



## Group Decision-Making Using Improved Multi-Criteria Decision Making Methods for Credit Risk Analysis

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**Abstract.** Credit risk analysis is a core research issue in the field of financial risk management. This paper first investigates the analytic hierarchy process (AHP) as a method of measuring index weights for group decision-making (GDM). AHP for group decision-making (AHP-GDM) is then researched and applied, taking into full account the cognitive levels of different experts. Second, the concept of grey relational degree is introduced into the ideal solution of the technique for order of preference by similarity to ideal solution (TOPSIS). This concept fully considers the relative closeness of grey relational degree between alternatives and the "ideal" solution in order to strengthen their relationship. The AHP-GDM method overcomes the problem of subjectivity in measuring index weights, and the revised TOPSIS (R-TOPSIS) method heightens the effectiveness of assessment results. An illustrative case using data from Chinese listed commercial banks shows that the R-TOPSIS method is more effective than both TOPSIS and grey relational analysis (GRA) in credit risk evaluation. The two improved multi-criteria decision making (MCDM) methods are also applied to empirical research regarding the credit risk analysis of Chinese urban commercial banks. The results indicate the validity and effectiveness of both methods.

### 1. Introduction

Credit risk analysis is a core research issue in the field of financial risk management. In order to reduce the possibility of loss as much as possible, credit risk must be recognized, measured, and minimized [1, 2]. In this regard, many data mining and statistical analysis methods can be applied to credit risk analysis, such as logistic regression analysis, discriminate analysis, artificial neural networks, probit regression analysis, genetic algorithms, decision trees, linear programming, k-nearest neighbors, and support vector machines

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[3, 4]. However, there is little research that considers the use of multi-criteria decision making (MCDM) to analyze credit risk. Thus, this paper presents an analytic hierarchy process-group decision-making (AHP-GDM) method and a revised technique for order of preference by similarity to ideal solution (R-TOPSIS) method for credit risk analysis. The increasing complexity of real-world applications makes it difficult for a single decision-maker (DM) to focus on all related aspects of decision problems [5]. It is necessary to use collective wisdom in order to make decisions. AHP is a well-known technique for measuring index weights in decision problems. For instance, Weiss and Rao [6] applied AHP to address a number of design issues for large-scale systems; Wind and Saaty [7] used an AHP approach to solve a marketing application problem; and Lee and Kozar [8] employed AHP to identify the prioritization of alternative web sites and the relative importance of web site quality factors. This paper presents an AHP-GDM method for measuring the index weights of credit risk for Chinese urban commercial banks. In this regard, the method comprehensively considers the cognitive levels of different experts. TOPSIS is a well-known technique for solving MCDM problems. Natividade-Jesus et al. [9] used TOPSIS to develop and propose a decision support system in order to evaluate urban-built space from different perspectives. Kahraman et al. [10] conducted a synthesized method based on a hierarchical fuzzy TOPSIS approach to improve the effectiveness of decisions. Further, Yue [11] developed an expanded use of TOPSIS with interval numbers to measure the weights of DMs. In order to improve decision accuracy, this paper employs an R-TOPSIS method integrated with grey relational degree [12–14] for credit risk evaluation. An R-TOPSIS method can express the corresponding importance among alternatives to “ideal” solutions. Hence, the method used here considers the relative distance between alternatives and the “ideal” solution in order to strengthen their relationship. The rest of this paper is as follows. Section 2 reviews and introduces related works. Section 3 presents and illustrates some of the MCDM approaches such as AHP, grey relational analysis (GRA), and TOPSIS. Section 4 presents two improved MCDM methods: an AHP-GDM method and an R-TOPSIS method. In Section 5, the R-TOPSIS method is tested and verified through an illustrative example of 11 Chinese public-listed commercial banks. Section 6 applies the two improved MCDM methods to real-world credit risk evaluation problems. The results illustrate and verify that the methods are feasible and effective. Section 7 discusses the results and concludes the paper.

## 2. Related Works and Literature Review

AHP, proposed by Saaty [15], is considered a well-known technique to address multiple-criteria problems of prioritization and choice [16]. On the basis of pair-wise comparison values, a relative priority vector, which can express preference information from DMs, can be elicited by AHP. The AHP method, working with such pair-wise comparison values, can combine subjective and objective criteria to determine index weights. By determining the weights of indicators, it is reasonable to distinguish the relative importance of different indicators [17]. However, the AHP method is subjective when structuring the pair-wise judgment matrix according to a single expert. To overcome this shortcoming, the current paper presents and applies the AHP-GDM method.

Some literature considers the AHP-GDM method. Chwolka and Raith [18] used a utilitarian-weighted arithmetic mean method to extend group preference aggregations in the AHP for multi-criteria decision problems. Escobar and Moreno-Jimenez [19] presented a new procedure, the aggregation of individual preference (AIP), to address multi-actor decision-making problems by using AHP as methodological support. Cho and Cho [20] used an inconsistency ratio of judgment matrix as a group evaluation quality to propose a loss function approach for the aggregation of group preference. As indicated, the aforementioned literature focuses on aggregate group preference. However, few studies consider experts weights with index weights. In this paper, the AHP-GDM method, using experts weights with index weights, is proposed in order to measure index weights in decision problems and to consider the cognitive levels of different experts.

GRA, based on quantitative research and qualitative analysis, is a basic approach of grey system theory [21]. This theory is based on grey space, which can process incomplete and inaccurate information [22]. In this context, grey relation is related to uncertain relations among elements or things. The GRA method has been developed as a widely applied technique in data processing, decision analysis, and modelling, as well as control and prediction [22–25]. The advantage of GRA is its use of a small amount of data combined

with a simple calculation and high precision. These advantages are especially helpful when information is poor. TOPSIS was developed by Hwang and Yoon [26] to select an optimal solution for addressing MCDM problems. TOPSIS is based on the principle that chosen alternatives should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution; namely, TOPSIS can find the best alternatives by minimizing the distance to the positive ideal solution and maximizing the distance to the negative ideal solution [27].

However, in an MCDM process, there is usually very limited statistical data, combined with human factors, and much of the data do not have the typical distribution rule. Thus, it is difficult to ensure the accuracy of results through direct sample data analysis. In contrast, when using "poor information," the GRA method, especially grey relational degree, can achieve satisfactory results because GRA has advantages with regard to a small amount of sample data, a simple principle, convenient operation, easy data-mining rules, and so on. Thus, in this paper, we make full use of these advantages and introduce the concept of grey relational degree into TOPSIS in order to present an R-TOPSIS method for credit risk analysis. This method considers the relative distance between alternatives and the "ideal" solution in order to strengthen their relationship.

### 3. MCMD Methods

#### 3.1. AHP

As discussed in Section 2, AHP, proposed by Saaty [15], is considered a well-known technique for modelling unstructured multi-criteria problems in management, society, the economy, and politics [16]. On the basis of pair-by-pair comparison values, a relative priority vector, which can express preference information, can be elicited by AHP [9]. The pair-wise comparison values can then be accepted according to a suitable scale, completed by expert scoring [28–31]. The calculation steps are as follows [32, 33].

- (1) Identify a decision problem. The decision problem is usually identified in the topmost level of a hierarchical structure.
- (2) Structure the hierarchy. In the solution process, the typical hierarchical structure usually has three levels: the alternative, criteria, and objective levels.
- (3) Construct a pair-wise comparison matrix. The pair-wise comparison matrix is constructed by expert scoring based on a 19 scale measurement.
- (4) Determine criteria weights. The criteria weights can be determined by the eigenvector method according to the formula:  $AW = \lambda_{max}w$  where  $\lambda_{max}$  is a maximum eigenvalue and  $w$  is an eigenvector with regard to  $\lambda_{max}$ .
- (5) Test consistency. When making pair-wise comparisons, the DMs may make inconsistent judgments. The consistency test is determined and calculated by two parameters of the consistency index  $CI$  and consistency ratio  $CR$ . If  $CR$  does not exceed 0.10, it is acceptable; otherwise, the pair-wise comparison judgment matrix is inconsistent.
- (6) Derive a priority vector of alternatives according to criteria weights.

#### 3.2. GRA

GRA is a basic approach of grey system theory, which addresses incomplete and imprecise information in grey systems [21]. It is an aspect of grey theory based on grey space [22]. In this context, grey relation refers to uncertain relations among elements or things. The GRA method, based on quantitative research and qualitative analysis, has been developed as a widely applied technique in data processing, decision analysis, and modelling, as well as control and prediction [22–25]. The advantages of GRA are its use of a small amount of data combined with a simple calculation and high precision. These are useful factors in situations where information is poor. Specifically, GRA can be calculated as follows [25].

Assume there is an initial matrix  $R$ :

$$R = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \tag{1}$$

(1) Normalize the initial matrix  $R$  and obtain the normalized matrix  $R'$ :

$$R' = \begin{bmatrix} x'_{11} & x'_{12} & \cdots & x'_{1n} \\ x'_{21} & x'_{22} & \cdots & x'_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ x'_{m1} & x'_{m2} & \cdots & x'_{mn} \end{bmatrix} \tag{2}$$

(2) Generate the reference sequence  $x'_0$ :

$$x'_0 = (x'_0(1), x'_0(2), \dots, x'_0(n)) \tag{3}$$

$x'_0(j)$  is the normalized and largest value in the  $j$ th factor.

(3) Compute all differences  $\Delta_{0i}(j)$  between all the normalized sequences and the reference sequences  $x'_0$ :

$$\Delta_{0i}(j) = |x'_0(j) - x'_{ij}| \tag{4}$$

$$\Delta = \begin{bmatrix} \Delta_{01}(1) & \Delta_{01}(2) & \cdots & \Delta_{01}(n) \\ \Delta_{02}(1) & \Delta_{02}(2) & \cdots & \Delta_{02}(n) \\ \vdots & \vdots & \cdots & \vdots \\ \Delta_{0m}(1) & \Delta_{0m}(2) & \cdots & \Delta_{0m}(n) \end{bmatrix} \tag{5}$$

(4) Calculate the grey coefficient  $r_{0i}(j)$ :

$$r_{0i}(j) = \frac{\min_i \min_j \Delta_{0i}(j) + \delta \times \max_i \max_j \Delta_{0i}(j)}{\Delta_{0i}(j) + \delta \times \max_i \max_j \Delta_{0i}(j)} \tag{6}$$

when  $\delta$  is an identification coefficient, its value is usually assigned to 0.5 in order to provide good stability and moderate distinguishing effects.

(5) Obtain the value of grey relational degree  $b_i$ :

$$b_i = \sum_{j=1}^n w_j \times r_{0i}(j) \tag{7}$$

where  $w_j$  is criteria weights and  $\sum_{j=1}^n w_j = 1$ . The larger the  $b_i$ , the better the chosen alternative.

### 3.3. TOPSIS

TOPSIS, proposed by Hwang and Yoon[26], is a well-known technique for solving MCDM problems [27]. The principle of TOPSIS is that the chosen alternatives should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution [34]. All the chosen alternatives can be ordered and evaluated by their relative closeness to the ideal solution. The detailed steps are as follows [35].

(1) Obtain a standardized matrix . The standardized value  $a_{ij}$  can be obtained as:

$$a_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m (x_{ij})^2}} (1 \leq i \leq m, 1 \leq j \leq n) \tag{8}$$

(2) Compute the weighted standardized matrix  $D$ :

$$D = (a_{ij} \times w_j) (1 \leq i \leq m, 1 \leq j \leq n) \tag{9}$$

where  $w_j$  is the criteria weight and  $\sum_{i=1}^n w_j = 1$ .

(3) Compute the positive ideal solution  $V^*$  and the negative ideal solution  $V^-$ :

$$V^* = \{v_1^*, v_2^*, \dots, v_n^*\} = \{\max_i v_{ij} | j \in J, \min_i v_{ij} | j \in J'\} \tag{10}$$

$$V^- = \{v_1^-, v_2^-, \dots, v_n^-\} = \{\min_i v_{ij} | j \in J, \max_i v_{ij} | j \in J'\} \tag{11}$$

(4) Compute the separation measures, applying m-dimensional Euclidean distance:

$$S_i^+ = \sqrt{\sum_{j=1}^n (V_i^j - V^*)^2} (1 \leq i \leq m, 1 \leq j \leq n) \tag{12}$$

$$S_i^- = \sqrt{\sum_{j=1}^n (V_i^j - V^-)^2} (1 \leq i \leq m, 1 \leq j \leq n) \tag{13}$$

(5) Compute the relative closeness:

$$Y_i = \frac{S_i^-}{S_i^+ + S_i^-} (1 \leq i \leq m) \tag{14}$$

When  $Y_i \in (0, 1)$ ,  $Y_i$  is closer to 1, the alternative is closer to the ideal solution.

(6) Rank priority:

The larger the  $Y_i$ , the better the chosen alternative.

#### 4. Improved MCDM Methods

##### 4.1. AHP-GDM

The increasing complexity of real-world applications makes it difficult for a single DM to focus on all related aspects of decision problems [5]. Thus, it is necessary to use collective wisdom to make decisions. AHP is a well-known technique to measure index weights in decision problems. In addition, AHP advocates GDM, whereby a group of collective DMs can take advantage of their knowledge and experience to decompose a complicated decision problem into a hierarchical structure and then solve the problem according to the traditional steps of AHP [36, 37]. Thus, this paper presents an AHP-GDM method to measure the index weights of the credit risk of Chinese urban commercial banks, taking into full consideration the cognitive levels of different experts. This approach consists of three steps. First, original index weights of various indicators are calculated by the AHP method. Second, the experts weights are computed to emphasize the cognitive level of different experts. Finally, the index weights are calculated according to the experts weights and the original index weights. The detailed steps are presented and proposed in our prior paper [38].

4.2. R-TOPSIS Method

In a complicated multi-criteria problem, the larger the relational degree of the chosen alternative to the ideal solution, the closer the chosen alternative to the "ideal" solution [39]. In this paper, in view of the grey relational degree of GRA and the ideal solution of TOPSIS, an R-TOPSIS method is presented for credit risk analysis. This considers the relative closeness of grey relational degree between alternatives and the "ideal" solution in order to strengthen their relationship. First, in this method the concept of grey relational degree is introduced into TOPSIS to measure the relation of the chosen alternative to the ideal solution. Second, the relative closeness of the grey relational degree of each chosen alternative to the positive ideal solution and negative ideal solution are computed. Finally, the optimal alternative is identified through the relative closeness of grey relational degree. The detailed steps are as follows.

- (1) Obtain the standardized matrix . The standardized value can be obtained as:

$$a_{ij} = \frac{r_{ij}}{\sqrt{\sum_{i=1}^m (r_{ij})^2}} (1 \leq i \leq m, 1 \leq j \leq n) \tag{15}$$

- (2) Compute the weighted standardized matrix  $D$ :

$$D = \begin{bmatrix} d_{11} & \cdots & d_{1j} & \cdots & d_{1n} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ d_{i1} & \cdots & d_{ij} & \cdots & d_{in} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ d_{m1} & \cdots & d_{mj} & \cdots & d_{mn} \end{bmatrix} \tag{16}$$

When  $d_{ij} = (a_{ij} \times w_j)$  ( $1 \leq i \leq m, 1 \leq j \leq n$ ),  $w_j$  is the criteria weight and  $\sum_{j=1}^n w_j = 1$ .

- (3) Compute the positive ideal solution  $V^*$  and the negative ideal solution  $V^-$ :

$$V^* = \{v_1^*, v_2^*, \dots, v_n^*\} = \{\max_i v_{ij} | j \in J, \min_i v_{ij} | j \in J'\} \tag{17}$$

$$V^- = \{v_1^-, v_2^-, \dots, v_n^-\} = \{\min_i v_{ij} | j \in J, \max_i v_{ij} | j \in J'\} \tag{18}$$

- (4) Compute the grey correlation coefficient of the  $i$ th alternative to the positive ideal solution to the  $j$ th criterion:

$$r_{ij}^+ = \frac{m + \delta \times M}{\Delta_i(j) + \delta \times M} (1 \leq j \leq n) \tag{19}$$

When ,  $\Delta_i(j) = |v_i^*(j) - v_i(j)|$ ,  $m = \min_i \min_j \Delta_i(j)$ ,  $M = \max_i \max_j \Delta_i(j)$ ,  $\delta$  is an identification coefficient and its value is usually assigned to 0.5. A grey correlation coefficient matrix  $R^+$  can then be obtained:

$$R^+ = \begin{bmatrix} r_{11}^+ & r_{12}^+ & \cdots & r_{1n}^+ \\ r_{21}^+ & r_{22}^+ & \cdots & r_{2n}^+ \\ \vdots & \vdots & \cdots & \vdots \\ r_{m1}^+ & r_{m2}^+ & \cdots & r_{mn}^+ \end{bmatrix} \tag{20}$$

The grey relational degree of the  $i$ th alternative to the positive ideal solution related to the  $j$ th criterion is:

$$R_i^+ = \frac{\sum_{j=1}^n r_{ij}^+}{n} (1 \leq i \leq m) \tag{21}$$

- (5) Similarly, compute the grey correlation coefficient of the  $i$ th  $i_i$ h alternative to the negative ideal solution related to the  $j$ th  $j_j$ h criterion:

$$r_{ij}^- = \frac{m + \delta \times M}{\Delta_i(j) + \delta \times M} (1 \leq j \leq n) \tag{22}$$

When,  $\Delta_i(j) = |v_i^*(j) - v_i(j)|$ ,  $m = \min_i \min_j \Delta_i(j)$ ,  $M = \max_i \max_j \Delta_i(j)$ ,  $\delta$  is an identification coefficient and its value is usually assigned to 0.5. A grey correlation coefficient matrix  $R^-$  can then be obtained:

$$R^- = \begin{bmatrix} r_{11}^- & r_{12}^- & \cdots & r_{1n}^- \\ r_{21}^- & r_{22}^- & \cdots & r_{2n}^- \\ \vdots & \vdots & \cdots & \vdots \\ r_{m1}^- & r_{m2}^- & \cdots & r_{mn}^- \end{bmatrix} \tag{23}$$

The grey relational degree of the  $i$ th alternative to the negative ideal solution related to the  $j$ th criterion is:

$$R_i^- = \frac{\sum_{j=1}^n r_{ij}^-}{n} (1 \leq i \leq m) \tag{24}$$

- (6) Compute the relative closeness of the grey relational degree:

$$Y_i = \frac{R_i^-}{R_i^+ + R_i^-} (1 \leq i \leq m) \tag{25}$$

When  $Y_i \in (0, 1)$ ,  $Y_i$  is closer to 1, the alternative is closer to the ideal solution.

- (7) Rank priority:

The larger the  $Y_i$ , the better the chosen alternative.

### 5. An Illustrative Case

This section introduces the illustrative case used to verify the effectiveness and feasibility of the R-TOPSIS method for credit risk analysis. The data for the empirical study originates from the 2007 financial data of 11 Chinese listed commercial banks. All appropriate ethical considerations have been followed regarding the use of this data.

#### 5.1. Credit Risk Index System of Chinese Listed Commercial Banks

A company's or a bank's financial performance can be represented by various factors from its financial statements [40, 41]. These financial factors, such as salary, income, expenditure, and balance, can be used to illustrate potential credit risk status. Thus, the process of credit risk evaluation involves multiple criteria. Consequently, the purpose of this subsection is to present the use of financial data to verify the effectiveness and feasibility of the R-TOPSIS method for credit risk evaluation. In this illustrative case, 12 financial factors are used as measurements of Chinese listed commercial banks credit risk. These factors contain benefit criteria and cost criteria. The selection principle of the index is based on the financial literature [42–46] and the opinion of three experts in credit risk analysis, consisting of a senior banker, a senior financial manager, and a credit risk assessment professor. In addition, the comprehensiveness principle, representative principle, and available principle are followed. The details of the index system of Chinese listed commercial banks and their financial data are presented in Table 1.

Table 1: Financial Data and Index System

Indicator (%)	Indicator (%)	Minimum Value	Maximum Value	Goal
Equity/total assets		2.24	8.23	Maximize
Value of fixed assets/total assets		0.44	1.32	Maximize
Year-end deposits/total assets		59.36	83.03	Maximize
Average amount of lendable funds/total assets		60.77	88.69	Maximize
Year-end loans/total assets		38.96	61.21	Maximize
Amount of investment and securities/total assets		12.51	37.53	Maximize
Non-interest income/total assets		0.12	0.55	Maximize
Interest income/total assets		2.96	5.12	Maximize
Interest expense/total assets		0.02	1.89	Maximize
Operating expenses/total assets		0.16	1.76	Maximize
Net profit/total assets		0.35	1.16	Maximize
Non-performing loans rate		1.15	5.62	Minimize

This data set contains many features. Most of the indicators are benefit criteria; only one is a cost criterion.

5.2. Empirical Process

In this subsection, we propose our research framework and describe our empirical process to verify the R-TOPSIS method. Compared with expert score ranking, the calculation results show that the R-TOPSIS method can be applied to analyze commercial bank credit risk and can improve assessment accuracy. The evaluation procedure for credit risk assessment is presented in Figure 1.

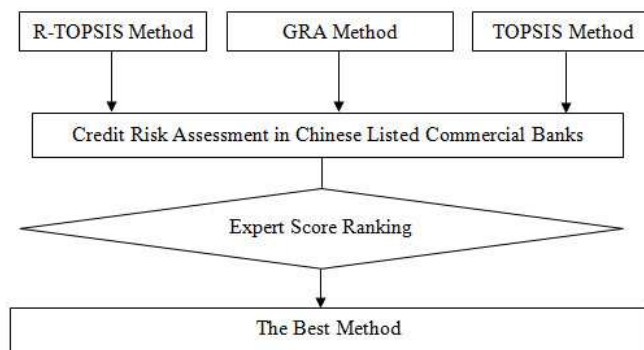


Figure 1: Evaluation Procedure for Credit Risk Assessment

TOPSIS, GRA, and the R-TOPSIS method are used respectively to evaluate the credit risk of Chinese listed commercial banks. The evaluation object is 11 of such banks: the Industrial and Commercial Bank of China (ICBC), the Bank of China (BOC), the China Construction Bank (CCB), the Bank of Communications



(BOCOM), the China Citic Bank (CITIC), the Huaxia Bank (HXB), the China Minsheng Bank (CMSB), the Shenzhen Development Bank (SDB), the China Merchants Bank (CMB), the Industrial Bank (CIB), and the Shanghai Pudong Development Bank (SPDB). Three experts in credit risk analysis, as described in 5.1, assess the ranking of commercial bank credit risk. The three experts opinions are then combined by linear weighted average. The expert score ranking of the banks is 3, 1, 2, 4, 5, 11, 6, 8, 7, 10, and 9, corresponding with the ICBC, BOC, CBC, BOCOM, CITIC, HXB, CMSB, SDB, CMBC, CIB, and SPDB respectively. The evaluation results are presented in Table 2.

Table 2: Credit Risk Level of Chinese Listed Commercial Banks

Bank	R-TOPSIS Method		TOPSIS Method		GRA Method		Expert Score Ranking
	Relative Closeness	Ranking	Relative Closeness	Ranking	Value	Ranking	
ICBC	0.4541	3	0.9858	1	0.9191	1	3
BOC	0.5365	1	0.1945	2	0.4260	4	1
CBC	0.5171	2	0.1093	3	0.4678	2	2
BOCOM	0.4463	5	0.0529	4	0.4194	6	4
CITIC	0.4538	4	0.0467	6	0.4291	3	5
HXB	0.3589	10	0.0375	9	0.3914	10	11
CMSB	0.4321	6	0.0440	7	0.4151	7	6
SDB	0.3357	11	0.0294	11	0.3981	8	8
CMBC	0.4171	7	0.0491	5	0.4260	5	7
CIB	0.4149	8	0.0354	10	0.3864	11	10
SPDB	0.3961	9	0.0379	8	0.3926	9	9

5.3. Results Analysis

From Table 2, it is difficult to compare the results of the three evaluation methods intuitively; thus, a visual analysis of the tables results is presented in Figure 2.

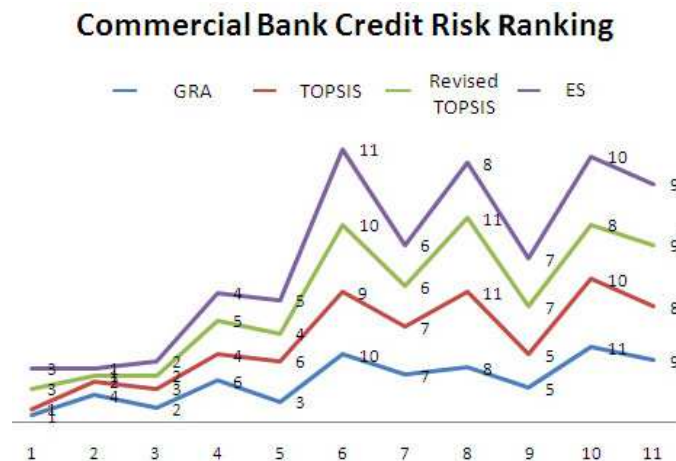


Figure 2: Diagram illustrations of the proposed operations

In Figure 2, ES represents expert score ranking from the three experts. According to Figure 2, BOC has the best credit ranking of all the commercial banks. The second best is CBC, the third is ICBC, and the worst is SDB, followed by HXB. According to the opinions of the experts, BOC, CBC, and ICBC have low credit risk levels, while HXB and CIB have high credit risk levels. From a comparison of fitting trends with

the expert score rankings, a conclusion can be drawn that the evaluation results of the R-TOPSIS method are superior to the TOPSIS method and the GRA method.

## 6. Credit Risk Analysis using the Improved Models

### 6.1. Problem Descriptions

In the Chinese banking industry, the development status of urban commercial banks has a major impact on the country’s economy; thus, the banks credit risk management is a matter of concern in the banking sector. As communication and cooperation strengthen throughout the worlds economies, Chinese urban commercial banks must establish credit risk management systems, management philosophies, and decision-making methods that correspond with their strategic development in order to improve profits and growth. In this regard, MCDM methods, based on multiple conflicting criteria, can be used to identify the most feasible solutions or alternatives. When addressing practical problems such as credit scoring, the selection of an index system and the determination of index weights are extremely important. Thus, in this section, the index system of Chinese urban commercial banks and the index weights are first discussed.

### 6.2. Credit Risk Index System of Chinese Urban Commercial Banks

The credit risks of Chinese urban commercial banks suggest that the banks suffer from losses due to many incertitude factors because a gap exists between bank expectations and actual results. If a commercial bank has a large credit risk, the loan principal and interest cannot be recovered on schedule. Consequently, once the amount of cash available fails to meet the demand for withdrawals, the banks normal business is threatened. This situation can even lead to bankruptcy. Thus, research on the credit risks of urban commercial banks is greatly significant. This paper adopts MCDM methods to address credit risk problems [50–54]. The core feature of such methods is to obtain a scientific and effective index system and the corresponding index weights. In this paper, a credit risk index system for Chinese urban commercial banks is established, structured by eight aspects of four dimensions, which include benefit criteria and cost criteria. The selection of the index system is based on the literature [47–49] and the opinions of three experts in credit risk analysis. The established index system is presented in Table 3.

Table 3: Credit Risk Index System of Chinese Urban Commercial Banks

Security	Liquidity	Profitability	Development Ability
Bad loan ratio (A <sub>1</sub> )	Current ratio (A <sub>3</sub> )	Return on equity (A <sub>5</sub> )	Deposit growth rate (A <sub>7</sub> )
Core capital adequacy ratio (A <sub>2</sub> )	Loan to deposit ratio (A <sub>4</sub> )	Earnings per share (A <sub>6</sub> )	Loan growth rate (A <sub>8</sub> )

#### 6.2.1. Security

Security is a major criterion for measuring the risk ability of Chinese urban commercial banks. In this regard, two main indicators can be applied to reflect the safety levels of such banks: core capital adequacy ratio and bad loan ratio.

- (1) Bad loan ratio is defined as the ratio of non-performing loans to total loans. The smaller the bad loan rate, the better the quality of the credit assets. Consequently, the smaller the bad loan rate, the more secure a bank becomes. Thus, bad loan ratio is a negative indicator.
- (2) Core capital adequacy ratio is defined as the ratio of core capital to total risk-weighted assets. The larger the ratio, the better the security.

### 6.2.2. *Liquidity*

Chinese urban commercial banks need to have liquidity in order to respond to any customer withdrawals or loan demands. Current ratio and loan to deposit ratio reflect the liquidity of commercial banks.

- (1) Current ratio is the ratio of current assets to current debts. It measures repayment liabilities in terms of the corporate current assets that can be converted into cash before short-term debt is mature. The larger the ratio, the better the liquidity.
- (2) Loan to deposit ratio is the ratio of a commercial banks outstanding loans to outstanding deposits at the end of the year. The larger the ratio, the worse the liquidity. Thus, loan to deposit ratio is a negative indicator.

### 6.2.3. *Profitability*

Chinese urban commercial banks, as business units, pursue maximum profits with common enterprise, an approach that is a source of intrinsic motivation for commercial business. Return on equity and earnings per share reflect the profitability of Chinese urban commercial banks.

- (1) Return on equity is known as return on shareholders equity, which is the ratio of net profit to average shareholders' equity. This indicator reflects equity earnings levels in order to measure the efficiency of banks using their own capital. The larger the index values, the higher the investment benefits.
- (2) Earnings per share are also known as after-tax profits per share, which is the ratio of after-tax profits to total equity. This is an important indicator that is used to determine equity investment value, and reflects bank profitability. The larger the index values, the higher the investment benefits.

### 6.2.4. *Development Ability*

Development ability refers to the growth ability of commercial banks. It reflects a banks anti-risk capability regarding all aspects of banking trends. The main indicators that reflect the development level of Chinese urban commercial banks include deposit growth rate and loan growth rate.

- (1) Deposit growth rate is the ratio of the amount of deposit growth in the current year to deposits in the prior year. Deposit growth rate reflects the expansion of the scale of a bank's capital, which is a key indicator that is used to measure changes in bank scale and growth status. The larger the value, the better the development.
- (2) Loan growth rate is the ratio of the amount of bank loan growth in the current year to loans in the prior year. Loan growth rate reflects the expansion of a banks credit operation, which is an important indicator that is used to measure a banks growth conditions. The larger the value, the better the development.

## 6.3. *Index Weights*

In this subsection, the index weights of the credit risks of Chinese urban commercial banks are calculated by the AHP-GDM method. The specific processes are as follows. First, the pair-wise comparison matrix is collected by expert scoring from the field of credit risk analysis. Second, original index weights of various indicators are calculated by the AHP method. Third, the experts weights are computed to emphasize the cognitive levels of the different experts. Finally, the index weights are calculated according to the experts weights and the original index weights. The specific evaluation processes are as follows.

- (1) Determine a pair-wise comparison matrix. An important process of AHP is pair-wise comparison. The pair-wise comparison matrix can be obtained by comparing pair-to-pair credit risk evaluation indicators of Chinese urban commercial banks, as established by the three chosen experts. The evaluation results are presented in Tables 4, 5, and 6.

Table 4: Pair-wise Comparison Matrix of Expert 1

	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	A <sub>4</sub>	A <sub>5</sub>	A <sub>6</sub>	A <sub>7</sub>	A <sub>8</sub>
A <sub>1</sub>	1	1/2	1	3	2	1/3	1	3
A <sub>2</sub>	2	1	2	4	3	1/2	2	4
A <sub>3</sub>	1	1/2	1	4	2	1/3	2	3
A <sub>4</sub>	1/3	1/3	1/3	1	1/2	1/5	1/3	1
A <sub>5</sub>	1/2	1/3	1/2	2	1	1/3	1/2	2
A <sub>6</sub>	3	2	3	5	4	1	3	5
A <sub>7</sub>	1	1/2	1/2	3	2	1/3	1	3
A <sub>8</sub>	1/3	1/3	1/3	1	1/2	1/5	1/3	1

Table 5: Pair-wise Comparison Matrix of Expert 2

	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	A <sub>4</sub>	A <sub>5</sub>	A <sub>6</sub>	A <sub>7</sub>	A <sub>8</sub>
A <sub>1</sub>	1	1	1/2	2	2	1/3	1	3
A <sub>2</sub>	1	1	1/2	2	2	1/3	3	3
A <sub>3</sub>	2	2	1	3	3	1/2	2	4
A <sub>4</sub>	1/2	1/2	1/3	1	2	1/3	1/2	2
A <sub>5</sub>	1/2	1/2	1/3	1/2	1	1/3	1/2	2
A <sub>6</sub>	3	3	2	4	4	1	3	5
A <sub>7</sub>	1	1/3	1/2	2	2	1/3	1	3
A <sub>8</sub>	1/3	1/3	1/3	1/2	1/2	1/5	1/3	1

Table 6: Pair-wise Comparison Matrix of Expert 3

	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	A <sub>4</sub>	A <sub>5</sub>	A <sub>6</sub>	A <sub>7</sub>	A <sub>8</sub>
A <sub>1</sub>	1	2	1/2	3	2	1/3	1/2	2
A <sub>2</sub>	1/2	1	1/2	3	2	1/3	1/2	2
A <sub>3</sub>	2	2	1	4	3	1/2	3	3
A <sub>4</sub>	1/3	1/3	1/3	1	1/2	1/5	1/3	1/2
A <sub>5</sub>	1/2	1/2	1/2	2	1	1/3	1/3	3
A <sub>6</sub>	3	3	2	5	4	1	2	4
A <sub>7</sub>	2	2	1/3	4	3	1/2	1	3
A <sub>8</sub>	1/2	1/2	1/3	2	1/3	1/3	1/3	1

- (2) Determine the original index weights of various indicators by the AHP method and conduct the consistency test. The evaluation results are presented in Table 7. The calculated consistency ratios of the pair-wise comparison decision matrix of each of the three experts are 0.0149, 0.0250, and 0.0378. These results are all less than 0.1, which indicates that the pair-wise comparison decision matrix of each expert has satisfied the consistency test. This indicates that the original index weights obtained by expert scoring are effective.

Table 7: The Original Index Weights of Each Expert

Indicator	Original Index Weights		
	Expert 1	Expert 2	Expert 3
Bad loan ratio	0.1155	0.1115	0.1119
Core capital adequacy ratio	0.1937	0.1336	0.0945
Current ratio	0.1332	0.1898	0.2020
Loan to deposit ratio	0.0414	0.0712	0.0378
Return on equity	0.0685	0.0595	0.0729
Earnings per share	0.2970	0.2926	0.2722
Deposit growth rate	0.1082	0.1019	0.1546
Loan growth rate	0.0426	0.0399	0.0542

- (3) Determine the experts weights. According to the formula in a prior paper [38], each experts weight can be calculated as follows:

$$P_1 = 0.8701, P_2 = 0.7999, P_3 = 0.7255$$

For standardization, the experts weights can be obtained as:

$$P_1^* = 0.3632, P_2^* = 0.3339, P_3^* = 0.3029$$

- (4) Determine the index weights. Based on the original weights and the experts weights, the index weights can be obtained as:

$$W^* = (0.1131, 0.1436, 0.1728, 0.0503, 0.0669, 0.2880, 0.1201, 0.0452)$$

The index weights of the credit risks of Chinese urban commercial bank are determined here in order to illustrate the feasibility and effectiveness of the AHP-GDM method. The research results indicate that this AHP-GDM method is feasible and effective. In this method, qualitative factors and quantitative factors are addressed in a unified framework. This provides a comparatively easy way to deal with decision-making problems.

#### 6.4. Alternative Evaluation

In this subsection, an empirical study is conducted according to the R-TOPSIS method for credit risk analysis. The 2007 data is derived from 12 Chinese urban commercial banks and includes benefit criteria and cost criteria, as presented in Table 8. The 12 banks are the Bank of Hangzhou, the Bank of Jiangsu, the Bank of Wenzhou, the Bank of Shanghai, the Qilu Bank, the Weihai City Commercial Bank, the Fujian Haixia Bank, the Jiaxing City Commercial Bank, the Fuzhou City Commercial Bank, the Taizhou Bank, the Bank of Nanchang, and the Huishang Bank. These cover parts of six provinces and one city in East China.

Table 8: Financial Data of Chinese Urban Commercial Banks

Indicator (%)	Minimum Value	Maximum Value	Goal
Bad loan ratio	0.43	2.87	Minimize
Core capital adequacy ratio	7.11	13.61	Maximize
Current ratio	38.21	72.33	Maximize
Loan to deposit ratio	61.08	72.96	Minimize
Return on equity	2.79	42.35	Maximize
Earnings per share	0.03	1.18	Maximize
Deposit growth rate	4.29	40.33	Maximize
Loan growth rate	7.3	43.04	Maximize

Table 9: Credit Risk Levels of Chinese Urban Commercial Banks

Bank	Relative Closeness	Ranking
Bank of Hangzhou	0.5323	2
Bank of Jiangsu	0.4443	8
Bank of Wenzhou	0.4285	10
Bank of Shanghai	0.4902	3
Qilu Bank	0.4819	5
Weihai City Commercial Bank	0.4397	9
Fujian Haixia Bank	0.4249	11
Jiaxing City Commercial Bank	0.419	12
Fuzhou City Commercial Bank	0.4819	6
Taizhou Bank	0.5637	1
Bank of Nanchang	0.4491	7
Huishang Bank	0.4823	4

In Table 9, the rankings for credit risk are 2, 8, 10, 3, 5, 9, 11, 12, 6, 1, 7, and 4 with regard to the Bank of Hangzhou, the Bank of Jiangsu, the Bank of Wenzhou, the Bank of Shanghai, the Qilu Bank, the Weihai City Commercial Bank, the Fujian Haixia Bank, the Jiaxing City Commercial Bank, the Fuzhou City Commercial Bank, the Taizhou Bank, the Bank of Nanchang, and the Huishang Bank respectively. The Taizhou Bank has the best credit ranking of the Chinese urban commercial banks studied here. The Bank of Hangzhou has the second best and the Bank of Shanghai has the third. The Jiaxing City Commercial Bank has the worst ranking. In other words,, the banks that have low credit risk levels are the Taizhou Bank, the Bank of Hangzhou, and the Bank of Shanghai. The banks that have the highest credit risk levels are the Jiaxing City Commercial Bank and the Fujian Haixia Bank. Further, from Table 9 we can see that the Taizhou Bank, which has the lowest credit risk, has an evaluation value of 0.5637, and the Jiaxing City Commercial Bank, which has the highest credit risk, has an evaluation value of 0.4190. Through calculation, we can further establish that the evaluation value of the lowest credit risk is superior to the highest credit risk by 34.54

## 7. Discussion and Conclusion

AHP is widely used in decision process and preference analysis to address complex multiple-criteria problems with qualitative and quantitative facts. In this paper, a method of measuring index weights regarding group decision-making is researched based on AHP. When group decision-making is undergone, the views and opinions of every group member must be respected; further, no one in the group should have dictatorial powers. Thus, group decision analysis is complex. If a problem is to be effectively resolved through group decision-making, the group must select the appropriate decision methods.

TOPSIS is a well-known MCDM technique. Its principle is that the chosen alternative should have the farthest distance from the negative ideal solution and the shortest distance from the positive ideal solution.

Traditional TOPSIS does not consider a relative relational degree between alternatives and the negative and positive ideal solutions, although this could be important to the decision process. GRA originates from the concept of grey space, which can handle incomplete and inaccurate information. It uses grey relational degree to measure the strength of the relationship between two factors. Thus, this paper introduces grey relational degree into the calculation of the distance from the "ideal" solutions of TOPSIS. In an environment with poor information, we can obtain more efficient, practical, and useful results and conclusions.

This paper verifies the effectiveness of the R-TOPSIS method for credit risk analysis through a case study. An illustrative example of credit risk evaluation for 11 Chinese listed commercial banks using 2007 data demonstrates the feasibility and practicability of the revised method for credit risk analysis in real-world applications. The results also indicate that the accuracy of the R-TOPSIS method is superior to that of the TOPSIS method and GRA method for the credit risk analysis of commercial banks. Further, we use two improved MCDM methods to evaluate the credit risks of Chinese urban commercial banks. This approach has two steps. First, an AHP-GDM method is used to measure index weights. Second, an R-TOPSIS method is used to analyze the credit risk of 12 Chinese urban commercial banks from six provinces and one city in East China. Through calculation, the evaluation value of the lowest credit risk is seen to be superior to the highest credit risk by 34.54. This paper makes two major contributions. First, a method of measuring index weights in GDM, based on AHP, is researched. This method fully considers the cognitive levels of different experts. The particular advantage of this AHP-GDM method is that it can overcome the problem of subjectivity in measuring index weights. Second, the concept of grey relational degree is introduced into the ideal solution of TOPSIS. This fully considers the relative closeness of grey relational degree between alternatives and the "ideal" solution in order to strengthen their relationship. The R-TOPSIS method heightens the effectiveness of assessment results.

The main limitation of this paper is that it focuses for its empirical studies on Chinese commercial banks. Future research could use the new methods suggested here in different credit risk analysis contexts and thereby illustrate the methods generalizability.

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