, 11)/07/01	$4k$	31/(28/19, 24)/24/16	$2k$
27/(09/02, 14)/06/01	$4k$	31/(28/19, 23)/23/16	$2k$
27/(09/02, 13)/05/01	$4k$	31/(28/19, 22)/10/02	$2k$
27/(09/02, 12)/06/01	$4k$	31/(28/19, 21)/09/02	$2k$
27/(09/02, 11)/05/01	$4k$	31/(28/19, 29)/14/04	$4k$
27/(14/04, 22)/07/01	$4k-2$	31/(28/19, 29)/13/04	$4k$
27/(14/04, 21)/06/01	$4k-2$	31/(28/19, 28)/24/16	$4k$
27/(14/04, 12)/06/01	$4k-2$	31/(28/19, 28)/23/16	$4k$
27/(14/04, 11)/07/01	$4k-2$	31/(28/19, 27)/10/02	$4k$
27/(13/04, 22)/08/01	$4k-2$	31/(28/19, 27)/09/02	$4k$
27/(13/04, 21)/05/01	$4k-2$	31/(28/19, 31)/24/16	$4k-2$

27/(13/04, 12)/08/01	$4k-2$
27/(13/04, 11)/05/01	$4k-2$
27/(12/03, 22)/08/01	$4k-2$
27/(12/03, 21)/06/01	$4k-2$
27/(12/03, 14)/08/01	$4k-2$
27/(12/03, 13)/06/01	$4k-2$
27/(11/03, 22)/07/01	$4k-2$
27/(11/03, 21)/05/01	$4k-2$
27/(11/03, 14)/07/01	$4k-2$
27/(11/03, 13)/05/01	$4k-2$

31/(31/30, 29)/26/17	$2k$
31/(31/30, 29)/25/17	$2k$
31/(31/30, 28)/24/16	$2k$
31/(31/30, 28)/23/16	$2k$
31/(31/30, 27)/22/15	$2k$
31/(31/30, 27)/21/15	$2k$
31/(27/18, 31)/22/15	$4k-2$
31/(27/18, 31)/21/15	$4k-2$
31/(27/18, 29)/12/03	$4k-2$
31/(27/18, 29)/11/03	$4k-2$
31/(27/18, 28)/14/04	$4k-2$
31/(27/18, 28)/13/04	$4k-2$
31/(26/17, 31)/26/17	$4k-2$
31/(26/17, 29)/26/17	$4k-2$
31/(26/17, 28)/14/04	$4k-2$
31/(26/17, 27)/14/04	$4k-2$
31/(26/17, 24)/06/01	$4k-2$
31/(26/17, 23)/07/01	$4k-2$
31/(26/17, 22)/07/01	$4k-2$
31/(26/17, 21)/06/01	$4k-2$
31/(25/17, 31)/25/17	$4k-2$
31/(25/17, 29)/25/17	$4k-2$
31/(25/17, 28)/13/04	$4k-2$
31/(25/17, 27)/13/04	$4k-2$
31/(25/17, 24)/08/01	$4k-2$
31/(25/17, 23)/05/01	$4k-2$
31/(25/17, 22)/08/01	$4k-2$
31/(25/17, 21)/05/01	$4k-2$
31/(24/16, 31)/24/16	$4k-2$
31/(24/16, 29)/12/03	$4k-2$
31/(24/16, 28)/24/16	$4k-2$
31/(24/16, 27)/12/03	$4k-2$
31/(24/16, 26)/06/01	$4k-2$
31/(28/19, 31)/23/16	$4k-2$
31/(28/19, 29)/10/02	$4k-2$
31/(28/19, 29)/09/02	$4k-2$
31/(28/19, 27)/14/04	$4k-2$
31/(28/19, 27)/13/04	$4k-2$
31/(27/18, 26)/14/04	$2k$
31/(27/18, 25)/13/04	$2k$
31/(27/18, 24)/12/03	$2k$
31/(27/18, 23)/11/03	$2k$
31/(27/18, 22)/22/15	$2k$
31/(27/18, 21)/21/15	$2k$
31/(27/18, 29)/14/04	$4k$
31/(27/18, 29)/13/04	$4k$
31/(27/18, 28)/12/03	$4k$
31/(27/18, 28)/11/03	$4k$
31/(27/18, 27)/22/15	$4k$
31/(27/18, 27)/21/15	$4k$
31/(24/16, 25)/08/01	$4k-2$
31/(24/16, 22)/08/01	$4k-2$
31/(24/16, 21)/06/01	$4k-2$
31/(23/16, 31)/23/16	$4k-2$
31/(23/16, 29)/11/03	$4k-2$
31/(23/16, 28)/23/16	$4k-2$
31/(23/16, 27)/11/03	$4k-2$
31/(23/16, 26)/07/01	$4k-2$
31/(23/16, 25)/05/01	$4k-2$
31/(23/16, 22)/07/01	$4k-2$
31/(23/16, 21)/05/01	$4k-2$
31/(22/15, 31)/22/15	$4k-2$
31/(22/15, 29)/10/02	$4k-2$
31/(22/15, 28)/10/02	$4k-2$
31/(22/15, 27)/22/15	$4k-2$
31/(22/15, 26)/07/01	$4k-2$
31/(22/15, 25)/08/01	$4k-2$
31/(22/15, 24)/08/01	$4k-2$
31/(22/15, 23)/07/01	$4k-2$
31/(21/15, 31)/21/15	$4k-2$
31/(21/15, 29)/09/02	$4k-2$
31/(21/15, 28)/09/02	$4k-2$
31/(21/15, 27)/21/15	$4k-2$
31/(21/15, 26)/06/01	$4k-2$
31/(21/15, 25)/05/01	$4k-2$
31/(21/15, 24)/06/01	$4k-2$
31/(21/15, 23)/05/01	$4k-2$

$D_{4k-2} \times C_2 = D_2k - 1 \times C_2^2$

$2k$	90	4	144
$4k$	54	6	240
$4k-2$	150		

In transition from the "middle" (in this case coinciding with the "strong") equality criterion $(G/(H_4/H_3, H_2)/H_1/H)$ to the the "weak" equality criterion $(G/H_1/H)$, we have the following results:

$2k$	36	4	36
$4k-2$	4	6	40

If the symbols $G/H_1/H$ are reduced to G/H, then:

$2k$	15	4	15
$4k-2$	1	6	16

Table 3.7

$C_n \times C_2^2$

30/(30, 20, 19)/01	$2k$	30/(17, 30, 18)/01	$4k-2$
30/(30, 20, 18)/01	$2k$	30/(17, 30, 16)/01	$4k-2$
30/(30, 19, 18)/01	$2k$	30/(17, 18, 16)/01	$4k-2$
30/(30, 20, 16)/01	$2k$	30/(17, 20, 19)/01	$4k-2$
30/(30, 20, 15)/01	$2k$	30/(17, 20, 16)/01	$4k-2$
30/(30, 16, 15)/01	$2k$	30/(17, 19, 16)/01	$4k-2$
30/(30, 19, 17)/01	$2k$	30/(17, 20, 18)/01	$4k-2$
30/(30, 19, 15)/01	$2k$	30/(17, 20, 15)/01	$4k-2$
30/(30, 17, 15)/01	$2k$	30/(17, 18, 15)/01	$4k-2$
30/(30, 18, 17)/01	$2k$	30/(16, 30, 20)/01	$4k-2$
30/(30, 18, 16)/01	$2k$	30/(16, 30, 17)/01	$4k-2$
30/(30, 17, 16)/01	$2k$	30/(16, 20, 17)/01	$4k-2$
30/(20, 19, 18)/01	$2k$	30/(16, 30, 18)/01	$4k-2$
30/(20, 19, 17)/01	$2k$	30/(16, 30, 15)/01	$4k-2$
30/(20, 18, 17)/01	$2k$	30/(16, 18, 15)/01	$4k-2$
30/(20, 17, 16)/01	$2k$	30/(16, 20, 19)/01	$4k-2$
30/(20, 17, 15)/01	$2k$	30/(16, 20, 15)/01	$4k-2$
30/(20, 16, 15)/01	$2k$	30/(16, 19, 15)/01	$4k-2$
30/(19, 20, 18)/01	$2k$	30/(16, 19, 18)/01	$4k-2$
30/(19, 20, 16)/01	$2k$	30/(16, 19, 17)/01	$4k-2$
30/(19, 18, 16)/01	$2k$	30/(16, 18, 17)/01	$4k-2$
30/(19, 17, 16)/01	$2k$	30/(15, 30, 20)/01	$4k-2$
30/(19, 17, 15)/01	$2k$	30/(15, 30, 17)/01	$4k-2$
30/(19, 16, 15)/01	$2k$	30/(15, 20, 17)/01	$4k-2$
30/(18, 20, 19)/01	$2k$	30/(15, 30, 19)/01	$4k-2$
30/(18, 20, 15)/01	$2k$	30/(15, 30, 16)/01	$4k-2$
30/(18, 19, 15)/01	$2k$	30/(15, 19, 16)/01	$4k-2$
30/(18, 17, 16)/01	$2k$	30/(15, 20, 18)/01	$4k-2$

30/(18, 17, 15)/01	$2k$	30/(15, 20, 16)/01	$4k-2$
30/(18, 16, 15)/01	$2k$	30/(15, 18, 16)/01	$4k-2$
30/(20, 30, 19)/01	$4k-2$	30/(15, 19, 18)/01	$4k-2$
30/(20, 30, 16)/01	$4k-2$	30/(15, 19, 17)/01	$4k-2$
30/(20, 19, 16)/01	$4k-2$	30/(15, 18, 17)/01	$4k-2$
30/(20, 30, 18)/01	$4k-2$	30/(20, 20, 19)/01	$4k$
30/(20, 30, 15)/01	$4k-2$	30/(20, 20, 15)/01	$4k$
30/(20, 18, 15)/01	$4k-2$	30/(20, 19, 15)/01	$4k$
30/(19, 30, 20)/01	$4k-2$	30/(20, 20, 18)/01	$4k$
30/(19, 30, 17)/01	$4k-2$	30/(20, 20, 16)/01	$4k$
30/(19, 20, 17)/01	$4k-2$	30/(20, 18, 16)/01	$4k$
30/(19, 30, 18)/01	$4k-2$	30/(19, 20, 19)/01	$4k$
30/(19, 30, 15)/01	$4k-2$	30/(19, 20, 15)/01	$4k$
30/(19, 18, 15)/01	$4k-2$	30/(19, 19, 15)/01	$4k$
30/(18, 30, 20)/01	$4k-2$	30/(19, 19, 18)/01	$4k$
30/(18, 30, 17)/01	$4k-2$	30/(19, 19, 17)/01	$4k$
30/(18, 20, 17)/01	$4k-2$	30/(19, 18, 17)/01	$4k$
30/(18, 30, 19)/01	$4k-2$	30/(18, 20, 18)/01	$4k$
30/(18, 30, 16)/01	$4k-2$	30/(18, 20, 16)/01	$4k$
30/(18, 19, 16)/01	$4k-2$	30/(18, 18, 16)/01	$4k$
30/(17, 30, 19)/01	$4k-2$	30/(18, 19, 18)/01	$4k$
30/(17, 30, 15)/01	$4k-2$	30/(18, 19, 17)/01	$4k$
30/(17, 19, 15)/01	$4k-2$	30/(18, 18, 17)/01	$4k$

$C_{4k-2} \times C_2^2 = C_{2k-1} \times C_2^3$

$2k$	30	2	28
$4k-2$	54	4	48
$4k$	18	6	84

In transition from the "middle" to the "strong" equality criterion, by permuting the corresponding subgroups of the index 2, from every group two groups can be derived. According to the "weak" equality criterion (G/H), there is the only one group 30/01 for every n=2k.

Table 3.8

$D_n \times C_2^2$

31/(31/30, 29, 28)/08/01	$2k$	31/(31/30, 29, 21)/05/01	$2k$
31/(31/30, 29, 27)/08/01	$2k$	31/(31/30, 28, 27)/05/01	$2k$
31/(31/30, 29, 24)/08/01	$2k$	31/(31/30, 28, 25)/05/01	$2k$
31/(31/30, 29, 22)/08/01	$2k$	31/(31/30, 28, 21)/05/01	$2k$
31/(31/30, 28, 27)/08/01	$2k$	31/(31/30, 27, 25)/05/01	$2k$
31/(31/30, 28, 25)/08/01	$2k$	31/(31/30, 27, 23)/05/01	$2k$
31/(31/30, 28, 22)/08/01	$2k$	31/(31/30, 25, 23)/05/01	$2k$
31/(31/30, 27, 25)/08/01	$2k$	31/(31/30, 25, 21)/05/01	$2k$

31/(31/30, 27, 24)/08/01	$2k$
31/(31/30, 25, 24)/08/01	$2k$
31/(31/30, 25, 22)/08/01	$2k$
31/(31/30, 24, 22)/08/01	$2k$
31/(31/30, 29, 28)/07/01	$2k$
31/(31/30, 29, 27)/07/01	$2k$
31/(31/30, 29, 23)/07/01	$2k$
31/(31/30, 29, 22)/07/01	$2k$
31/(31/30, 28, 27)/07/01	$2k$
31/(31/30, 28, 26)/07/01	$2k$
31/(31/30, 28, 22)/07/01	$2k$
31/(31/30, 27, 26)/07/01	$2k$
31/(31/30, 27, 23)/07/01	$2k$
31/(31/30, 26, 23)/07/01	$2k$
31/(31/30, 26, 22)/07/01	$2k$
31/(31/30, 23, 22)/07/01	$2k$
31/(31/30, 29, 28)/06/01	$2k$
31/(31/30, 29, 27)/06/01	$2k$
31/(31/30, 29, 24)/06/01	$2k$
31/(31/30, 29, 21)/06/01	$2k$
31/(31/30, 28, 27)/06/01	$2k$
31/(31/30, 28, 26)/06/01	$2k$
31/(31/30, 28, 21)/06/01	$2k$
31/(31/30, 27, 26)/06/01	$2k$
31/(31/30, 27, 24)/06/01	$2k$
31/(31/30, 26, 24)/06/01	$2k$
31/(31/30, 26, 21)/06/01	$2k$
31/(31/30, 24, 21)/06/01	$2k$
31/(31/30, 29, 28)/05/01	$2k$
31/(31/30, 29, 27)/05/01	$2k$
31/(31/30, 29, 23)/05/01	$2k$
31/(29/20, 31, 28)/07/01	$4k-2$
31/(29/20, 31, 23)/07/01	$4k-2$
31/(29/20, 28, 23)/07/01	$4k-2$
31/(29/20, 31, 27)/07/01	$4k-2$
31/(29/20, 31, 22)/07/01	$4k-2$
31/(29/20, 27, 22)/07/01	$4k-2$
31/(29/20, 31, 28)/06/01	$4k-2$
31/(29/20, 31, 24)/06/01	$4k-2$
31/(29/20, 28, 24)/06/01	$4k-2$
31/(29/20, 31, 27)/06/01	$4k-2$
31/(29/20, 31, 21)/06/01	$4k-2$
31/(29/20, 27, 21)/06/01	$4k-2$
31/(29/20, 31, 28)/05/01	$4k-2$
31/(29/20, 31, 23)/05/01	$4k-2$
31/(31/30, 23, 21)/05/01	$2k$
31/(29/20, 28, 27)/08/01	$2k$
31/(29/20, 28, 25)/08/01	$2k$
31/(29/20, 27, 25)/08/01	$2k$
31/(29/20, 25, 24)/08/01	$2k$
31/(29/20, 25, 22)/08/01	$2k$
31/(29/20, 24, 22)/08/01	$2k$
31/(29/20, 28, 27)/07/01	$2k$
31/(29/20, 28, 26)/07/01	$2k$
31/(29/20, 27, 26)/07/01	$2k$
31/(29/20, 26, 23)/07/01	$2k$
31/(29/20, 26, 22)/07/01	$2k$
31/(29/20, 23, 22)/07/01	$2k$
31/(29/20, 28, 27)/06/01	$2k$
31/(29/20, 28, 26)/06/01	$2k$
31/(29/20, 27, 26)/06/01	$2k$
31/(29/20, 26, 24)/06/01	$2k$
31/(29/20, 26, 21)/06/01	$2k$
31/(29/20, 24, 21)/06/01	$2k$
31/(29/20, 28, 27)/05/01	$2k$
31/(29/20, 28, 25)/05/01	$2k$
31/(29/20, 27, 25)/05/01	$2k$
31/(29/20, 25, 23)/05/01	$2k$
31/(29/20, 25, 21)/05/01	$2k$
31/(29/20, 23, 21)/05/01	$2k$
31/(29/20, 31, 28)/08/01	$4k-2$
31/(29/20, 31, 24)/08/01	$4k-2$
31/(29/20, 28, 24)/08/01	$4k-2$
31/(29/20, 31, 27)/08/01	$4k-2$
31/(29/20, 31, 22)/08/01	$4k-2$
31/(29/20, 27, 22)/08/01	$4k-2$
31/(28/19, 26, 21)/06/01	$2k$
31/(28/19, 24, 21)/06/01	$2k$
31/(28/19, 29, 27)/05/01	$2k$
31/(28/19, 29, 23)/05/01	$2k$
31/(28/19, 27, 23)/05/01	$2k$
31/(28/19, 25, 23)/05/01	$2k$
31/(28/19, 25, 21)/05/01	$2k$
31/(28/19, 23, 21)/05/01	$2k$
31/(28/19, 31, 29)/08/01	$4k-2$
31/(28/19, 31, 25)/08/01	$4k-2$
31/(28/19, 29, 25)/08/01	$4k-2$
31/(28/19, 31, 27)/08/01	$4k-2$
31/(28/19, 31, 22)/08/01	$4k-2$
31/(28/19, 27, 22)/08/01	$4k-2$

31/(29/20,28,23)/05/01	$4k-2$	31/(28/19,31,29)/07/01	$4k-2$
31/(29/20,31,27)/05/01	$4k-2$	31/(28/19,31,26)/07/01	$4k-2$
31/(29/20,31,21)/05/01	$4k-2$	31/(28/19,29,26)/07/01	$4k-2$
31/(29/20,27,21)/05/01	$4k-2$	31/(28/19,31,27)/07/01	$4k-2$
31/(29/20,29,28)/08/01	$4k$	31/(28/19,31,22)/07/01	$4k-2$
31/(29/20,29,22)/08/01	$4k$	31/(28/19,27,22)/07/01	$4k-2$
31/(29/20,28,22)/08/01	$4k$	31/(28/19,31,29)/06/01	$4k-2$
31/(29/20,29,27)/08/01	$4k$	31/(28/19,31,26)/06/01	$4k-2$
31/(29/20,29,24)/08/01	$4k$	31/(28/19,29,26)/06/01	$4k-2$
31/(29/20,27,24)/08/01	$4k$	31/(28/19,31,27)/06/01	$4k-2$
31/(29/20,29,28)/07/01	$4k$	31/(28/19,31,21)/06/01	$4k-2$
31/(29/20,29,22)/07/01	$4k$	31/(28/19,27,21)/06/01	$4k-2$
31/(29/20,28,22)/07/01	$4k$	31/(28/19,31,29)/05/01	$4k-2$
31/(29/20,29,27)/07/01	$4k$	31/(28/19,31,25)/05/01	$4k-2$
31/(29/20,29,23)/07/01	$4k$	31/(28/19,29,25)/05/01	$4k-2$
31/(29/20,27,23)/07/01	$4k$	31/(28/19,31,27)/05/01	$4k-2$
31/(29/20,29,28)/06/01	$4k$	31/(28/19,31,21)/05/01	$4k-2$
31/(29/20,29,21)/06/01	$4k$	31/(28/19,27,21)/05/01	$4k-2$
31/(29/20,28,21)/06/01	$4k$	31/(28/19,29,28)/08/01	$4k$
31/(29/20,29,27)/06/01	$4k$	31/(28/19,29,22)/08/01	$4k$
31/(29/20,29,24)/06/01	$4k$	31/(28/19,28,22)/08/01	$4k$
31/(29/20,27,24)/06/01	$4k$	31/(28/19,28,27)/08/01	$4k$
31/(29/20,29,28)/05/01	$4k$	31/(28/19,28,25)/08/01	$4k$
31/(29/20,29,21)/05/01	$4k$	31/(28/19,27,25)/08/01	$4k$
31/(29/20,28,21)/05/01	$4k$	31/(28/19,29,28)/07/01	$4k$
31/(29/20,29,27)/05/01	$4k$	31/(28/19,29,22)/07/01	$4k$
31/(29/20,29,23)/05/01	$4k$	31/(28/19,28,22)/07/01	$4k$
31/(29/20,27,23)/05/01	$4k$	31/(28/19,28,27)/07/01	$4k$
31/(28/19,29,27)/08/01	$2k$	31/(28/19,28,26)/07/01	$4k$
31/(28/19,29,24)/08/01	$2k$	31/(28/19,27,26)/07/01	$4k$
31/(28/19,27,24)/08/01	$2k$	31/(28/19,29,28)/06/01	$4k$
31/(28/19,25,24)/08/01	$2k$	31/(28/19,29,21)/06/01	$4k$
31/(28/19,25,22)/08/01	$2k$	31/(28/19,28,21)/06/01	$4k$
31/(28/19,24,22)/08/01	$2k$	31/(28/19,28,27)/06/01	$4k$
31/(28/19,29,27)/07/01	$2k$	31/(28/19,28,26)/06/01	$4k$
31/(28/19,29,23)/07/01	$2k$	31/(28/19,27,26)/06/01	$4k$
31/(28/19,27,23)/07/01	$2k$	31/(28/19,29,28)/05/01	$4k$
31/(28/19,26,23)/07/01	$2k$	31/(28/19,29,21)/05/01	$4k$
31/(28/19,26,22)/07/01	$2k$	31/(28/19,28,21)/05/01	$4k$
31/(28/19,23,22)/07/01	$2k$	31/(28/19,28,27)/05/01	$4k$
31/(28/19,29,27)/06/01	$2k$	31/(28/19,28,25)/05/01	$4k$
31/(28/19,29,24)/06/01	$2k$	31/(28/19,27,25)/05/01	$4k$
31/(28/19,27,24)/06/01	$2k$	31/(27/18,29,28)/08/01	$2k$
31/(28/19,26,24)/06/01	$2k$	31/(27/18,29,22)/08/01	$2k$
31/(27/18,28,22)/08/01	$2k$	31/(27/18,29,27)/06/01	$4k$

$31/(27/18,25,24)/08/01$	$2k$
$31/(27/18,25,22)/08/01$	$2k$
$31/(27/18,24,22)/08/01$	$2k$
$31/(27/18,29,28)/07/01$	$2k$
$31/(27/18,29,22)/07/01$	$2k$
$31/(27/18,28,22)/07/01$	$2k$
$31/(27/18,26,23)/07/01$	$2k$
$31/(27/18,26,22)/07/01$	$2k$
$31/(27/18,23,22)/07/01$	$2k$
$31/(27/18,29,28)/06/01$	$2k$
$31/(27/18,29,21)/06/01$	$2k$
$31/(27/18,28,21)/06/01$	$2k$
$31/(27/18,26,24)/06/01$	$2k$
$31/(27/18,26,21)/06/01$	$2k$
$31/(27/18,24,21)/06/01$	$2k$
$31/(27/18,29,28)/05/01$	$2k$
$31/(27/18,29,21)/05/01$	$2k$
$31/(27/18,28,21)/05/01$	$2k$
$31/(27/18,25,23)/05/01$	$2k$
$31/(27/18,25,21)/05/01$	$2k$
$31/(27/18,23,21)/05/01$	$2k$
$31/(27/18,31,29)/08/01$	$4k-2$
$31/(27/18,31,25)/08/01$	$4k-2$
$31/(27/18,29,25)/08/01$	$4k-2$
$31/(27/18,31,28)/08/01$	$4k-2$
$31/(27/18,31,24)/08/01$	$4k-2$
$31/(27/18,28,24)/08/01$	$4k-2$
$31/(27/18,31,29)/07/01$	$4k-2$
$31/(27/18,31,26)/07/01$	$4k-2$
$31/(27/18,29,26)/07/01$	$4k-2$
$31/(27/18,31,28)/07/01$	$4k-2$
$31/(27/18,31,23)/07/01$	$4k-2$
$31/(27/18,28,23)/07/01$	$4k-2$
$31/(27/18,31,29)/06/01$	$4k-2$
$31/(27/18,31,26)/06/01$	$4k-2$
$31/(27/18,29,26)/06/01$	$4k-2$
$31/(27/18,31,28)/06/01$	$4k-2$
$31/(27/18,31,24)/06/01$	$4k-2$
$31/(27/18,28,24)/06/01$	$4k-2$
$31/(27/18,31,29)/05/01$	$4k-2$
$31/(27/18,31,25)/05/01$	$4k-2$
$31/(27/18,29,25)/05/01$	$4k-2$
$31/(27/18,31,28)/05/01$	$4k-2$
$31/(27/18,31,23)/05/01$	$4k-2$
$31/(27/18,28,23)/05/01$	$4k-2$
$31/(27/18,29,24)/06/01$	$4k$
$31/(27/18,27,24)/06/01$	$4k$
$31/(27/18,28,27)/06/01$	$4k$
$31/(27/18,28,26)/06/01$	$4k$
$31/(27/18,27,26)/06/01$	$4k$
$31/(27/18,29,27)/05/01$	$4k$
$31/(27/18,29,23)/05/01$	$4k$
$31/(27/18,27,23)/05/01$	$4k$
$31/(27/18,28,27)/05/01$	$4k$
$31/(27/18,28,25)/05/01$	$4k$
$31/(27/18,27,25)/05/01$	$4k$
$31/(26/17,31,28)/06/01$	$4k-2$
$31/(26/17,31,24)/06/01$	$4k-2$
$31/(26/17,28,24)/06/01$	$4k-2$
$31/(26/17,31,27)/06/01$	$4k-2$
$31/(26/17,31,21)/06/01$	$4k-2$
$31/(26/17,27,21)/06/01$	$4k-2$
$31/(26/17,29,28)/06/01$	$4k-2$
$31/(26/17,29,21)/06/01$	$4k-2$
$31/(26/17,28,21)/06/01$	$4k-2$
$31/(26/17,29,27)/06/01$	$4k-2$
$31/(26/17,29,24)/06/01$	$4k-2$
$31/(26/17,27,24)/06/01$	$4k-2$
$31/(26/17,31,28)/07/01$	$4k-2$
$31/(26/17,31,22)/07/01$	$4k-2$
$31/(26/17,28,22)/07/01$	$4k-2$
$31/(26/17,31,27)/07/01$	$4k-2$
$31/(26/17,31,23)/07/01$	$4k-2$
$31/(26/17,27,23)/07/01$	$4k-2$
$31/(26/17,29,28)/07/01$	$4k-2$
$31/(26/17,29,23)/07/01$	$4k-2$
$31/(26/17,28,23)/07/01$	$4k-2$
$31/(26/17,29,27)/07/01$	$4k-2$
$31/(26/17,29,22)/07/01$	$4k-2$
$31/(26/17,27,22)/07/01$	$4k-2$
$31/(25/17,31,28)/08/01$	$4k-2$
$31/(25/17,31,24)/08/01$	$4k-2$
$31/(25/17,28,24)/08/01$	$4k-2$
$31/(25/17,31,27)/08/01$	$4k-2$
$31/(25/17,31,22)/08/01$	$4k-2$
$31/(25/17,27,22)/08/01$	$4k-2$
$31/(25/17,29,28)/08/01$	$4k-2$
$31/(25/17,29,22)/08/01$	$4k-2$
$31/(25/17,28,22)/08/01$	$4k-2$
$31/(25/17,29,27)/08/01$	$4k-2$

31/(27/18, 29, 27)/08/01	$4k$
31/(27/18, 29, 24)/08/01	$4k$
31/(27/18, 27, 24)/08/01	$4k$
31/(27/18, 28, 27)/08/01	$4k$
31/(27/18, 28, 25)/08/01	$4k$
31/(27/18, 27, 25)/08/01	$4k$
31/(27/18, 29, 27)/07/01	$4k$
31/(27/18, 29, 23)/07/01	$4k$
31/(27/18, 27, 23)/07/01	$4k$
31/(27/18, 28, 27)/07/01	$4k$
31/(27/18, 28, 26)/07/01	$4k$
31/(27/18, 27, 26)/07/01	$4k$
31/(25/17, 29, 23)/05/01	$4k-2$
31/(25/17, 27, 23)/05/01	$4k-2$
31/(24/16, 31, 29)/08/01	$4k-2$
31/(24/16, 31, 22)/08/01	$4k-2$
31/(24/16, 29, 22)/08/01	$4k-2$
31/(24/16, 31, 27)/08/01	$4k-2$
31/(24/16, 31, 25)/08/01	$4k-2$
31/(24/16, 27, 25)/08/01	$4k-2$
31/(24/16, 29, 28)/08/01	$4k-2$
31/(24/16, 29, 25)/08/01	$4k-2$
31/(24/16, 28, 25)/08/01	$4k-2$
31/(24/16, 28, 27)/08/01	$4k-2$
31/(24/16, 28, 22)/08/01	$4k-2$
31/(24/16, 27, 22)/08/01	$4k-2$
31/(24/16, 31, 29)/06/01	$4k-2$
31/(24/16, 31, 21)/06/01	$4k-2$
31/(24/16, 29, 21)/06/01	$4k-2$
31/(24/16, 31, 27)/06/01	$4k-2$
31/(24/16, 31, 26)/06/01	$4k-2$
31/(24/16, 27, 26)/06/01	$4k-2$
31/(24/16, 29, 28)/06/01	$4k-2$
31/(24/16, 29, 26)/06/01	$4k-2$
31/(24/16, 28, 26)/06/01	$4k-2$
31/(24/16, 28, 27)/06/01	$4k-2$
31/(24/16, 28, 21)/06/01	$4k-2$
31/(24/16, 27, 21)/06/01	$4k-2$
31/(23/16, 31, 29)/07/01	$4k-2$
31/(23/16, 31, 26)/07/01	$4k-2$
31/(23/16, 29, 26)/07/01	$4k-2$
31/(23/16, 31, 27)/07/01	$4k-2$
31/(23/16, 31, 22)/07/01	$4k-2$
31/(23/16, 27, 22)/07/01	$4k-2$
31/(23/16, 29, 28)/07/01	$4k-2$
31/(25/17, 29, 24)/08/01	$4k-2$
31/(25/17, 27, 24)/08/01	$4k-2$
31/(25/17, 31, 28)/05/01	$4k-2$
31/(25/17, 31, 23)/05/01	$4k-2$
31/(25/17, 28, 23)/05/01	$4k-2$
31/(25/17, 31, 27)/05/01	$4k-2$
31/(25/17, 31, 21)/05/01	$4k-2$
31/(25/17, 27, 21)/05/01	$4k-2$
31/(25/17, 29, 28)/05/01	$4k-2$
31/(25/17, 29, 21)/05/01	$4k-2$
31/(25/17, 28, 21)/05/01	$4k-2$
31/(25/17, 28, 27)/05/01	$4k-2$
31/(23/16, 27, 21)/05/01	$4k-2$
31/(22/15, 31, 29)/08/01	$4k-2$
31/(22/15, 31, 25)/08/01	$4k-2$
31/(22/15, 29, 25)/08/01	$4k-2$
31/(22/15, 31, 28)/08/01	$4k-2$
31/(22/15, 31, 24)/08/01	$4k-2$
31/(22/15, 28, 24)/08/01	$4k-2$
31/(22/15, 29, 27)/08/01	$4k-2$
31/(22/15, 29, 24)/08/01	$4k-2$
31/(22/15, 27, 24)/08/01	$4k-2$
31/(22/15, 28, 27)/08/01	$4k-2$
31/(22/15, 28, 25)/08/01	$4k-2$
31/(22/15, 27, 25)/08/01	$4k-2$
31/(22/15, 31, 29)/07/01	$4k-2$
31/(22/15, 31, 26)/07/01	$4k-2$
31/(22/15, 29, 26)/07/01	$4k-2$
31/(22/15, 31, 28)/07/01	$4k-2$
31/(22/15, 31, 23)/07/01	$4k-2$
31/(22/15, 28, 23)/07/01	$4k-2$
31/(22/15, 29, 27)/07/01	$4k-2$
31/(22/15, 29, 23)/07/01	$4k-2$
31/(22/15, 27, 23)/07/01	$4k-2$
31/(22/15, 28, 27)/07/01	$4k-2$
31/(22/15, 28, 26)/07/01	$4k-2$
31/(22/15, 27, 26)/07/01	$4k-2$
31/(21/15, 31, 29)/06/01	$4k-2$
31/(21/15, 31, 26)/06/01	$4k-2$
31/(21/15, 29, 26)/06/01	$4k-2$
31/(21/15, 31, 28)/06/01	$4k-2$
31/(21/15, 31, 24)/06/01	$4k-2$
31/(21/15, 28, 24)/06/01	$4k-2$
31/(21/15, 29, 27)/06/01	$4k-2$
31/(21/15, 29, 24)/06/01	$4k-2$

31/(23/16, 29, 22)/07/01	$4k-2$	31/(21/15, 27, 24)/06/01	$4k-2$
31/(23/16, 28, 22)/07/01	$4k-2$	31/(21/15, 28, 27)/06/01	$4k-2$
31/(23/16, 28, 27)/07/01	$4k-2$	31/(21/15, 28, 26)/06/01	$4k-2$
31/(23/16, 28, 26)/07/01	$4k-2$	31/(21/15, 27, 26)/06/01	$4k-2$
31/(23/16, 27, 26)/07/01	$4k-2$	31/(21/15, 31, 29)/05/01	$4k-2$
31/(23/16, 31, 29)/05/01	$4k-2$	31/(21/15, 31, 25)/05/01	$4k-2$
31/(23/16, 31, 21)/05/01	$4k-2$	31/(21/15, 29, 25)/05/01	$4k-2$
31/(23/16, 29, 21)/05/01	$4k-2$	31/(21/15, 31, 28)/05/01	$4k-2$
31/(23/16, 31, 27)/05/01	$4k-2$	31/(21/15, 31, 23)/05/01	$4k-2$
31/(23/16, 31, 25)/05/01	$4k-2$	31/(21/15, 28, 23)/05/01	$4k-2$
31/(23/16, 27, 25)/05/01	$4k-2$	31/(21/15, 29, 27)/05/01	$4k-2$
31/(23/16, 29, 28)/05/01	$4k-2$	31/(21/15, 29, 23)/05/01	$4k-2$
31/(23/16, 29, 25)/05/01	$4k-2$	31/(21/15, 27, 23)/05/01	$4k-2$
31/(23/16, 28, 25)/05/01	$4k-2$	31/(21/15, 28, 27)/05/01	$4k-2$
31/(23/16, 28, 27)/05/01	$4k-2$	31/(21/15, 28, 25)/05/01	$4k-2$
31/(23/16, 28, 21)/05/01	$4k-2$	31/(21/15, 27, 25)/05/01	$4k-2$

$D_{4k-2} \times C_2^2 = D_{2k-1} \times C_2^3$

$2k$	120	4	192
$4k-2$	216	6	336
$4k$	72		

In transition from the "middle" to the "strong" equality criterion, by permuting the corresponding subgroups of the index 2, from every group two groups can be derived. According to the "weak" equality criterion ($G/H_1/H$), there are four groups 31/08/01, 31/07/01, 31/06/01, 31/05/01 for every $n = 2k$, and the only one group G/H.

Table 3.9

Using some additional data pointing to the section of the corresponding subgroups, every extended group/subgroup symbol uniquely defines the colored symmetry group. By additional data given in parentheses () are denoted sections indicating to Table 2.2, and by [] sections indicating to Table 3.5.

$Dn(2n)xC22$			
21/(15, 21, 09)(06)/01	$2k-1$	31/(30, 31, 21)/15	$2k-1$
21/(15, 21, 09)(05)/01	$2k-1$	31/(30, 29, 28)(10)/02	$2k-1$
21/(15, 21, 06)/01	$2k-1$	31/(30, 29, 28)(09)/02	$2k-1$
21/(15, 21, 05)/01	$2k-1$	31/(30, 29, 27)(12)/03	$2k-1$
21/(15, 09, 06)/01	$2k-1$	31/(30, 29, 27)(11)/03	$2k-1$
21/(15, 09, 05)/01	$2k-1$	31/(30, 29, 26)/17	$2k-1$
		31/(30, 29, 25)/17	$2k-1$
27/(18, 22, 14)/01	$2k-1$	31/(30, 29, 24)/03	$2k-1$
27/(18, 22, 13)/01	$2k-1$	31/(30, 29, 23)/03	$2k-1$
27/(18, 22, 12)/01	$2k-1$	31/(30, 29, 22)/02	$2k-1$

27/(18,22,11)/01	$2k-1$	31/(30,29,21)/02	$2k-1$
27/(18,21,14)/01	$2k-1$	31/(30,28,27)(14)/04	$2k-1$
27/(18,21,13)/01	$2k-1$	31/(30,28,27)(13)/04	$2k-1$
27/(18,21,12)/01	$2k-1$	31/(30,28,26)/04	$2k-1$
27/(18,21,11)/01	$2k-1$	31/(30,28,25)/04	$2k-1$
27/(18,14,12)/01	$2k-1$	31/(30,28,24)/16	$2k-1$
27/(18,14,11)/01	$2k-1$	31/(30,28,23)/16	$2k-1$
27/(18,13,12)/01	$2k-1$	31/(30,28,22)/02	$2k-1$
27/(18,13,11)/01	$2k-1$	31/(30,28,21)/02	$2k-1$
		31/(30,27,26)/04	$2k-1$
31/(30,31,29)(26)/17	$2k-1$	31/(30,27,25)/04	$2k-1$
31/(30,31,29)(25)/17	$2k-1$	31/(30,27,24)/03	$2k-1$
31/(30,31,28)(24)/16	$2k-1$	31/(30,27,23)/03	$2k-1$
31/(30,31,28)(23)/16	$2k-1$	31/(30,27,22)/15	$2k-1$
31/(30,31,27)(22)/15	$2k-1$	31/(30,27,21)/15	$2k-1$
31/(30,31,27)(21)/15	$2k-1$	31/(30,26,24)/01	k
31/(30,31,26)/17	$2k-1$	31/(30,26,23)/01	k
31/(30,31,25)/17	$2k-1$	31/(30,26,22)/01	k
31/(30,31,24)/16	$2k-1$	31/(30,26,21)/01	k
31/(30,31,23)/16	$2k-1$	31/(30,25,24)/01	k
31/(30,31,22)/15	$2k-1$	31/(30,25,23)/01	k
31/(30,25,22)/01	k	31/(20,29,24)/01	$4k$
31/(30,25,21)/01	k	31/(20,29,23)/01	$4k$
31/(30,24,22)/01	k	31/(20,29,22)/01	$4k$
31/(30,24,21)/01	k	31/(20,29,21)/01	$4k$
31/(30,23,22)/01	k	31/(20,28,27)[03,02](14)/01	$4k$
31/(30,23,21)/01	k	31/(20,28,27)[03,02](13)/01	$4k$
31/(30,29,28)(10)/01	$2k$	31/(20,28,26)[03]/01	$4k$
31/(30,29,28)(09)/01	$2k$	31/(20,28,25)[03]/01	$4k$
31/(30,29,27)(12)/01	$2k$	31/(20,28,22)/01	$4k$
31/(30,29,27)(11)/01	$2k$	31/(20,28,21)/01	$4k$
31/(30,29,24)/01	$2k$	31/(20,27,26)[02]/01	$4k$
31/(30,29,23)/01	$2k$	31/(20,27.25)[02]/01	$4k$
31/(30,29,22)/01	$2k$	31/(20,27,24)/01	$4k$
31/(30,29,21)/01	$2k$	31/(20,27,23)/01	$4k$
31/(30,28,27)(14)/01	$2k$	31/(19,26,24)/01	$2k$
31/(30,28,27)(13)/01	$2k$	31/(19,26,23)/01	$2k$
31/(30,28,26)/01	$2k$	31/(19,26,22)/01	$2k$
31/(30,28,25)/01	$2k$	31/(19,26,21)/01	$2k$
31/(30,28,22)/01	$2k$	31/(19,25,24)/01	$2k$
31/(30,28,21)/01	$2k$	31/(19,25,23)/01	$2k$
31/(30,27,26)/01	$2k$	31/(19,25,22)/01	$2k$
31/(30,27,25)/01	$2k$	31/(19,25,21)/01	$2k$
31/(30,27,24)/01	$2k$	31/(19,24,22)/01	$2k$
31/(30,27,23)/01	$2k$	31/(19,24,21)/01	$2k$

31/(20,26,24)/01	$2k$	31/(19,23,22)/01	$2k$
31/(20,26,23)/01	$2k$	31/(19,23,21)/01	$2k$
31/(20,26,22)/01	$2k$	31/(19,31,29)(26)/01	$4k-2$
31/(20,26,21)/01	$2k$	31/(19,31,29)(25)/01	$4k-2$
31/(20,25,24)/01	$2k$	31/(19,31,27)(22)/01	$4k-2$
31/(20,25,23)/01	$2k$	31/(19,31,27)(21)/01	$4k-2$
31/(20,25,22)/01	$2k$	31/(19,31,26)/01	$4k-2$
31/(20,25,21)/01	$2k$	31/(19,31,25)/01	$4k-2$
31/(20,24,22)/01	$2k$	31/(19,31,22)/01	$4k-2$
31/(20,24,21)/01	$2k$	31/(19,31,21)/01	$4k-2$
31/(20,23,22)/01	$2k$	31/(19,29,27)[02,04](12)/01	$4k-2$
31/(20,23,21)/01	$2k$	31/(19,29,27)[02,04](11)/01	$4k-2$
31/(20,31,28)(24)/01	$4k-2$	31/(19,29,26)/01	$4k-2$
31/(20,31,28)(23)/01	$4k-2$	31/(19,29,25)/01	$4k-2$
31/(20,31,27)(22)/01	$4k-2$	31/(19,29,24)[02]/01	$4k-2$
31/(20,31,27)(21)/01	$4k-2$	31/(19,29,23)[02]/01	$4k-2$
31/(20,31,24)/01	$4k-2$	31/(19,27,24)/01	$4k-2$
31/(20,31,23)/01	$4k-2$	31/(19,27,23)/01	$4k-2$
31/(20,31,22)/01	$4k-2$	31/(19,27,22)/01	$4k-2$
31/(20,31,21)/01	$4k-2$	31/(19,27,21)/01	$4k-2$
31/(20,28,27)[02,03](14)/01	$4k-2$	31/(19,29,28)(10)/01	$4k$
31/(20,28,27)[02,03](13)/01	$4k-2$	31/(19,29,28)(09)/01	$4k$
31/(20,28,26)[02]/01	$4k-2$	31/(19,29,27)[04,02](12)/01	$4k$
31/(20,28,25)[02]/01	$4k-2$	31/(19,29,27)[04,02](11)/01	$4k$
31/(20,28,24)/01	$4k-2$	31/(19,29,24)[04]/01	$4k$
31/(20,28,23)/01	$4k-2$	31/(19,29,23)[04]/01	$4k$
31/(20,27,26)[03]/01	$4k-2$	31/(19,29,22)/01	$4k$
31/(20,27,25)[03]/01	$4k-2$	31/(19,29,21)/01	$4k$
31/(20,27,22)/01	$4k-2$	31/(19,28,27)(14)/01	$4k$
31/(20,27,21)/01	$4k-2$	31/(19,28,27)(13)/01	$4k$
31/(20,29,28)(10)/01	$4k$	31/(19,28,26)/01	$4k$
31/(20,29,28)(09)/01	$4k$	31/(19,28,25)/01	$4k$
31/(20,29,27)(12)/01	$4k$	31/(19,28,22)/01	$4k$
31/(20,29,27)(11)/01	$4k$	31/(19,28,21)/01	$4k$
31/(19,27,26)/01	$4k$	31/(17,31,27)(22)/01	$4k-2$
31/(19,27,25)/01	$4k$	31/(17,31,27)(21)/01	$4k-2$
31/(19,27,24)/01	$4k$	31/(17,31,24)/01	$4k-2$
31/(19,27,23)/01	$4k$	31/(17,31,23)/01	$4k-2$
31/(19,26,24)/01	$2k$	31/(17,31,22)/01	$4k-2$
31/(19,26,23)/01	$2k$	31/(17,31,21)/01	$4k-2$
31/(19,26,22)/01	$2k$	31/(17,29,28)(10)/01	$4k-2$
31/(19,26,21)/01	$2k$	31/(17,29,28)(09)/01	$4k-2$
31/(18,26,24)/01	$2k$	31/(17,29,27)(12)/01	$4k-2$
31/(18,26,23)/01	$2k$	31/(17,29,27)(11)/01	$4k-2$
31/(18,26,22)/01	$2k$	31/(17,29,24)/01	$4k-2$

31/(18, 26, 21)/01	$2k$	31/(17, 29, 23)/01	$4k-2$
31/(18, 25, 24)/01	$2k$	31/(17, 29, 22)/01	$4k-2$
31/(18, 25, 23)/01	$2k$	31/(17, 29, 21)/01	$4k-2$
31/(18, 25, 22)/01	$2k$	31/(17, 28, 24)/01	$4k-2$
31/(18, 25, 21)/01	$2k$	31/(17, 28, 23)/01	$4k-2$
31/(18, 24, 22)/01	$2k$	31/(17, 28, 22)/01	$4k-2$
31/(18, 24, 21)/01	$2k$	31/(17, 28, 21)/01	$4k-2$
31/(18, 23, 22)/01	$2k$	31/(17, 27, 24)/01	$4k-2$
31/(18, 23, 21)/01	$2k$	31/(17, 27, 23)/01	$4k-2$
31/(18, 31, 29)(26)/01	$4k-2$	31/(17, 27, 22)/01	$4k-2$
31/(18, 31, 29)(25)/01	$4k-2$	31/(17, 27, 21)/01	$4k-2$
31/(18, 31, 28)(24)/01	$4k-2$	31/(16, 31, 29)(26)/01	$4k-2$
31/(18, 31, 28)(23)/01	$4k-2$	31/(16, 31, 29)(25)/01	$4k-2$
31/(18, 31, 26)/01	$4k-2$	31/(16, 31, 27)(22)/01	$4k-2$
31/(18, 31, 25)/01	$4k-2$	31/(16, 31, 27)(21)/01	$4k-2$
31/(18, 31, 24)/01	$4k-2$	31/(16, 31, 26)/01	$4k-2$
31/(18, 31, 23)/01	$4k-2$	31/(16, 31, 25)/01	$4k-2$
31/(18, 29, 28)[03, 04](10)/01	$4k-2$	31/(16, 31, 22)/01	$4k-2$
31/(18, 29, 28)[03, 04](09)/01	$4k-2$	31/(16, 31, 21)/01	$4k-2$
31/(18, 29, 26)/01	$4k-2$	31/(16, 29, 28)(10)/01	$4k-2$
31/(18, 29, 25)/01	$4k-2$	31/(16, 29, 28)(09)/01	$4k-2$
31/(18, 29, 22)[03]/01	$4k-2$	31/(16, 29, 26)/01	$4k-2$
31/(18, 29, 21)[03]/01	$4k-2$	31/(16, 29, 25)/01	$4k-2$
31/(18, 28, 26)/01	$4k-2$	31/(16, 29, 22)/01	$4k-2$
31/(18, 28, 25)/01	$4k-2$	31/(16, 29, 21)/01	$4k-2$
31/(18, 28, 22)/01	$4k-2$	31/(16, 28, 27)(14)/01	$4k-2$
31/(18, 28, 21)/01	$4k-2$	31/(16, 28, 27)(13)/01	$4k-2$
31/(18, 29, 28)[04, 03](10)/01	$4k$	31/(16, 28, 26)/01	$4k-2$
31/(18, 29, 28)[04, 03](09)/01	$4k$	31/(16, 28, 25)/01	$4k-2$
31/(18, 29, 27)(12)/01	$4k$	31/(16, 28, 22)/01	$4k-2$
31/(18, 29, 27)(11)/01	$4k$	31/(16, 28, 21)/01	$4k-2$
31/(18, 29, 24)/01	$4k$	31/(16, 27, 26)/01	$4k-2$
31/(18, 29, 23)/01	$4k$	31/(16, 27, 25)/01	$4k-2$
31/(18, 29, 22)[04]/01	$4k$	31/(16, 27, 22)/01	$4k-2$
31/(18, 29, 21)[04]/01	$4k$	31/(16, 27, 21)/01	$4k-2$
31/(18, 28, 27)(14)/01	$4k$	31/(15, 31, 29)(26)/01	$4k-2$
31/(18, 28, 27)(13)/01	$4k$	31/(15, 31, 29)(25)/01	$4k-2$
31/(18, 28, 26)/01	$4k$	31/(15, 31, 28)(24)/01	$4k-2$
31/(18, 28, 25)/01	$4k$	31/(15, 31, 28)(23)/01	$4k-2$
31/(18, 28, 22)/01	$4k$	31/(15, 31, 26)/01	$4k-2$
31/(18, 28, 21)/01	$4k$	31/(15, 31, 25)/01	$4k-2$
31/(18, 27, 26)/01	$4k$	31/(15, 31, 24)/01	$4k-2$
31/(18, 27, 25)/01	$4k$	31/(15, 31, 23)/01	$4k-2$
31/(18, 27, 24)/01	$4k$	31/(15, 29, 27)(12)/01	$4k-2$
31/(18, 27, 23)/01	$4k$	31/(15, 29, 27)(11)/01	$4k-2$

31/(17,31,28)(24)/01	$4k-2$	31/(15,29,26)/01	$4k-2$
31/(17,31,28)(23)/01	$4k-2$	31/(15,29,25)/01	$4k-2$
31/(15,29,24)/01	$4k-2$	31/(15,28,24)/01	$4k-2$
31/(15,29,23)/01	$4k-2$	31/(15,28,23)/01	$4k-2$
31/(15,28,27)(14)/01	$4k-2$	31/(15,27,26)/01	$4k-2$
31/(15,28,27)(13)/01	$4k-2$	31/(15,27,25)/01	$4k-2$
31/(15,28,26)/01	$4k-2$	31/(15,27,24)/01	$4k-2$
31/(15,28,25)/01	$4k-2$	31/(15,27,23)/01	$4k-2$

$2k-1$	108	2	192
k	12	3	120
$2k$	54	4	120
$4k-2$	126	6	192
$4k$	54		

In transition from the "middle" to the "strong" equality criterion, by permuting the corresponding subgroups of the index 2, from every group two groups can be derived.

According to the "weak" equality criterion (G/H), we have the following results:

$2k-1$	15	2	1
k	1	3	16
		4	1
		6	1

Table 3.10

By additional data given in parentheses ¡¿ are denoted the sections corresponding to Table 2.3, and by () the sections corresponding to Table 2.2.

$D_{n(2n)} \times C_2^3$

$31/(30,31,29,28) < 08 > /01$	$2k-1$	$31/(30,31,28,27) < 06 > /01$	$2k-1$
$31/(30,31,29,28) < 07 > /01$	$2k-1$	$31/(30,31,28,27) < 05 > /01$	$2k-1$
$31/(30,31,29,28) < 06 > /01$	$2k-1$	31/(30,31,28,26)(24)/01	$2k-1$
$31/(30,31,29,28) < 05 > /01$	$2k-1$	31/(30,31,28,25)(24)/01	$2k-1$
$31/(30,31,29,27) < 08 > /01$	$2k-1$	31/(30,31,28,22)(24)/01	$2k-1$
$31/(30,31,29,27) < 07 > /01$	$2k-1$	31/(30,31,28,21)(24)/01	$2k-1$
$31/(30,31,29,27) < 06 > /01$	$2k-1$	31/(30,31,28,26)(23)/01	$2k-1$
$31/(30,31,29,27) < 05 > /01$	$2k-1$	31/(30,31,28,25)(23)/01	$2k-1$
31/(30,31,29,24)(26)/01	$2k-1$	31/(30,31,28,22)(23)/01	$2k-1$
31/(30,31,29,23)(26)/01	$2k-1$	31/(30,31,28,21)(23)/01	$2k-1$
31/(30,31,29,22)(26)/01	$2k-1$	31/(30,31,27,26)(22)/01	$2k-1$
31/(30,31,29,21)(26)/01	$2k-1$	31/(30,31,27,25)(22)/01	$2k-1$
31/(30,31,29,24)(25)/01	$2k-1$	31/(30,31,27,24)(22)/01	$2k-1$
31/(30,31,29,23)(25)/01	$2k-1$	31/(30,31,27,23)(22)/01	$2k-1$
31/(30,31,29,22)(25)/01	$2k-1$	31/(30,31,27,26)(21)/01	$2k-1$
31/(30,31,29,21)(25)/01	$2k-1$	31/(30,31,27,25)(21)/01	$2k-1$

31/(30,31,28,27) < 08 > /01	$2k-1$	31/(30,31,27,24)(21)/01	$2k-1$
31/(30,31,28,27) < 07 > /01	$2k-1$	31/(30,31,27,23)(21)/01	$2k-1$
31/(30,31,26,24)/01	$2k-1$	31/(30,29,25,22)/01	$2k-1$
31/(30,31,26,23)/01	$2k-1$	31/(30,29,25,21)/01	$2k-1$
31/(30,31,26,22)/01	$2k-1$	31/(30,29,24,22)/01	$2k-1$
31/(30,31,26,21)/01	$2k-1$	31/(30,29,24,21)/01	$2k-1$
31/(30,31,25,24)/01	$2k-1$	31/(30,29,23,22)/01	$2k-1$
31/(30,31,25,23)/01	$2k-1$	31/(30,29,23,21)/01	$2k-1$
31/(30,31,25,22)/01	$2k-1$	31/(30,28,27,24)(14)/01	$2k-1$
31/(30,31,25,21)/01	$2k-1$	31/(30,28,27,23)(14)/01	$2k-1$
31/(30,31,24,22)/01	$2k-1$	31/(30,28,27,22)(14)/01	$2k-1$
31/(30,31,24,21)/01	$2k-1$	31/(30,28,27,21)(14)/01	$2k-1$
31/(30,31,23,22)/01	$2k-1$	31/(30,28,27,24)(13)/01	$2k-1$
31/(30,31,23,21)/01	$2k-1$	31/(30,28,27,23)(13)/01	$2k-1$
31/(30,29,28,27) < 08 > /01	$2k-1$	31/(30,28,27,22)(13)/01	$2k-1$
31/(30,29,28,27) < 07 > /01	$2k-1$	31/(30,28,27,21)(13)/01	$2k-1$
31/(30,29,28,27) < 06 > /01	$2k-1$	31/(30,28,26,24)/01	$2k-1$
31/(30,29,28,27) < 05 > /01	$2k-1$	31/(30,28,26,23)/01	$2k-1$
31/(30,29,28,26)(10)/01	$2k-1$	31/(30,28,26,22)/01	$2k-1$
31/(30,29,28,25)(10)/01	$2k-1$	31/(30,28,26,21)/01	$2k-1$
31/(30,29,28,24)(10)/01	$2k-1$	31/(30,28,25,24)/01	$2k-1$
31/(30,29,28,23)(10)/01	$2k-1$	31/(30,28,25,23)/01	$2k-1$
31/(30,29,28,26)(09)/01	$2k-1$	31/(30,28,25,22)/01	$2k-1$
31/(30,29,28,25)(09)/01	$2k-1$	31/(30,28,25,21)/01	$2k-1$
31/(30,29,28,24)(09)/01	$2k-1$	31/(30,28,24,22)/01	$2k-1$
31/(30,29,28,23)(09)/01	$2k-1$	31/(30,28,24,21)/01	$2k-1$
31/(30,29,27,26)(12)/01	$2k-1$	31/(30,28,23,22)/01	$2k-1$
31/(30,29,27,25)(12)/01	$2k-1$	31/(30,28,23,21)/01	$2k-1$
31/(30,29,27,22)(12)/01	$2k-1$	31/(30,27,26,24)/01	$2k-1$
31/(30,29,27,21)(12)/01	$2k-1$	31/(30,27,26,23)/01	$2k-1$
31/(30,29,27,26)(11)/01	$2k-1$	31/(30,27,26,22)/01	$2k-1$
31/(30,29,27,25)(11)/01	$2k-1$	31/(30,27,26,21)/01	$2k-1$
31/(30,29,27,22)(11)/01	$2k-1$	31/(30,27,25,24)/01	$2k-1$
31/(30,29,27,21)(11)/01	$2k-1$	31/(30,27,25,23)/01	$2k-1$
31/(30,29,26,24)/01	$2k-1$	31/(30,27,25,22)/01	$2k-1$
31/(30,29,26,23)/01	$2k-1$	31/(30,27,25,21)/01	$2k-1$
31/(30,29,26,22)/01	$2k-1$	31/(30,27,24,22)/01	$2k-1$
31/(30,29,26,21)/01	$2k-1$	31/(30,27,24,21)/01	$2k-1$
31/(30,29,25,24)/01	$2k-1$	31/(30,27,23,22)/01	$2k-1$
31/(30,29,25,23)/01	$2k-1$	31/(30,27,23,21)/01	$2k-1$

$2k-1$ 112 3 112

In transition from the "middle" to the "strong" equality criterion, by permuting the corresponding subgroups of the index 2, from every group six groups can be derived. According to the "weak" equality criterion (G/H), there is the only one group 31/01 for every $n = 2k - 1$.

The results obtained imply the complete survey of the group/subgroup relations of the symmetry groups of bands. In the spirit of [4] Table 4.1 gives the index of a subgroup in the group, and Table 4.2 the quotient group $P \cong G/H$.

In Table 4.1 the groups C_2 and C_2^2 can be replaced, respectively, by D_N, where $[G : H] = N$ (Table 4.2). The italic indexes in Table 4.2 indicate that H is not a normal subgroup of G.

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Table 4.1

	01	02	03	04
01	C_k			
02	C_{2k}	C_{2k-1}		
03	C_{2k}		C_{2k-1}	
04	C_{2k}			C_{2k-1}
05	$D_{k(2k)}$			
06	$D_{k(2k)}$			
07	$D_{k(2k)}$			
08	$D_{k(2k)}$			
09	$D_{2k(4k)}$	$D_{2k-1(4k-2)}$		
10	$D_{2k(4k)}$	$D_{2k-1(4k-2)}$		
11	$D_{2k(4k)}$		$D_{2k-1(4k-2)}$	
12	$D_{2k(4k)}$		$D_{2k-1(4k-2)}$	
13	$D_{2k(4k)}$			$D_{2k-1(4k-2)}$
14	$D_{2k(4k)}$			$D_{2k-1(4k-2)}$
15	$C_k \times C_2$	C_{2k}		
16	$C_k \times C_2$		C_{2k}	
17	$C_k \times C_2$			C_{2k}
18	$C_{2k} \times C_2$	C_{4k}	C_{4k-2}	C_{4k-2}
19	$C_{2k} \times C_2$	C_{4k-2}	C_{4k}	C_{4k-2}
20	$C_{2k} \times C_2$	C_{4k-2}	C_{4k-2}	C_{4k}
21	$D_{k(2k)} \times C_2$	$D_{2k(4k)}$		
22	$D_{k(2k)} \times C_2$	$D_{2k(4k)}$		
23	$D_{k(2k)} \times C_2$		$D_{2k(4k)}$	
24	$D_{k(2k)} \times C_2$		$D_{2k(4k)}$	
25	$D_{k(2k)} \times C_2$			$D_{2k(4k)}$
26	$D_{k(2k)} \times C_2$			$D_{2k(4k)}$
27	$D_{2k(4k)} \times C_2$	$D_{4k(8k)}$	$D_{4k-2(8k-4)}$	$D_{4k-2(8k-4)}$
28	$D_{2k(4k)} \times C_2$	$D_{4k-2(8k-4)}$	$D_{4k(8k)}$	$D_{4k-2(8k-4)}$
29	$D_{2k(4k)} \times C_2$	$D_{4k-2(8k-4)}$	$D_{4k-2(8k-4)}$	$D_{4k(8k)}$
30	$C_{4k-2} \times C_2$	$C_{2k} \times C_2$	$C_{2k} \times C_2$	$C_{2k} \times C_2$
31	$D_{k(2k)} \times C_2^2$	$D_{2k(4k)} \times C_2$	$D_{2k(4k)} \times C_2$	$D_{2k(4k)} \times C_2$

05	06	07	08	09	10	11	12	13	14	15
C_2										
	C_2									
		C_2								
			C_2							
C_2	C_2			D_{2k-1}						
		C_2	C_2		D_{2k-1}					
C_2		C_2				D_{2k-1}				
	C_2		C_2				D_{2k-1}			
C_2			C_2					D_{2k-1}		
	C_2	C_2							D_{2k-1}	
										C_k
										C_{2k}
C_2	C_2			C_2						$D_{k(2k)}$
		C_2	C_2		C_2					$D_{k(2k)}$
C_2		C_2				C_2				
	C_2		C_2				C_2			
C_2			C_2					C_2		
	C_2	C_2							C_2	
C_2^2	C_2^2	C_2^2	C_2^2	D_{4k}	D_{4k}	C_2	C_2	C_2	C_2	$D_{2k(4k)}$
C_2^2	C_2^2	C_2^2	C_2^2	C_2	C_2	D_{4k}	D_{4k}	C_2	C_2	
C_2 2	C_2^2	C_2^2	C_2^2	C_2	C_2	C_2	C_2	D_{4k}	D_{4k}	
										$C_k \times C_2$
C_2^2	C_2^2	C_2 2	C_2^2	C_2^2	C_2^2	C_2^2	C_2^2	C_2^2	C_2^2	$D_{k(2k)} \times C_2$

	16	17	18	19	20
01					
02					
03					
04					
05					
06					
07					
08					
09					
10					
11					
12					
13					
14					
15					
16	C_k				
17		C_k			
18			C_{2k-1}		
19	C_{2k}			C_{2k-1}	
20		C_{2k}			C_{2k-1}
21					
22					
23	$D_{k(2k)}$				
24	$D_{k(2k)}$				
25		$D_{k(2k)}$			
26		$D_{k(2k)}$			
27			$D_{2k-1(4k-2)}$		
28	$D_{2k(4k)}$			$D_{2k-1(4k-2)}$	
29		$D_{2k(4k)}$			$D_{2k-1(4k-2)}$
30	$C_k \times C_2$	$C_k \times C_2$	C_{2k}	C_{2k}	C_{2k}
31	$D_{k(2k)} \times C_2$	$D_{k(2k)} \times C_2$	$D_{2k(4k)}$	$D_{2k(4k)}$	$D_{2k(4k)}$

21	22	23	24	25	26	27	28	29	30	31
C_2										
	C_2									
		C_2								
			C_2							
				C_2						
					C_2					
C_2	C_2					D_{2k-1}				
		C_2	C_2				D_{2k-1}			
				C_2	C_2			D_{2k-1}		
									C_k	
C_2	C_2	C_2	C_2	C_2	C_2	C_2	C_2	C_2	$D_{k(2k)}$	C_2

Table 4.2

	01	02	03	04	05	06	07	08	09	10
01	k									
02	$2k$	$2k-1$								
03	$2k$		$2k-1$							
04	$2k$			$2k-1$						
05	$2k$				k					
06	$2k$					k				
07	$2k$						k			
08	$2k$							k		
09	$4k$	$4k-2$			$2k$	$2k$			$2k-1$	
10	$4k$	$4k-2$					$2k$	$2k$		$2k-1$
11	$4k$		$4k-2$		$2k$		$2k$			
12	$4k$		$4k-2$			$2k$		$2k$		
13	$4k$			$4k-2$	$2k$			$2k$		
14	$4k$			$4k-2$		$2k$	$2k$			
15	$2k$	$2k$								
16	$2k$		$2k$							
17	$2k$			$2k$						
18	$4k$	$4k$	$4k-2$	$4k-2$						
19	$4k$	$4k-2$	$4k$	$4k-2$						
20	$4k$	$4k-2$	$4k-2$	$4k$						
21	$4k$	$4k$			$2k$	$2k$			$2k$	
22	$4k$	$4k$					$2k$	$2k$		$2k$
23	$4k$		$4k$		$2k$		$2k$			
24	$4k$		$4k$			$2k$		$2k$		
25	$4k$			$4k$	$2k$			$2k$		
26	$4k$			$4k$		$2k$	$2k$			
27	$8k$	$8k$	$8k-4$	$8k-4$	$4k$	$4k$	$4k$	$4k$	$4k$	$4k$
28	$8k$	$8k-4$	$8k$	$8k-4$	$4k$	$4k$	$4k$	$4k$	$4k-2$	$4k-2$
29	$8k$	$8k-4$	$8k-4$	$8k$	$4k$	$4k$	$4k$	$4k$	$4k-2$	$4k-2$
30	$8k-4$	$4k$	$4k$	$4k$						
31	$8k$	$8k$	$8k$	$8k$	$4k$	$4k$	$4k$	$4k$	$4k$	$4k$

11	12	13	14	15	16	17	18	19	20	21
$2k-1$										
	$2k-1$									
		$2k-1$								
			$2k-1$							
				k						
					k					
						k				
				$2k$			$2k-1$			
					$2k$			$2k-1$		
						$2k$			$2k-1$	
				$2k$						k
				$2k$						
$2k$					$2k$					
	$2k$				$2k$					
		$2k$				$2k$				
			$2k$			$2k$				
$4k-2$	$4k-2$	$4k-2$	$4k-2$	$4k$			$4k-2$			$2k$
$4k$	$4k$	$4k-2$	$4k-2$		$4k$			$4k-2$		
$4k-2$	$4k-2$	$4k$	$4k$			$4k$			$4k-2$	
				$2k$	$2k$	$2k$	$2k$	$2k$	$2k$	
$4k4$	$k4$	$k4$	$k4$	$k4$	$k4$	$k4$	$k4$	$k4$	$k2$	k

	22	23	24	25	26	27	28	29	30	31
01										
02										
03										
04										
05										
06										
07										
08										
09										
10										
11										
12										
13										
14										
15										
16										
17										
18										
19										
20										
21										
22	k									
23		k								
24			k							
25				k						
26					k					
27	$2k$					$2k-1$				
28		$2k$	$2k$				$2k-1$			
29				$2k$	$2k$			$2k-1$		
30									k	
31	$2k$	$2k$	$2k$	$2k$	$2k$	$2k$	$2k$	$2k$	$2k$	k