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Extreme Return, Extreme Volatility and Investor Sentiment

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Abstract. The extreme return and extreme volatility have great influences on the investor sentiment in stock market. However, few researchers have taken the phenomenon into consideration. In this paper, we first distinguish the extreme situations from non-extreme situations. Then we use the ordinary generalized least squares and quantile regression methods to estimate a linear regression model by applying the standardized AAII, the return and volatility of SP 500. The results indicate that, except for extremely negative return, other return sequences can cause great changes in investor sentiment, and non-extreme return plays a leading role in affecting the overall American investor sentiment. Extremely positive (negative) return can rapidly improve (further reduce) the level of investor sentiment when investors encounter extremely pessimistic situations. The impact gradually decreases with improvement of the sentiment until the situation turns optimistic. In addition, we find that extreme and non-extreme volatility cannot affect the overall investor sentiment.

1. Introduction

Recently, investor sentiment has attracted considerable attention from academics. Most existing studies on investor sentiment have focused on how emotion affects return and volatility in the stock market [1-4]. Most scholars believe that investor sentiment can affect stock market return. Fisher and Statman [5] found that investor sentiment is positively related to the current month's stock return but is negatively related to future return. Brown and Cliff [6] and Schmeling [7] reached the same conclusion. Baker and Wurgler [8-9] have adopted the return rate, the discounting rate of the closed-end fund and the first-day return of IPOs, etc. as variables to test investor sentiment as predictor of stock return. Wen et al. [10-12] studied the relationship between investors' risk attitude and return from the perspective of risk preference. In addition, some literatures pay close attention to the issue which investor sentiment affects stock market volatility.

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Most empirical results have shown that investor sentiment can explain the stock market volatility. For example, Brown [13] found that investor sentiment had an impact on stock market fluctuations. Lee et al. [14] also supported this conclusion. Yu and Yuan [15] showed the investor sentiment play an important role in the relationship between return and volatility. Sayin et al. [16] found that the influence of investor sentiment on stock market volatility is vary in different industries.

Furthermore, some scholars tested whether the investor sentiment affected by the return and volatility of stock market. Solt and Statman [17] found that stock return can affect investor sentiment. Brown and Cliff [18] studied the relationship between investor sentiment and short-term stock market return. Their results exhibited that historical return are important factors that influence investor sentiment. What's more, they also found that investor sentiment and its change correlated strongly with stock return at the same time, but the former couldn't forecast future short-term result of stock market correctly. Wang et al. [19] found that most emotional behaviors were caused by stock market return and volatility, but the inverse could not be verified. Kling and Gao [20] exhibited that high return in stock market can boost investor sentiment, while negative revenues cause a decline in sentiment. Although investor sentiment can not predict the future trend of the market, it does provide a better explanation for fluctuations. Most of the above studies supported the idea that stock market return can significantly affect investor sentiment, but whether volatility can change investor sentiment is uncertainty.

Lots of empirical studies and models focus on correlations among expected return and volatility [21-25]. However, little attention has been paid to the role of extreme stock market return on investor sentiment. Long ago, some scholars studied the extreme situation of stock market return. Parkinson [26] believed that an extreme situation contains much useful information which can further understanding of the stock market. Longin [27] first defined that the feature of extreme return is "the lowest return (minimum) or the highest return (maximum)" in the stock market. Besides, Many literatures (such as Hirschey [28]; Ning and Wirjanto [29]) have reported that extreme values of stock market return are very important, but they lacked in-depth analysis about influence of extreme return on stock market.

Summarizing the about literatures, we find that most researches support that the equity return can affect investor sentiment significantly. But as far as the influence of volatility on sentiment is concerned, there isn't an agreement. However, extreme situations often bring dramatic changes to many factors in stock market. Especially when stock prices drop sharply in extreme situations, investors tend to panic. In this paper, we attempt to analyze how return and volatility in stock market affect investor sentiment from the perspective of extreme situations. We will use the ordinary least squares (OLS) and quantile regression (QR) to analyze the above issues. By using these two regression methods, we expect that we can not only find the overall impact of extreme return and extreme volatility on investor sentiment, but also observe the roles of extreme return and extreme volatility at different levels of emotion.

The remainder of the paper is organized as follows. Section 2 is data description, including the investor sentiment index, return and volatility. In Section 3, we build two models. Section 4 estimates the models' parameter by using the ordinary least squares regression and quantile regression. Section 5 is the conclusion of this paper.

2. Data description

2.1. Description of Investor Sentiment Index

This paper selects the standardized American Association of Individual Investors (AAII) as the investor sentiment index. The AAII is commonly used in the study of the American stock market, which can accurately reflect the investor sentiment of whole market. The sample period is from January 4, 1996 to June 20, 2013, and includes 911 weekly observations. The data are from the website of American Association of Individual Investors (http://www.aaii.com/).

The descriptive statistics of investor sentiment are shown in Table 1.

Table1. Descriptive statistics of investor sentiment index

	Mean	Std.	Max	Min	Jarque-Bera
Sent	0.000	1.000	2.7859	-3.2371	2.9200

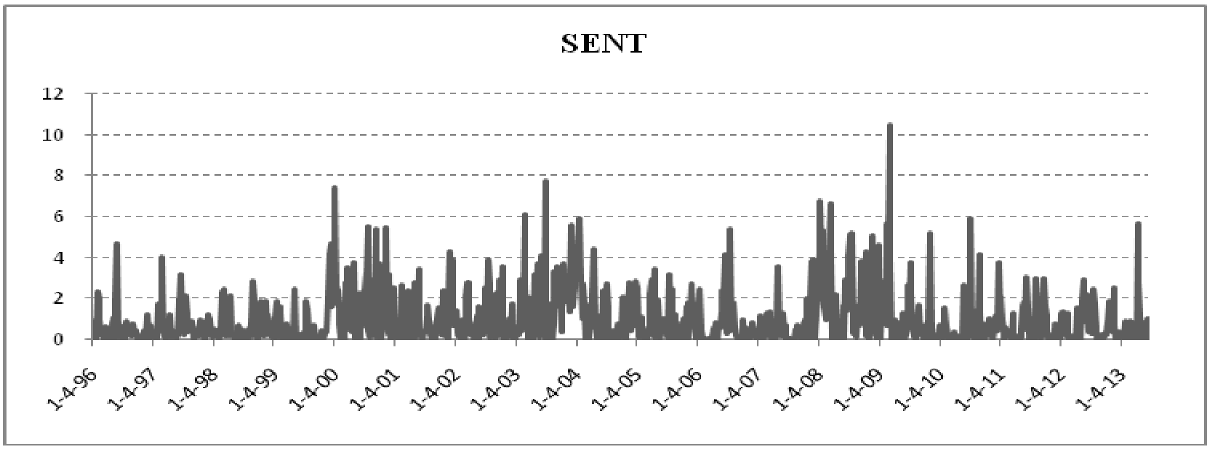

Figure 1. Square of Investor sentiment index.

It can be seen from Figure 1 that the change of investor sentiment is very obvious in American stock market. Its change is relatively small at the period from 1996 to 2000 (about 200 weeks), while this change has increased since 2000 and then gradually dropped from about 2004 onwards. This phenomenon may be associated with the lifting of price limits in U.S. in 2000. The change of investor sentiment reached a new high in 2008. It may be a result of the American subprime mortgage crisis. This crisis not only led to the global financial crisis but also had a severe influence on the domestic investor sentiment.

2.2. Return and Volatility Data Processing

We choose the SP 500 weekly closing price to compute the return of stock market, $r_t = 100(\ln(P_t) - \ln(P_{t-1})$. We couldn't observe the real volatility, so we adopt the commonly used GARCH model (Bollerslev [30]) and SV model (Taylor [31]) to extract volatility sequences in this paper. The ideas of the above two methods are as follows:

(1) Generalized autoregressive conditional heteroskedasticity model.

$$y_t = \gamma x_t + u_t \qquad \delta_t^2 = \omega + \alpha u_{t-1}^2 + \beta \delta_{t-1}^2 \tag{1}$$

In the formula, x_t is the number of vectors of explanatory variables while γ is the coefficient vector. In formula (1), the above formula is called mean value equation. The formula below is a forecasting variance equation based on the previous information. Therefore,it is called conditional variance (this equation is also known as the conditional variance equation). We use the above mentioned method to extract the δ_t^2 as volatility sequences.

(2) Stochastic Volatility model (SV models)

$$y_t = \epsilon_t e^{h_t/2} \qquad h_t = \alpha + \beta h_{t-1} + \eta_t \tag{2}$$

where y_t is the return at time after eliminating the mean value. That is, in the formula $y_t = R_t - \mu_t$, μ_t is the mean value of R_t. β, as continuous parameter, reflects the impact of prior volatility on the current volatility. h_t is the logarithm of volatility. The volatility sequence $exp(h_t)$ is the data we need.

In this paper, we apply the above two methods to extract the volatility sequences from return sequences, which are represented by δ_g^2, δ_s^2. After the correlation test, we find that their correlation has reached 0.8171, which is a strong correlation. It shows that both of them are stable and can be used as volatility for further explanation and description.

Table 2. Descriptive statistics of SP500 index–return rate and volatility

	Mean	Std.	Max	Min	Jarque-Bera
r	0.1039	2.5386	10.182	-16.451	792.71***
δ_g^2	7.0151	7.6591	74.989	1.3362	19736***
δ_s^2	5.6222	4.7923	34.674	0.7931	2999.2***

According to Longin's [27] explanation for the definition of extreme return, we believe that extreme return is within a certain return range and it is beyond the threshold value. The threshold value could be set as a certain percentage or standard deviation multiple. In this paper, we adopt the standard deviation multiple as the threshold value. Firstly, we define the stock market return adjusted to zero as the cut-off point. If it is above zero or equals zero, the return is positive. If it is below zero, the return is negative. If deviated from the value of the zero up and down for a standard deviation, it is either extremely positive or extremely negative return. If deviations from the zero value are less than a standard deviation, they are defined as the non-extremely positive return and the non-extremely negative return. In this paper, $r_{(e+)}$, $r_{(+)}$, $r_{(-)}$, $r_{(e-)}$ stand for the extremely positive return, the non-extremely positive return, the non-extremely negative return and extremely negative return, respectively.

According to the above definition of an extreme situation, we use the volatility sequences extracted by GARCH and SV models for classification of volatility. Since the volatility sequences are positive, here we divide volatility into two parts: extreme volatility and non-extreme volatility. If deviation from the mean value is above an average standard, we define it as extreme volatility. The others are called non-extreme volatility. We adopt δ_{g1}^2, δ_{g2}^2 (δ_{s1}^2, δ_{s2}^2) as the extreme volatility and non-extreme volatility obtained from GARCH(SV) model.

2.3. *ADF Test*

ADF tests were conducted on variables in order to avoid spurious regression. The stationarity is a specific statistical terminology used to describe time series. If the following three conditions are met: the time series are generated by a random process; the mean and variance of this time series are constant and independent of time *t*; and the covariance is only related with the interval time, the time series are a stationary stochastic process. Instability can usually be eliminated through the difference process. Table 3 shows the results of ADF test.

Table 3. ADF tests for the variables

	ADF test		ADF test
sent	-9.8468***	δ_g^2	-8.027***
r	-34.083***	δ_{g1}^2	-10.251***
$r_{(e+)}$	-30.617***	δ_{g2}^2	-9.7258***
$r_{(+)}$	-30.074***	δ_s^2	-5.4241***
$r_{(-)}$	-30.181***	δ_{s1}^2	-5.4292***
$r_{(e-)}$	-13.881***	δ_{s2}^2	-7.3704***

According to the above ADF test, we find that the investor sentiment index, return and volatility in American stock market are significant at 0.01 confidence level. This means that all variables are stationary. Therefore, the variables can be adopted as independent variables for further study of their impact on investor sentiment.

2.4. *Granger Causality Test*

We need to confirm the causality between investor sentiment and return (volatility) before empirical analysis. Then, we use the investor sentiment indexes and return indexes (which are tested by unit root) to make the Granger causality test in order to know the causality between them. Table 4 describes the results of Granger causality test between investor sentiment and return. Table 5 shows the results of the Granger causality test between investor sentiment and volatility by two different methods. In this part, we choose lag results of Granger test based on the lowest AIC value.

Table 4. Granger causality test of investor sentiment index and return

Null hypothesis	Prob.	Null hypothesis	Prob.
sent does not granger cause r	0.8427	r is not granger caused by *sent*	1.E-09***
sent does not granger cause $r_{(e+)}$	2.E-06***	$r_{(e+)}$ is not granger caused by *sent*	0.0009***
sent does not granger cause $r_{(+)}$	0.0888*	$r_{(+)}$ is not granger caused by *sent*	7.E-05***
sent does not granger cause $r_{(-)}$	0.0306**	$r_{(-)}$ is not granger caused by *sent*	0.0005***
sent does not granger cause $r_{(e-)}$	0.0003***	$r_{(e-)}$ is not granger caused by *sent*	0.0073***

Table 4 is the result of granger causality test for investor sentiment index and return. From this table, we can find that factors such as sequence of investor sentiment and extreme return (non-extreme return) are interacted except that the investor sentiment sequence couldn't affect the unclassified return sequence. Since the return in both extreme and non-extreme situations can reject the hypothesis at a significance level of 0.1, we believe that return sequence can affect the investor sentiment. That is to say, return can be used as independent variables to be introduced into the model. Then we can make an in-depth analysis of the different return's influence on investor sentiment. Our conclusion is consistent with Fisher and Statman (2000) who concluded that investor sentiment and return interact with each other. Our conclusion also agrees with Brown and Cliff (2004) in that the return is the causing factor affecting investor sentiment.

Table 5. Granger causality test of investor sentiment index and volatility

Null hypothesis	Prob. (GARCH)	Prob. (SV)
sent does not granger cause δ^2	3.E-06***	0.0002***
δ^2 does not granger cause *sent*	0.3747	0.0061***
sent does not granger cause δ_1^2	0.0003***	0.0540*
δ_1^2 does not granger cause *sent*	0.0531*	0.0142**
sent does not granger cause δ_2^2	0.3709	0.4276
δ_2^2 does not granger cause *sent*	0.0027***	0.8777

In Table 5, δ^2, δ_1^2 and δ_2^2 are the whole volatility, extreme volatility and non-extreme volatility, respectively. The result of Granger causality test for investor sentiment and volatility are shown in Table 5. The majority of volatility can change the investor sentiment, except that the entire volatility sequence in GARCH or non-extreme volatility sequence in SV model are not the granger cause of the investor sentiment. The difference of this result may be related to the characteristics of the model. However, both the above conclusions indicate that the majority of volatility is the Granger cause of the investor sentiment. It shows that both extreme volatility and non-extreme volatility can be used as independent variables applied to the model for further study of their respective impacts on investor sentiment.

3. Methodology

Linear regression analysis is frequently applied in economic analysis. It is a statistical method to analyze the cause-effect relationship between two or more variables. It is also suitable for this paper to study the cause-effect relationship between extreme return/volatility and investor sentiment.

Then we incorporate the return sequence and volatility sequence into models for empirical analysis. The above Granger causality test shows that return and volatility are the cause of investor sentiment. And weekly closing price (usually Friday closing price) can only affect the investor sentiment during the next week. Therefore, we select the return and volatility of first-order lag as the independent variables to observe their impact on investor sentiment. Usually, the time series have auto-regression effect. Different investor sentiment variables bearing different order lags in turn are added for obtaining least squares regression of the current investor sentiment. We find that investor sentiment with two order lags is always stable and significant, while sentiments with third or more order lags are not stable and significant. Therefore, only investor sentiment bearing two order lags as control variables is added in order to improve the accuracy of the model. As a result, regression equations of return and volatility are expressed as follows.

$$sent_t = u_t + ar_{(e+)t-1} + br_{(+)t-1} + cr_{(-)t-1} + dr_{(e-)t-1} + \sum_{k=1}^{2} e_k sent_{t-k} + \epsilon_t, k = 1, 2 \tag{3}$$

$$sent_t = \omega_t + \alpha\delta^2_{1,t-1} + \beta\delta^2_{2,t-1} + \sum_{k=1}^{2} \gamma_k sent_{t-k} + \epsilon_t, k = 1, 2 \tag{4}$$

where $sent_t$ is the investor sentiment in the American stock market, stands for the intercept of the regression equation, and $r_{(e+),t-1}$, $r_{(+),t-1}$, $r_{(-),t-1}$, $r_{(e-),t-1}$ in regression Equation (3) are represented as independent variables. a, b, c, d are expressed as the degree of sensitivity of $sent_t$ to extremely positive return, non-extremely positive return, non-extremely negative return and extremely negative return, respectively. δ^2_{t-1} is the dependent variable in Equation (4). Coefficients α and β stand for the degree of sensitivity of to extreme and non-extreme volatility, respectively. δ^2_{t-k} is the lag factors of investor sentiment index and control variables of the regression equation. Coefficients e_k and γ_k mean the degree to which the investor sentiment with time lag affects the current investor sentiment index. k is expressed as the lag intervals of investor sentiment, ϵ_t is the residual error of the regression equation.

After building the model, we use the least squares method and the quantile regression (Koenker and Bassett[32]) approach to fit the model. In fact, QLS is a kind of linear conditional expectation: $E[Y|X] = \int_0^1 Q_Y(\theta|x), d\theta$ (Bessett et al.[33]). In other words, OLS is a summary of all quantile regression information, in which a part of the distributed information is hidden. However, quantile regression can adequately reflect the information with conditional distribution, such as the important information of the ending part.

Because OLS and QR have their own characteristics and the emphasis is also different, we use these two methods to study the effects of extreme return and extreme volatility on investor sentiment. OLS can analyze the impact of different return ratios and volatility on the whole investor sentiment. During this process, we can learn the influence of each variable on investor sentiment and the degree of its influence. QR can improve the results of OLS because it can be used to analyze the influence of return and volatility on investor sentiment at different levels. Combination of the two methods can be much more complete and accurate than only one model.

4. Empirical Analysis

4.1. Results of OLS

(1) Estimation results for the effect of return on investor sentiment

The above data processing and model building are the foundation of the empirical process. We apply Equation (3) to analyze the effect of return on investor sentiment. The regression results are listed Table 6.

Table 6. Estimation results for the effect of return on investor sentiment

	a	b	c	d	e_1	e_1	R^2
Coefficient	0.0567***	0.1202***	0.0984**	0.0412	0.4704***	0.2530***	0.5041
t value	3.1213	2.8485	2.1919	1.5900	12.779	7.0233	

It has been tested that the return is Granger causes of investor sentiment. In this section, with the results of this model, we can make an in-depth analysis of the degree of influence return has on investor sentiment. Table 6 shows that the extremely positive return, non-extremely positive return and non-extremely negative return have significant effects on investor sentiment, while extremely negative return has little effect on investor sentiment. At the same time, investors' current mood affected by lag factors of investor sentiment is decreased gradually, and the investor sentiment can be digested slowly by itself with time.

From the point of influence coefficient, we can find that effect of only the extremely negative return on investor sentiment is not significant, while non-extreme return can make the mood change significantly. That is, the influence of non-extreme return on investor sentiment is dominant in U.S. stock market. Since institutional investors dominate the U.S. stock market, the number of individual investors is relatively small. As we all know, institutional investors have abundant resources, strong research and development ability, and appropriate risk-control measures. Institutional investors have much more advantages than general individual investors. Therefore, their research results are often similar to the true values of stocks. Compared with the influence brought by one-off and short time good or bad news, some factors such as the profits of a company and so on which affect the value result in a greater impact of non-extreme return on investor sentiment. The results show that the investor type may affect the effect of extreme and non-extreme return on investor sentiment.

Besides, the extremely positive return has a greater effect on investor sentiment than extremely negative return. Because the extremely positive earnings make a positive effect on investor sentiment, investors often think that the economic prospects are good while risks are low and return are relatively high in stock market. To the contrary, extremely negative earnings often make investors panic, especially when a separate event causes return to be extremely negative. When investors can't judge the prospects of the whole stock market, some investors become pessimistic and believe that the stock market in the future will be in a depression. Of course, there are some other investors who are optimistic about the future of the stock market. They believe that individual events will not have long-term impact on the stock market. Therefore, effects of extremely negative return on investor sentiment are much more complicated. The OLS method may not accurately obtain the regular results of extremely negative return affecting investor sentiment.

(2) Estimation results for the effect of volatility on investor sentiment This section introduces Formula (4) for model-fitting analysis of volatility and investor sentiment in order to study the impact of extreme and non-extreme volatility on investor sentiment. The regression results of model are as follows.

Table 7. Estimation results for the effect of volatility on investor sentiment

	α	β	$gamma_1$	$gamma_2$	R^2
GARCH	0.0004	0.0130	0.5373***	0.2081***	0.4811
t value	0.1435	1.4554	14.832	5.8172	
SV	-0.0092*	-0.0118	0.1971***	0.2265***	0.4815
t value	-1.7524	-1.4600	5.5046	4.3165	

In Table 7, regression results of the two kinds of volatility on investor sentiment are almost same. Extreme and non-extreme volatility have hardly any influence on investor sentiment (The influence of the extreme volatility extracted by SV model on investor sentiment is also not significant at a confidence level of 0.05). What's more, the role of the lagged factors of investor sentiment in this model is identical with its role in the model of return on investor sentiment. Their influence on investor sentiment can be digested slowly by itself with time. Influence of volatility on investor sentiment is not significant. One possible reason is the difference between the two models. Influence of extreme volatility extracted by SV model is significant only at a confidence level of 0.1. On the other hand, volatility itself often links with return and risk rather than investor sentiment. We can not directly find the effect of volatility on investor sentiment. Next, in this paper, we intend to analyze whether volatility has different effects on investor sentiment at different levels by introducing the quantile regression method (QR). We hope this method can replenish and improve the results of OLS.

4.2. Results of QR

(1) Estimation results for the effect of return on investor sentiment

In this section, we use quantile regression to estimate Equation (3). τ is represented as the quantile point. The results are shown in Table 8 (the ten percentile result):

Table 8. Estimation results for the effect of return on investor sentiment

τ	$r_{(e+)}$	$r_{(+)}$	$r_{(-)}$	$r_{(e-)}$
0.1	0.0839***	0.0958***	0.1639*	0.1026***
0.2	0.0788***	0.0875*	0.1563***	0.0941***
0.3	0.0694***	0.0799	0.1536***	0.0989***
0.4	0.0475***	0.0780	0.1640***	0.0923***
0.5	0.0394	0.0753	0.1660***	0.0721**
0.6	0.0318	0.0289	0.1571***	0.0512
0.7	0.0400	0.1054	0.1509***	0.0345
0.8	0.0639***	0.1855***	0.0718	0.0220
0.9	0.0383	0.1737***	0.0580	-0.0106

In order to understand and analyze Table 8, we change the above table to a graph. Figures 2a 2b, 2c and 2d exhibit the influence of extremely positive return, non-extremely positive return, non-extremely negative return and extremely negative return on investor sentiment.

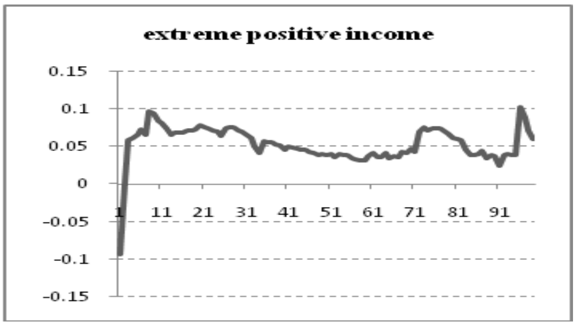

Figure 2*a*.

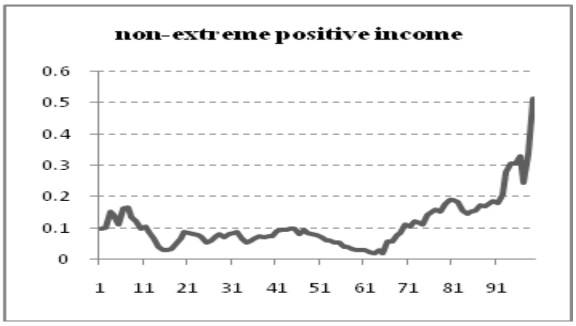

Figure 2*b*.

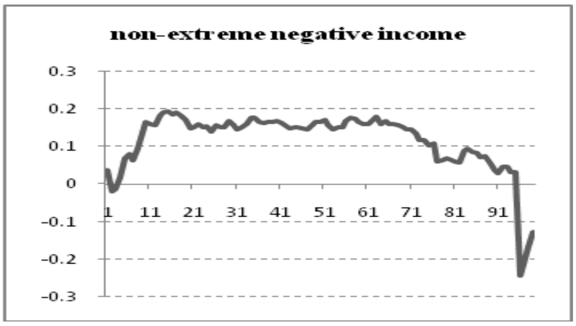

Figure 2*c*.

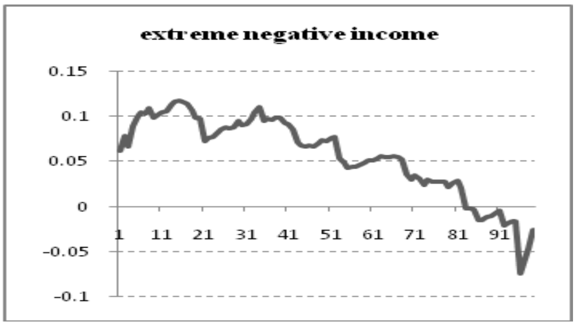

Figure 2*d*.

Firstly, we analyze the influence of extremely positive and extremely negative return on investor sentiment. Table 8 shows that they are remarkable when the investor sentiment stays at the low quantile point while not significant at the high quantile point. From Figures 2a and 2d, when the value of is below 0.5 (that is to say, the part before the value of 51 in the abscissa in Figures), we can find that the results of extremely positive and extremely negative return on pessimistic emotion show a trend which first increases and then decreases.

Because extremely positive and extremely negative returns are divided into positive and negative values, their roles in investor sentiment are different. The former first shows a sharp increase. It means that the extremely positive return can quickly improve investor's confidence when investor sentiment is extreme pessimism. This makes investors believe that the market will reverse. Therefore, the extremely positive return is an important reason to improve investors's mood from the bottom. However, extremely negative return can accelerate the decrease of investor sentiment, making investors much more pessimistic when they are already extremely pessimistic. Extremely negative return is the "chief culprits" which make the investor sentiment decline rapidly. When we compare the steepness in Figures 2a and 2d, we find that the extremely positive return have greater impact on investor sentiment than extremely negative return. At the same time, investors are desperate for a good performance of the stock market and want to recover from the extreme pessimism. In the following downward trend, we find that the influence of extremely positive return and extremely negative return gradually decreases with the decline of investors' pessimism. From Figures 2a to 2d, we find that the marginal impact gradually decreases when compared to the level of extreme pessimism. It indicates that the influence of extreme return in non-extremely pessimistic situation is relatively smaller than that in extreme pessimism. At this time, the degree of investors' rationality gradually improves.

Also, we need to be aware that in pessimistic conditions the influence of extremely negative return on investor sentiment shows a trend which first increases and then decreases as a whole with some small fluctuations in the process. However, these small fluctuations further explain the result based on the above OLS method. That is, investors often find it difficult to describe the whole stock market's prospects. Especially in pessimism, investors are intensely irrational. This leads to two opposite views. Therefore, the extremely negative return has more complicated influence on investor sentiment.

Neither extremely positive nor extremely negative return can play a significant role in optimistic sentiment (the part that the value of is greater than 0.5). Table 8 indicates that quantile regression results are not significant when the value of τ is greater than 0.5. The reason may be that in optimistic conditions investors always have strong confidence in the prospects of the stock market. When there is extremely optimistic sentiment, they are not sensitive to what's happening in the stock market. This results in extreme return not having significant affect on investor sentiment. However, this situation is often the precursor of a stock market bubble.

Secondly, the effect of non-extremely positive and non-extremely negative return on investor sentiment is different. But both are significant in extreme pessimism. From Figures 2b and 2c, as a whole, the former shows a downward trend while the latter is upward. Due to the different values of non-extremely positive return and non-extremely negative return, the two different trends show the same results. That is, non-extremely positive return and non-extremely negative return can further decrease investor sentiment when investors fall into extreme pessimism. This confirms that investors find it very difficult to recover from extreme pessimism. Non-extremely positive and non-extremely negative returns play a role in the middle and higher positions, respectively, of quantile τ when investors are non-extremely pessimistic. It shows that extreme optimism in the market is sensitive to non-extremely positive return. Investors believe that the market is booming at this time. The good and fundamental information about the value of a listed company brings the non-extremely positive return and may further improve investor sentiment. When investor sentiment is in a relatively stable condition, investors are often loss averse (Kahneman and Tversky [34]). When facing equal profit and loss, the non-extremely negative return lead to much more fluctuation in sentiment. That means compared with non-extremely positive return, and non-extremely negative return often make investors more sensitive.

(2) Estimation results for the effect of volatility on investor sentiment

In order to analyze the influence of extreme volatility on investor sentiment more accurately, we select two kinds of volatility models for quantile regression on investor sentiment. The results of quantile regression are shown in Table 9.

Table 9. Estimation results for the effect of volatility on investor sentiment

τ	δ_{g1}^2	δ_{g2}^2	δ_{s1}^2	δ_{s2}^2
0.1	-0.0047	-0.0026	-0.0278***	-0.0230
0.2	-0.001	0.0006	-0.0149	-0.0161
0.3	0.0001	-0.0008	-0.0148*	-0.0084
0.4	-0.0006	-0.0061	-0.0171**	-0.0175
0.5	0.0038	0.0256***	-0.0140	-0.0133
0.6	0.0030	0.0169*	-0.0025	-0.0037
0.7	0.0045	0.0262***	-0.0004	0.0025
0.8	0.0030	0.0339***	0.0013	0.0108
0.9	0.0034	0.0215*	0.0040	0.0096

Furthermore, we convert the content of the above tables into figures. Figures 3a and 3b (4a and 4b) represent the quantile regression results of the extreme volatility (the non-extreme volatility).

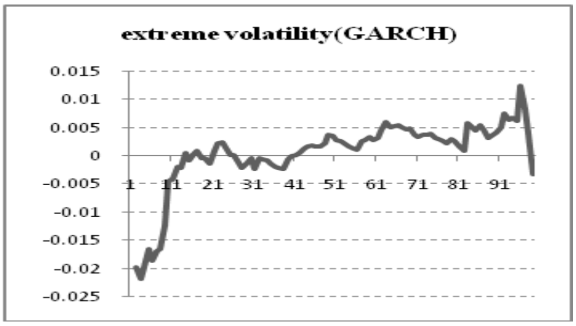

Figure 3*a*.

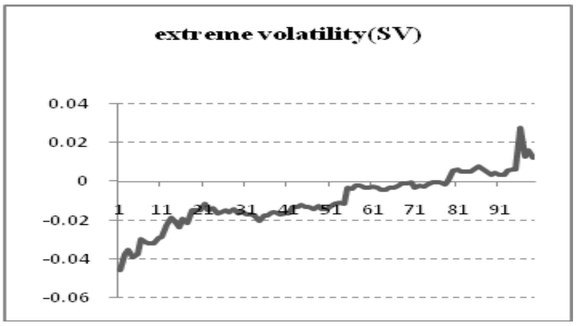

Figure 3*b*.

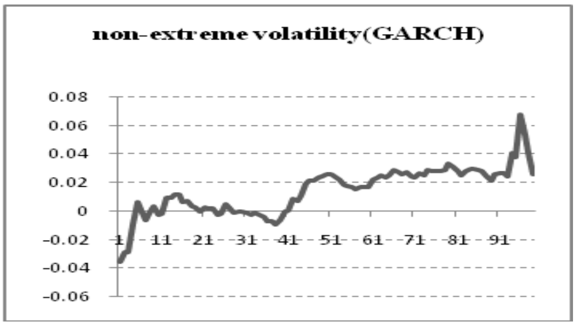

Figure 4*a*.

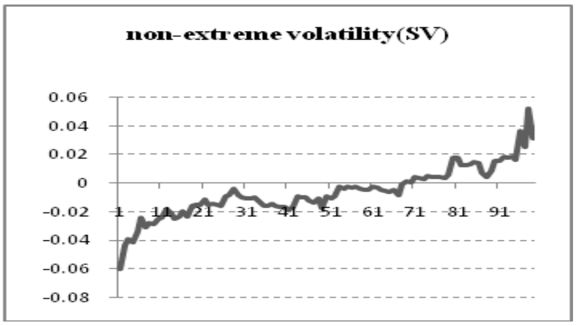

Figure 4*b*.

In order to understand and analyze Table 8, we change the above table to a graph. Figures 2a 2b, 2c and 2d exhibit the influence of extremely positive return, non-extremely positive return, non-extremely negative return and extremely negative return on investor sentiment.

Figures 3a and 3b show that the effect of extreme volatility on investor sentiment in the two different models tends to be the same. With the growing of τ, the effect of extreme volatility on investor sentiment gradually moves from negative to positive and keeps increasing. However, there is a large difference in the statistical significance of the regression results (Table 9). Different volatility models have different results. It cannot be concluded that extreme volatility influences investor sentiment significantly. Similarly, we cannot conclude that non-extreme volatility consistently affects investor sentiment (Figures 4a and 4b). The above conclusions are consistent with the OLS, that is, volatility cannot explain the investor sentiment effectively. Although Wang et al. [19] have found that stock market volatility can impact investor sentiment, we prove that the explanatory ability of volatility for investor sentiment is limited because we cannot conclude consistency of extreme volatility affecting investor sentiment in extreme and non-extreme situations.

4.3. Robustness

In this paper, we simultaneously apply the following classification methods and data to the stability test. We use a 0.5 and 1.5 standard deviation as the dividing principle for the stability analysis. SP 500 index is replaced by Dow Jones index and New York stock exchange composite index. The above stability test concludes that the effect of return rate series on investor sentiment is consistent with the above empirical study, while the effect of volatility on investor sentiment follows an erratic pattern. As for empirical results of the previous and the last stage, although there are differences in the statistical significance individually, it is normal and doesn't affect the whole results in this study. Therefore, empirical results of this paper have good robustness.

5. Conclusion

We choose the investor sentiment index, the return and volatility of SP 500 index to analyze the impact of extreme return and extreme volatility on investor sentiment. We first distinguish the extreme situation from non-extreme situation. Then we use the ordinary generalized least squares and quantile regression to estimate the linear regression model which can analyze the relationship between investor sentiment and return (or volatility) in stock market. We get the following conclusions.

(1) In the OLS results, except for the extremely negative earnings, other return rate series significantly lead to fluctuations of investor sentiment. And the non-extreme return plays a dominant role in affecting investor sentiment of the American stock market.

(2) QR results show both extremely positive and extremely negative returns have their influences on pessimistic emotions. Though the trend is the same, the impacts of extremely positive and extremely negative returns on pessimism are not the same. The extremely positive return can accelerate and quickly strengthen investor confidence while the extremely negative return can accelerate the reduction of investor sentiment. Their impacts are gradually mitigated with the decrease of the degree of investor pessimism. However, extremely positive and extremely negative returns do not play an important role in optimistic sentiment.

(3) Volatility cannot impact investor sentiment effectively and steadily. Its explanatory power for investor sentiment is limited.

Through the above results, we found that non-extreme earnings remain dominant in the American market, but the extreme positive return does work at different levels of emotion. Especially when investors are extremely pessimistic, extreme negative return makes investors panic and leads to wild fluctuation in financial markets. In order to prevent sharp fluctuations in the stock market, we should improve investor's trading skills and reduce the change in investor sentiment. This paper investigates the relationship between return (volatility) and investor sentiment in the stock market based on low-frequency data. In future works, we will use high-frequency data to compute the return (volatility) of financial assets (see [35-38]), and study the relationship between return (volatility) and investor sentiment in other markets such as the energy market, foreign exchange market, and bond market.

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